FUNCTION

1. If $g(x) = x^2 + x - 1$ and $(gof)(x) = 4x^2 - 10x + 5$, then $f\left(\frac{5}{4}\right)$ is equal to

(1) $\frac{3}{2}$ (2) $-\frac{1}{2}$

(3) $-\frac{3}{2}$ (4) $\frac{1}{2}$

2. Let $f : (1,3) \to R$ be a function defined by $f(x) = \frac{x[x]}{1+x^2}$, where [x] denotes the greatest integer $\le$ x. Then the range of f is

(1) $\left(\frac{3}{5}, \frac{4}{5}\right)$ (2) $\left(\frac{2}{5}, \frac{3}{5}\right] \cup \left(\frac{3}{4}, \frac{4}{5}\right)$

(3) $\left(\frac{2}{5}, \frac{4}{5}\right]$ (4) $\left(\frac{2}{5}, \frac{1}{2}\right) \cup \left(\frac{3}{5}, \frac{4}{5}\right]$

3. Let $f : R \to R$ be such that for all $x \in R$ $(2^{1+x} + 2^{1-x})$, $f(x)$ and $(3^x + 3^{-x})$ are in A.P., then the minimum value of $f(x)$ is

(1) 0 (2) 3

(3) 2 (4) 4

4. The inverse function of

$$f(x) = \frac{8^{2x} - 8^{-2x}}{8^{2x} + 8^{-2x}}, x \in (-1,1), \text{ is}$$

(1) $\frac{1}{4}(\log_8 e)\log_e\left(\frac{1-x}{1+x}\right)$

(2) $\frac{1}{4}\log_e\left(\frac{1-x}{1+x}\right)$

(3) $\frac{1}{4}(\log_8 e)\log_e\left(\frac{1+x}{1-x}\right)$

(4) $\frac{1}{4}\log_e\left(\frac{1+x}{1-x}\right)$

5. Let $f : R \to R$ be a function which satisfies $f(x + y) = f(x) + f(y) \forall x,y \in R$. If $f(1) = 2$ and $g(n) = \sum_{k=1}^{(n-1)} f(k), n \in N$ then the value of n, for which $g(n) = 20$, is :

(1) 5 (2) 9

(3) 20 (4) 4

6. Let [t] denote the greatest integer $\le$ t. Then the equation in x, $[x]^2 + 2[x + 2] - 7 = 0$ has :

(1) no integral solution

(2) exactly four integral solutions

(3) exactly two solutions

(4) infinitely many solutions

7. Let A = {a, b, c} and B = {1, 2, 3, 4}. Then the number of elements in the set $C = \{f : A \to B \mid 2 \in f(A) \text{ and } f \text{ is not one-one}\}$ is ________.

8. For a suitably chosen real constant a, let a function, $f : R - \{-a\} \to R$ be defined by $f(x) = \frac{a-x}{a+x}$. Further suppose that for any real number $x \ne -a$ and $f(x) \ne -a$, $(fof)(x) = x$. Then $f\left(-\frac{1}{2}\right)$ is equal to :

(1) $\frac{1}{3}$ (2) 3

(3) –3 (4) $-\frac{1}{3}$

9. Suppose that a function $f : R \to R$ satisfies $f(x + y) = f(x)f(y)$ for all $x, y \in R$ and $f(1) = 3$. If $\sum_{i=1}^{n} f(i) = 363$, then n is equal to ________.

SOLUTION

1. NTA Ans. (2)

Sol. $g(x) = x^2 + x - 1$

$g(f(x)) = 4x^2 - 10x + 5$

$= (2x - 2)^2 + (2 - 2x) - 1$

$= (2 - 2x)^2 + (2 - 2x) - 1$

$\Rightarrow f(x) = 2 - 2x$

$f\left(\frac{5}{4}\right) = \frac{-1}{2}$

2. NTA Ans. (4)

Sol. $f(x) = \begin{cases} \frac{x}{x^2+1} & ; \quad x \in (1,2) \\ \frac{2x}{x^2+1} & ; \quad x \in [2,3) \end{cases}$

$f(x)$ is decreasing function

$\therefore f(x) \in \left(\frac{2}{5}, \frac{1}{2}\right) \cup \left(\frac{3}{5}, \frac{4}{5}\right]$

(4) Option

3. NTA Ans. (2)

Sol. $f(x) = \frac{2(2^x + 2^{-x}) + (3^x + 3^{-x})}{2} \geq 3$

(A.M $\geq$ G.M)

4. NTA Ans. (3)

Sol. $f(x) = y = \frac{8^{4x} - 1}{8^{4x} + 1} = 1 - \frac{2}{8^{4x} + 1}$

so, $8^{4x} + 1 = \frac{2}{1-y} \Rightarrow 8^{4x} = \frac{1+y}{1-y}$

$\Rightarrow x = \ell n\left(\frac{1+y}{1-y}\right) \times \frac{1}{4\ell n 8} = f^{-1}(y)$

Hence, $f^{-1}(x) = \frac{1}{4}\log_8 e\,\ell n\left(\frac{1+x}{1-x}\right)$

5. Official Ans. by NTA (1)

Sol. $f(x + y) = f(x) + f(y)$

$\Rightarrow f(n) = nf(1)$

$f(n) = 2n$

$g(n) = \sum_{k=1}^{n-1} 2n = 2\left(\frac{(n-1)n}{2}\right) = n(n - 1)$

$g(n) = 20 \Rightarrow n(n - 1) = 20$

$n = 5$

6. Official Ans. by NTA (4)

Sol. $[x]^2 + 2[x + 2] - 7 = 0$

$\Rightarrow [x]^2 + 2[x] + 4 - 7 = 0$

$\Rightarrow [x] = 1, -3$

$\Rightarrow x \in [1, 2) \cup [-3, -2)$

7. Official Ans. by NTA (19.00)

Sol. $C = \{f : A \to B \mid 2 \in f(A)$ and f is not one-one$\}$

Case-I : If $f(x) = 2\ \forall\ x \in A$ then number of function = 1

Case-II : If $f(x) = 2$ for exactly two elements then total number of many-one function = ${}^3C_2\ {}^3C_1 = 9$

Case-III : If $f(x) = 2$ for exactly one element then total number of many-one functions $= {}^3C_1\ {}^3C_1 = 9$

Total = 19

8. Official Ans. by NTA (2)

Sol. $f(x)=\dfrac{a-x}{a+x}\quad x\in R-\{-a\}\to R$

$$f(f(x))=\frac{a-f(x)}{a+f(x)}=\frac{a-\left(\frac{a-x}{a+x}\right)}{a+\left(\frac{a-x}{a+x}\right)}$$

$$f(f(x))=\frac{(a^2-a)+x(a+1)}{(a^2+a)+x(a-1)}=x$$

$\Rightarrow (a^2-a)+x(a+1)=(a^2+a)x+x^2(a-1)$

$\Rightarrow a(a-1)+x(1-a^2)-x^2(a-1)=0$

$\Rightarrow a=1$

$f(x)=\dfrac{1-x}{1+x},$

$$f\left(\frac{-1}{2}\right)=\frac{1+\frac{1}{2}}{1-\frac{1}{2}}=3$$

9. Official Ans. by NTA (5.00)

Sol. $f(x+y)=f(x)\,f(y)$

put $x=y=1\quad f(2)=(f(1))^2=3^2$

put $x=2,\ y=1\ f(3)=(f(1))^3=3^3$

$\vdots$

Similarly $f(x)=3^x$

$$\sum_{i=1}^{n}f(i)=363\Rightarrow\sum_{i=1}^{n}3^i=363$$

$(3+3^2+\ldots+3^n)=363$

$$\frac{3(3^n-1)}{2}=363$$

$3^n-1=242\Rightarrow 3^n=243$

$\Rightarrow n=5$