ELLIPSE

1. If $3x + 4y = 12\sqrt{2}$ is a tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{9}=1$ for some $a \in R$, then the distance between the foci of the ellipse is :

(1) 4 (2) $2\sqrt{7}$

(3) $2\sqrt{5}$ (4) $2\sqrt{2}$

2. If the distance between the foci of an ellipse is 6 and the distance between its directrices is 12, then the length of its latus rectum is :

(1) $\sqrt{3}$ (2) $2\sqrt{3}$

(3) $3\sqrt{2}$ (4) $\frac{3}{\sqrt{2}}$

3. Let the line $y = mx$ and the ellipse $2x^2 + y^2 = 1$ intersect at a ponit P in the first quadrant. If the normal to this ellipse at P meets the co-ordinate axes at $\left(-\frac{1}{3\sqrt{2}},0\right)$ and $(0,\beta)$, then β is equal to

(1) $\frac{2}{\sqrt{3}}$ (2) $\frac{2\sqrt{2}}{3}$

(3) $\frac{2}{3}$ (4) $\frac{\sqrt{2}}{3}$

4. The length of the minor axis (along y-axis) of an ellipse in the standard form is $\frac{4}{\sqrt{3}}$. If this ellipse touches the line, $x + 6y = 8$; then its eccentricity is :

(1) $\sqrt{\frac{5}{6}}$ (2) $\frac{1}{2}\sqrt{\frac{11}{3}}$

(3) $\frac{1}{3}\sqrt{\frac{11}{3}}$ (4) $\frac{1}{2}\sqrt{\frac{5}{3}}$

5. Let e_1 and e_2 be the eccentricities of the ellipse, $\frac{x^2}{25}+\frac{y^2}{b^2}=1(b<5)$ and the hyperbola, $\frac{x^2}{16}-\frac{y^2}{b^2}=1$ respectively satisfying $e_1e_2=1$. If α and β are the distances between the foci of the ellipse and the foci of the hyperbola respectively, then the ordered pair (α, β) is equal to :

(1) (8, 10) (2) (8, 12)

(3) $\left(\frac{20}{3},12\right)$ (4) $\left(\frac{24}{5},10\right)$

6. Let $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ $(a > b)$ be a given ellipse, length of whose latus rectum is 10. If its eccentricity is the maximum value of the function, $\phi(t)=\frac{5}{12}+t-t^2$, then a^2+b^2 is equal to :

(1) 126 (2) 135

(3) 145 (4) 116

7. Let $x = 4$ be a directrix to an ellipse whose centre is at the origin and its eccentricity is $\frac{1}{2}$. If $P(1, \beta), \beta > 0$ is a point on this ellipse, then the equation of the normal to it at P is :-

(1) $7x - 4y = 1$ (2) $4x - 2y = 1$

(3) $4x - 3y = 2$ (4) $8x - 2y = 5$

8. If the point P on the curve, $4x^2 + 5y^2 = 20$ is farthest from the point $Q(0, -4)$, then PQ^2 is equal to:

(1) 21 (2) 36

(3) 48 (4) 29

9. If the co-ordinates of two points A and B are $(\sqrt{7}, 0)$ and $(-\sqrt{7}, 0)$ respectively and P is any point on the conic, $9x^2 + 16y^2 = 144$, then PA + PB is equal to :

(1) 8 (2) 6

(3) 16 (4) 9

10. Which of the following points lies on the locus of the foot of perpendicular drawn upon any tangent to the ellipse, $\frac{x^2}{4}+\frac{y^2}{2}=1$ from any of its foci ?

(1) $(-1,\sqrt{3})$ (2) $(-1,\sqrt{2})$

(3) $(-2,\sqrt{3})$ (4) (1,2)

11. If the normal at an end of a latus rectum of an ellipse passes through an extremity of the minor axis, then the eccentricity e of the ellipse satisfies :

(1) $e^2 + 2e - 1 = 0$ (2) $e^2 + e - 1 = 0$

(3) $e^4 + 2e^2 - 1 = 0$ (4) $e^4 + e^2 - 1 = 0$

SOLUTION

1. NTA Ans. (2)

Sol. $3x + 4y = 12\sqrt{12}$ is tangent to $\frac{x^2}{a^2}+\frac{y^2}{9}=1$

$c^2 = m^2a^2 + b^2$

$\Rightarrow a^2 = 16$

$e = \sqrt{1-\frac{9}{16}} = \frac{\sqrt{7}}{4}$

Distance between focii = 2ae = $2\sqrt{7}$

2. NTA Ans. (3)

Sol. Given 2ae = 6 $\Rightarrow$ $\boxed{ae=3}$ ......(1)

and $\frac{2a}{e}=12 \Rightarrow \boxed{a=6e}$ ....(2)

from (1) and (2)

$6e^2 = 3 \Rightarrow \boxed{e=\frac{1}{\sqrt{2}}}$

$\Rightarrow \boxed{a=3\sqrt{2}}$

Now, $b^2 = a^2(1 - e^2)$

$\Rightarrow b^2 = 18\left(1-\frac{1}{2}\right)=9$

Length of L.R = $\frac{2(9)}{3\sqrt{2}}=3\sqrt{2}$

3. NTA Ans. (4)

Sol. Any normal to the ellipse is

$$\frac{x\sec\theta}{\sqrt{2}} - y\operatorname{cosec}\theta = -\frac{1}{2}$$

$$\Rightarrow \frac{x}{\left(\frac{-\cos\theta}{\sqrt{2}}\right)}+\frac{y}{\left(\frac{\sin\theta}{2}\right)}=1$$

$$\Rightarrow \frac{\cos\theta}{\sqrt{2}}=\frac{1}{3\sqrt{2}} \text{ and } \frac{\sin\theta}{2}=\beta$$

$$\Rightarrow \beta = \frac{\sqrt{2}}{3}$$

4. NTA Ans. (2)

Sol. Let $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$; a > b;

$2b=\frac{4}{\sqrt{3}} \Rightarrow b=\frac{2}{\sqrt{3}} \Rightarrow b^2=\frac{4}{3}$

tangent $y=\frac{-x}{6}+\frac{4}{3}$ compare with

$y = mx \pm \sqrt{a^2m^2+b^2}$

$\Rightarrow m = \frac{-1}{6} \Rightarrow \sqrt{\frac{a^2}{36}+\frac{4}{3}}=\frac{4}{3} \Rightarrow a = 4;$

$e=\sqrt{1-\frac{b^2}{a^2}}=\frac{1}{2}\sqrt{\frac{11}{3}}$

5. Official Ans. by NTA (1)

Sol. For ellipse $\frac{x^2}{25}+\frac{y^2}{b^2}=1$ (b < 5)

Let e_1 is eccentricity of ellipse

$\therefore b^2 = 25(1 - e_1^2)$

.... (1)

Again for hyperbola

$\frac{x^2}{16}-\frac{y^2}{b^2}=1$

Let e_2 is eccentricity of hyperbola.

$\therefore b^2 = 16(e_2^2 - 1)$

.... (2)

by (1) & (2)

$25(1 - e_1^2) = 16(e_2^2 - 1)$

Now $e_1.e_2 = 1$ (given)

$\therefore 25(1 - e_1^2) = 16\left(\frac{1-e_1^2}{e_1^2}\right)$

or $e_1 = \frac{4}{5}$ $\therefore e_2 = \frac{5}{4}$

Now distance between foci is 2ae

$\therefore$ distance for ellipse = $2 \times 5 \times \frac{4}{5} = 8 = \alpha$

distance for hyperbola = $2 \times 4 \times \frac{5}{4} = 10 = \beta$

$\therefore (\alpha, \beta) \equiv (8, 10)$

6. Official Ans. by NTA (1)

Sol. $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ $(a > b)$; $\frac{2b^2}{a} = 10 \Rightarrow b^2 = 5a$ ...(i)

Now, $\phi(t) = \frac{5}{12} + t - t^2 = \frac{8}{12} - \left(t - \frac{1}{2}\right)^2$

$\phi(t)_{max} = \frac{8}{12} = \frac{2}{3} = e \Rightarrow e^2 = 1 - \frac{b^2}{a^2} = \frac{4}{9}$

... (ii)

$\Rightarrow a^2 = 81$ (from (i) & (ii))

So, $a^2 + b^2 = 81 + 45 = 126$

7. Official Ans. by NTA (2)

Sol. Ellipse : $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

directrix : $x = \frac{a}{e} = 4$ & $e = \frac{1}{2}$

$\Rightarrow a = 2$ & $b^2 = a^2 (1-e^2) = 3$

$\Rightarrow$ Ellipse is $\frac{x^2}{4} + \frac{y^2}{3} = 1$

P is $\left(1, \frac{3}{2}\right)$

Normal is : $\frac{4x}{1} - \frac{3y}{3/2} = 4 - 3$

$\Rightarrow 4x - 2y = 1$

8. Official Ans. by NTA (2)

Sol. Given ellipse is $\frac{x^2}{5} + \frac{y^2}{4} = 1$

Let point P is $(\sqrt{5}\cos\theta, 2\sin\theta)$

$(PQ)^2 = 5\cos^2\theta + 4(\sin\theta + 2)^2$

$(PQ)^2 = \cos^2\theta + 16\sin\theta + 20$

$(PQ)^2 = -\sin^2\theta + 16\sin\theta + 21$

$= 85 - (\sin\theta - 8)^2$

will be maximum when $\sin\theta = 1$

$\Rightarrow (PQ)^2_{max} = 85 - 49 = 36$

9. Official Ans. by NTA (1)

Sol. $\frac{x^2}{16} + \frac{y^2}{9} = 1$

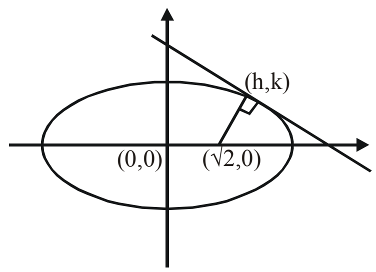
$a = 4;\ b = 3;\ e = \sqrt{\frac{16-9}{16}} = \frac{\sqrt{7}}{4}$

A and B are foci

$\Rightarrow PA + PB = 2a = 2 \times 4 = 8$

10. Official Ans. by NTA (1)

Sol. Let foot of perpendicular is (h,k)



$\frac{x^2}{4} + \frac{y^2}{2} = 1$ (Given)

$a = 2$, $b = \sqrt{2}$, $e = \sqrt{1 - \frac{2}{4}} = \frac{1}{\sqrt{2}}$

$\therefore$ Focus $(ae, 0) = (\sqrt{2}, 0)$

Equation of tangent

$y = mx + \sqrt{a^2m^2 + b^2}$

$y = mx + \sqrt{4m^2 + 2}$

Passes throguh (h,k)

$(k - mh)^2 = 4m^2 + 2$ ...(1)

line perpendicular to tangent will have slope

$-\frac{1}{m}$

$y - 0 = -\frac{1}{m}\left(x - \sqrt{2}\right)$

$my = -x + \sqrt{2}$

$(h + mk)^2 = 2$ ...(2)

Add equaiton (1) and (2)

$k^2(1 + m^2) + h^2(1 + m^2) = 4(1 + m^2)$

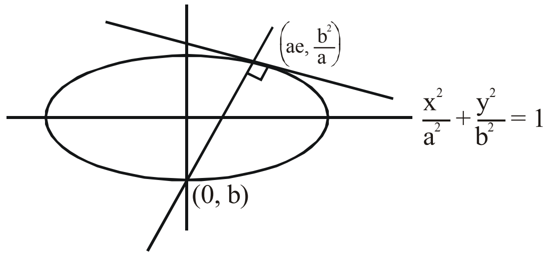
$h^2 + k^2 = 4$

$x^2 + y^2 = 4$ (Auxilary circle)

$\therefore \left(-1, \sqrt{3}\right)$ lies on the locus.

11. Official Ans. by NTA (4)

Sol. $\frac{a^2x}{x_1} - \frac{b^2y}{y_1} = a^2e^2$



$\frac{a^2x}{ae} - \frac{b^2y}{b^2}.a = a^2e^2$

$\frac{ax}{e} - ay = a^2e^2 \Rightarrow \frac{x}{e} - y = ae^2$

passes through (0, b)

$-b = ae^2 \Rightarrow b^2 = a^2e^4$

$a^2(1 - e^2) = a^2e^4 \Rightarrow e^4 + e^2 = 1$