DIFFERENTIAL EQUATION

1. Let y = y(x) be the solution curve of the differential equation, $(y^2 - x)\frac{dy}{dx} = 1$, satisfying y(0) = 1. This curve intersects the x-axis at a point whose abscissa is :

(1) 2 + e (2) 2
(3) 2 – e (4) –e

2. If y = y(x) is the solution of the differential equation, $e^y\left(\frac{dy}{dx} - 1\right) = e^x$ such that y(0) = 0, then y(1) is equal to :

(1) $2 + \log_e 2$ (2) 2e
(3) $\log_e 2$ (4) $1 + \log_e 2$

3. The differential equation of the family of curves, $x^2 = 4b(y + b)$, $b \in R$, is

(1) $x(y')^2 = x + 2yy'$

(2) $x(y')^2 = 2yy' - x$

(3) $xy'' = y'$

(4) $x(y')^2 = x - 2yy'$

4. Let y = y(x) be a solution of the differential equation, $\sqrt{1-x^2}\frac{dy}{dx} + \sqrt{1-y^2} = 0, |x| < 1$. If $y\left(\frac{1}{2}\right) = \frac{\sqrt{3}}{2}$, then $y\left(\frac{-1}{\sqrt{2}}\right)$ is equal to

(1) $-\frac{\sqrt{3}}{2}$ (2) $\frac{1}{\sqrt{2}}$

(3) $\frac{\sqrt{3}}{2}$ (4) $-\frac{1}{\sqrt{2}}$

5. If $\frac{dy}{dx} = \frac{xy}{x^2 + y^2}$; y(1) = 1; then a value of x satisfying y(x) = e is :

(1) $\sqrt{2}e$ (2) $\frac{e}{\sqrt{2}}$

(3) $\frac{1}{2}\sqrt{3}e$ (4) $\sqrt{3}e$

6. If $f'(x) = \tan^{-1}(\sec x + \tan x)$, $-\frac{\pi}{2} < x < \frac{\pi}{2}$, and $f(0) = 0$, then $f(1)$ is equal to :

(1) $\frac{\pi - 1}{4}$ (2) $\frac{\pi + 2}{4}$

(3) $\frac{\pi + 1}{4}$ (4) $\frac{1}{4}$

7. If for x ≥ 0, y = y(x) is the solution of the differential equation $(x + 1)dy = ((x + 1)^2 + y - 3)dx$, y(2) = 0, then y(3) is equal to ———.

8. Let y = y(x) be the solution of the differential equation, $\frac{2 + \sin x}{y + 1} \cdot \frac{dy}{dx} = -\cos x, y > 0, y(0) = 1$.

If $y(\pi) = a$ and $\frac{dy}{dx}$ at $x = \pi$ is b, then the ordered pair (a, b) is equal to :

(1) (2, 1) (2) $\left(2, \frac{3}{2}\right)$
(3) (1, –1) (4) (1, 1)

9. If a curve y = f(x), passing through the point (1,2), is the solution of the differential equation, $2x^2dy = (2xy + y^2)dx$, then $f\left(\frac{1}{2}\right)$ is equal to:

(1) $\frac{1}{1 - \log_e 2}$ (2) $\frac{1}{1 + \log_e 2}$

(3) $\frac{-1}{1 + \log_e 2}$ (4) $1 + \log_e 2$

10. The solution curve of the differential equation, $(1+e^{-x})(1+y^2)\frac{dy}{dx}=y^2$, which passes through the point (0, 1), is :

(1) $y^2 = 1 + y\log_e\left(\frac{1+e^x}{2}\right)$

(2) $y^2 + 1 = y\left(\log_e\left(\frac{1+e^x}{2}\right)+2\right)$

(3) $y^2 = 1 + y\log_e\left(\frac{1+e^{-x}}{2}\right)$

(4) $y^2 + 1 = y\left(\log_e\left(\frac{1+e^{-x}}{2}\right)+2\right)$

11. If $x^3dy + xy\,dx = x^2\,dy + 2y\,dx$; $y(2) = e$ and $x > 1$, then $y(4)$ is equal to :

(1) $\frac{3}{2}+\sqrt{e}$ (2) $\frac{3}{2}\sqrt{e}$

(3) $\frac{1}{2}+\sqrt{e}$ (4) $\frac{\sqrt{e}}{2}$

12. Let $y = y(x)$ be the solution of the differential equation, $xy' - y = x^2(x\cos x + \sin x)$, $x > 0$. If $y(\pi) = \pi$, then $y''\left(\frac{\pi}{2}\right)+y\left(\frac{\pi}{2}\right)$ is equal to :

(1) $2+\frac{\pi}{2}$ (2) $1+\frac{\pi}{2}$

(3) $1+\frac{\pi}{2}+\frac{\pi^2}{4}$ (4) $2+\frac{\pi}{2}+\frac{\pi^2}{4}$

13. The solution of the differential equation $\frac{dy}{dx}-\frac{y+3x}{\log_e(y+3x)}+3=0$ is :-

(where C is a constant of integration.)

(1) $x-2\log_e(y+3x)=C$

(2) $x-\log_e(y+3x)=C$

(3) $x-\frac{1}{2}(\log_e(y+3x))^2 = C$

(4) $y + 3x-\frac{1}{2}(\log_e x)^2 = C$

14. If $y = y(x)$ is the solution of the differential equation $\frac{5+e^x}{2+y}\cdot\frac{dy}{dx}+e^x=0$ satisfying $y(0) = 1$, then a value of $y(\log_e 13)$ is :

(1) 1 (2) –1

(3) 2 (4) 0

15. Let $y = y(x)$ be the solution of the differential equation $\cos x\frac{dy}{dx} + 2y\sin x = \sin 2x$, $x \in \left(0, \frac{\pi}{2}\right)$. If $y(\pi/3) = 0$, then $y(\pi/4)$ is equal to :

(1) $\sqrt{2}-2$ (2) $\frac{1}{\sqrt{2}} - 1$

(3) $2 - \sqrt{2}$ (4) $2 + \sqrt{2}$

16. Which of the following points lies on the tangent to the curve $x^4e^y + 2\sqrt{y+1} = 3$ at the point (1, 0)?

(1) (2, 2) (2) (–2, 6)

(3) (–2, 4) (4) (2, 6)

17. The general solution of the differential equation $\sqrt{1+x^2+y^2+x^2y^2} + xy\frac{dy}{dx} = 0$ is :

(where C is a constant of integration)

(1) $\sqrt{1+y^2} + \sqrt{1+x^2} = \frac{1}{2}\log_e\left(\frac{\sqrt{1+x^2}+1}{\sqrt{1+x^2}-1}\right) + C$

(2) $\sqrt{1+y^2} - \sqrt{1+x^2} = \frac{1}{2}\log_e\left(\frac{\sqrt{1+x^2}+1}{\sqrt{1+x^2}-1}\right) + C$

(3) $\sqrt{1+y^2} + \sqrt{1+x^2} = \frac{1}{2}\log_e\left(\frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}+1}\right) + C$

(4) $\sqrt{1+y^2} - \sqrt{1+x^2} = \frac{1}{2}\log_e\left(\frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}+1}\right) + C$

18. If $y = \left(\frac{2}{\pi}x - 1\right)\text{cosec}x$ is the solution of the differential equation, $\frac{dy}{dx} + p(x)y = \frac{2}{\pi}\text{cosec}x$, $0 < x < \frac{\pi}{2}$, then the function p(x) is equal to

(1) cotx (2) tanx

(3) cosecx (4) secx

SOLUTION

1. NTA Ans. (3)

Sol. $(y^2 - x)\dfrac{dy}{dx} = 1$

$\Rightarrow \dfrac{dx}{dy} + x = y^2$

I.F. $= e^{\int dy} = e^y$

Solution is given by

$x\,e^y = \int y^2 e^y dy + C$

$\Rightarrow x\,e^y = (y^2 - 2y + 2)e^y + C$

$x = 0,\ y = 1,$ gives $C = -e$

If $y = 0$, then $x = 2 - e$

2. NTA Ans. (4)

Sol. $e^y\dfrac{dy}{dx} - e^y = e^x$, Let $e^y = t$

$\Rightarrow e^y\dfrac{dy}{dx} = \dfrac{dt}{dx}$

$\dfrac{dt}{dx} - t = e^x$

I.F. $= e^{\int -dx} = e^{-x}$

$t\,e^{-x} = x + c \Rightarrow e^{y-x} = x + c$

$y(0) = 0 \Rightarrow c = 1$

$e^{y-x} = x + 1 \Rightarrow y(1) = 1 + \log_e 2$

3. NTA Ans. (1)

Sol. $2x = 4by' \Rightarrow y' = \dfrac{2x}{4b}$

Required D.E. is $x^2 = \dfrac{2x}{y'}y + \left(\dfrac{x}{y'}\right)^2$

$x(y')^2 = 2yy' + x$

(1) Option

4. NTA Ans. (2)

ALLEN Ans. **(BONUS)**

Note: As per the given informaton, x cannot be negative. So, it is invalid to ask y(x) for x < 0. Hence, it should be bonus but, NTA retained its answer as option (2).

Sol. $\dfrac{dy}{dx} = -\dfrac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$ so, $\dfrac{dy}{\sqrt{1-y^2}} + \dfrac{dx}{\sqrt{1-x^2}} = 0$

Integrating, $\sin^{-1}x + \sin^{-1}y = c$

so, $\dfrac{\pi}{6} + \dfrac{\pi}{3} = c$

Hence, $\sin^{-1}x + \sin^{-1}y = \dfrac{\pi}{2}$

Put $x = -\dfrac{1}{\sqrt{2}}, \sin^{-1}y = \dfrac{3\pi}{4}$ (Not possible)

5. NTA Ans. (4)

Sol. $\dfrac{dy}{dx} = \dfrac{xy}{x^2 + y^2}$

Let $y = vx$

$\dfrac{dy}{dx} = v + x.\dfrac{dv}{dx}$

$v + x\dfrac{dv}{dx} = \dfrac{xvx}{x^2 + v^2x^2} = \dfrac{v}{1+v^2}$

$x\dfrac{dv}{dx} = \dfrac{v}{1+v^2} - v = \dfrac{v - v - v^3}{1+v^2} = -\dfrac{v^3}{1+v^2}$

$\int \dfrac{1+v^2}{v^3}.dv = \int -\dfrac{dx}{x}$

$\Rightarrow \int v^{-3}.dv + \int \dfrac{1}{v}dv = -\int \dfrac{dx}{x}$

$\Rightarrow \dfrac{v^{-2}}{-2} + \ell n\,v = -\ell n\,x + \lambda$

$\Rightarrow -\dfrac{1}{2v^2} + \ell n\left(\dfrac{y}{x}\right) = -\ell n\,x + \lambda$

$\Rightarrow -\dfrac{1}{2}\dfrac{x^2}{y^2} + \ell n\,y - \ell n\,x = -\ell n\,x + \lambda$

$\Rightarrow -\dfrac{1}{2} + 0 = \lambda \Rightarrow \lambda = -\dfrac{1}{2}$

$$\Rightarrow -\frac{1}{2}\frac{x^2}{y^2}+\ell n\,y+\frac{1}{2}=0 \quad \text{at } y = e$$

$$\Rightarrow -\frac{1}{2}\frac{x^2}{e^2}+1+\frac{1}{2}=0 \Rightarrow \frac{x^2}{2e^2}=\frac{3}{2}\Rightarrow x^2=3e^2$$

$$\therefore x=\sqrt{3}e$$

6. NTA Ans. (3)

Sol. $f'(x) = \tan^{-1}(\sec x + \tan x)$

$$f'(x) = \tan^{-1}\left(\frac{1+\sin x}{\cos x}\right)=\tan^{-1}\left(\frac{1+\tan\frac{x}{2}}{1-\tan\frac{x}{2}}\right)$$

$$=\tan^{-1}\left(\tan\left(\frac{\pi}{4}+\frac{\pi}{2}\right)\right)$$

$$\because -\frac{\pi}{2}<x<\frac{\pi}{2}\Rightarrow 0<\frac{\pi}{4}+\frac{x}{2}<\frac{\pi}{2}$$

$$\Rightarrow f'(x)=\frac{\pi}{4}+\frac{x}{2}$$

$$\therefore f(x)=\frac{\pi}{4}.x+\frac{x^2}{4}+c$$

$\because f(0) = 0 \Rightarrow c = 0$

$$\Rightarrow f(x)=\frac{\pi}{4}x+\frac{x^2}{4}$$

$$\therefore f(1)=\frac{\pi+1}{4}$$

7. NTA Ans. (3.00)

Sol. $(x + 1)dy - ydx = ((x + 1)^2 - 3)dx$

$$\Rightarrow \frac{(x+1)dy-ydx}{(x+1)^2}=\left(1-\frac{3}{(x+1)^2}\right)dx$$

$$\Rightarrow d\left(\frac{y}{(x+1)}\right)=\left(1-\frac{3}{(x+1)^2}\right)dx$$

integrating both sides

$$\frac{y}{x+1}=x+\frac{3}{(x+1)}+C$$

Given $y(2) = 0 \Rightarrow c = -3$

$$\therefore\ y=(x+1)\left(x+\frac{3}{(x+1)}-3\right)$$

$$\therefore\ y(3)=3.00$$

8. Official Ans. by NTA (4)

Sol. $\frac{2+\sin x}{y+1}\frac{dy}{dx} = -\cos x,\ y > 0$

$$\Rightarrow \frac{dy}{y+1} = \frac{-\cos x}{2+\sin x}dx$$

By integrating both sides :

$$\ell n\,|y+1|=-\ell n\,|2+\sin x|+\ell nK$$

$$\Rightarrow y + 1 = \frac{K}{2+\sin x} \qquad (y + 1 > 0)$$

$$\Rightarrow y(x) = \frac{K}{2+\sin x} - 1$$

Given $y(0) = 1 \Rightarrow K = 4$

So, $y(x) = \frac{4}{2+\sin x} - 1$

$a = y(\pi) = 1$

$$b = \left.\frac{dy}{dx}\right]_{x=\pi} = \left.\frac{-\cos x}{2+\sin x}(y(x)+1)\right]_{x=\pi} = 1$$

So, $(a, b) = (1, 1)$

9. Official Ans. by NTA (2)

Sol. $2x^2dy = (2xy + y^2)\,dx$

$$\Rightarrow \frac{dy}{dx}=\frac{2xy+y^2}{2x^2}\ \{\text{Homogeneous D.E.}\}$$

$$\left\{\begin{array}{l}\text{let } y = xt\\ \Rightarrow \frac{dy}{dx}=t+x\frac{dt}{dx}\end{array}\right\}$$

$$\Rightarrow t + x\frac{dt}{dx} = \frac{2x^2t + x^2t^2}{2x^2}$$

$$\Rightarrow t + x\frac{dt}{dx} = t + \frac{t^2}{2}$$

$$\Rightarrow x\frac{dt}{dx} = \frac{t^2}{2}$$

$$\Rightarrow 2\int \frac{dt}{t^2} = \int \frac{dx}{x}$$

$$\Rightarrow 2\left(-\frac{1}{t}\right) = \ell n(x) + C \quad \left\{\text{Put } t = \frac{y}{x}\right\}$$

$$\Rightarrow -\frac{2x}{y} = \ell n x + C \quad \left\{\begin{matrix}\text{Put } x = 1 \,\&\, y = 2 \\ \text{then we get } C = -1\end{matrix}\right\}$$

$$\Rightarrow \frac{-2x}{y} = \ell n(x) - 1 \Rightarrow y = \frac{2x}{1 - \ell n x}$$

$$\Rightarrow f(x) = \frac{2x}{1 - \log_e x}$$

so, $\boxed{f\left(\frac{1}{2}\right) = \frac{1}{1 + \log_e 2}}$

10. Official Ans. by NTA (1)

Sol. $(1 + e^{-x})(1 + y^2)\frac{dy}{dx} = y^2$

$$\Rightarrow (1 + y^{-2})\, dy = \left(\frac{e^x}{1 + e^x}\right)dx$$

$$\Rightarrow \left(y - \frac{1}{y}\right) = \ell n(1 + e^x) + c$$

$\therefore$ It passes through (0, 1) $\Rightarrow c = -\ell n 2$

$$\Rightarrow y^2 = 1 + y\, \ell n\left(\frac{1 + e^x}{2}\right)$$

11. Official Ans. by NTA (2)

Sol. $x^3dy + xy\,dx = x^2\,dy + 2y\,dx$

$\Rightarrow dy(x^3 - x^2) = dx\,(2y - xy)$

$$\Rightarrow -\int \frac{1}{y}dy = \int \frac{x - 2}{x^2(x - 1)}dx$$

$$\Rightarrow -\ell ny = \int\left(\frac{A}{x} + \frac{B}{x^2} + \frac{C}{(x - 1)}\right)dx$$

Where A = 1, B = +2, C = –1

$$\Rightarrow -\ell ny = \ell n\, x - \frac{2}{x} - \ell n\,(x - 1) + \lambda$$

$\Rightarrow y(2) = e$

$\Rightarrow -1 = \ell n\ 2 - 1 - 0 + \lambda$

$\therefore \lambda = -\ \ell n 2$

$$\Rightarrow \ell n\, y = -\ \ell nx + \frac{2}{x} + \ell n(x - 1) + \ell n 2$$

Now put x = 4 in equation

$$\Rightarrow \ell n\, y = -\ell n 4 + \frac{1}{2} + \ell n 3 + \ell n 2$$

$$\Rightarrow \ell n\, y = \ell n\left(\frac{3}{2}\right) + \frac{1}{2}\ell n e$$

$$\Rightarrow y = \frac{3}{2}\sqrt{e}$$

12. Official Ans. by NTA (1)

Sol. $x\frac{dy}{dx} - y = x^2(x\cos x + \sin x),\ x > 0$

$$\frac{dy}{dx} - \frac{y}{x} = x(x\cos x + \sin x) \Rightarrow \frac{dy}{dx} + Py = Q$$

so, I.F. $= e^{\int -\frac{1}{x}dx} = \frac{1}{|x|} = \frac{1}{x}\ (x > 0)$

Thus, $\frac{y}{x} = \int \frac{1}{x}\big(x(x\cos x + \sin x)\big)dx$

$$\Rightarrow \frac{y}{x} = x\sin x + C$$

$\because y(\pi) = \pi \Rightarrow C = 1$

so, $y = x^2\sin x + x \Rightarrow (y)_{\pi/2} = \frac{\pi^2}{4} + \frac{\pi}{2}$

Also, $\frac{dy}{dx} = x^2\cos x + 2x\sin x + 1$

$\Rightarrow \frac{d^2y}{dx^2} = -x^2 \sin x + 4x\cos x + 2\sin x$

$\Rightarrow \left.\frac{d^2y}{dx^2}\right|_{\frac{\pi}{2}} = -\frac{\pi^2}{4} + 2$

Thus, $y_{\left(\frac{\pi}{2}\right)} + \frac{d^2y}{dx^2}_{\left(\frac{\pi}{2}\right)} = \frac{\pi}{2} + 2$

13. Official Ans. by NTA (3)

Sol. $\ell n(y + 3x) = z$ (let)

$\frac{1}{y+3x} \cdot \left(\frac{dy}{dx} + 3\right) = \frac{dz}{dx}$

..(1)

$\frac{dy}{dx} + 3 = \frac{y+3x}{\ell n(y+3x)}$ (given)

$\frac{dz}{dx} = \frac{1}{z}$

$\Rightarrow z\,dz = dx \Rightarrow \frac{z^2}{2} = x + C$

$\Rightarrow \frac{1}{2}\ell n^2(y+3x) = x+C$

$\Rightarrow x - \frac{1}{2}(\ell n(y + 3x))^2 = C$

14. Official Ans. by NTA (2)

Sol. $\frac{(5+e^x)}{2+y}\frac{dy}{dx} = -e^x$

$\int \frac{dy}{2+y} = \int \frac{-e^x}{e^x+5}dx$

$\ln(y + 2) = -ln(e^x + 5) + k$

$(y + 2)(e^x + 5) = C$

$\because y(0) = 1$

$\Rightarrow C = 18$

$y + 2 = \frac{18}{e^x+5}$

at $x = ln13$

$y + 2 = \frac{18}{13+5} = 1$

$\boxed{y = -1}$

15. Official Ans. by NTA (1)

Sol. $\cos x \frac{dy}{dx} + 2y \sin x = \sin 2x$

$\frac{dy}{dx} + \frac{2\sin x}{\cos x} y = 2 \sin x$

I.F. $= e^{\int 2\frac{\sin x}{\cos x}dx}$

$= e^{2\ln \sec x} = \sec^2 x$

$y \cdot \sec^2 x = \int 2\sin x \cdot \sec^2 x\,dx$

$y \sec^2 x = 2\int \tan x \sec x\,dx$

$y \sec^2 x = 2 \sec x + c$

At $x = \frac{\pi}{3}$, $y = 0$

$\Rightarrow 0 = 2\sec\frac{\pi}{3} + C \Rightarrow C = -4$

$\boxed{y\sec^2 x = 2\sec x - 4}$

Put $x = \frac{\pi}{4}$

$y \cdot 2 = 2\sqrt{2} - 4$

$y = \sqrt{2} - 2$

16. Official Ans. by NTA (2)

Sol. $x^4e^y + 2\sqrt{y+1} = 3$

d.w.r. to x

$x^4 e^y y' + e^y 4x^3 + \frac{2y'}{2\sqrt{y+1}} = 0$

at P(1, 0)

$y'_P + 4 + y'_P = 0$

$\Rightarrow y'_P = -2$

Tangent at P(1, 0) is

$y - 0 = -2(x - 1)$

$2x + y = 2$

(–2, 6) lies on it

17. Official Ans. by NTA (1)

Sol. $\sqrt{1+x^2+y^2+x^2y^2}+xy\frac{dy}{dx}=0$

$\Rightarrow\sqrt{(1+x)^2(1+y^2)}+xy\frac{dy}{dx}=0$

$\Rightarrow\sqrt{1+x^2}\sqrt{1+y^2}=-xy\frac{dy}{dx}$

$\Rightarrow\int\frac{ydy}{\sqrt{1+y^2}}=-\int\frac{\sqrt{1+x^2}}{x}dx$ ...(1)

Now put $1+x^2=u^2$ and $1+y^2=v^2$

$2xdx = 2udu$ and $2ydy = 2vdv$

$\Rightarrow xdx = udu$ and $ydy = vdv$

substitude these values in equation (1)

$\int\frac{vdv}{v}=-\int\frac{u^2.du}{u^2-1}$

$\Rightarrow\int dv=-\int\frac{u^2-1+1}{u^2-1}du$

$\Rightarrow v=-\int\left(1+\frac{1}{u^2-1}\right)du$

$\Rightarrow v=-u-\frac{1}{2}\log_e\left|\frac{u-1}{u+1}\right|+c$

$\Rightarrow\sqrt{1+y^2}=-\sqrt{1+x^2}+\frac{1}{2}\log_e\left|\frac{\sqrt{1+x^2}+1}{\sqrt{1+x^2}-1}\right|+c$

$\Rightarrow\sqrt{1+y^2}+\sqrt{1+x^2}=\frac{1}{2}\log_e\left|\frac{\sqrt{1+x^2}+1}{\sqrt{1+x^2}-1}\right|+c$

18. Official Ans. by NTA (1)

Sol. $y=\left(\frac{2x}{\pi}-1\right)\operatorname{cosec}x$

...(1)

$\frac{dy}{dx}=\frac{2}{\pi}\operatorname{cosec}x-\left(\frac{2x}{\pi}-1\right)\operatorname{cosec}x\cot x$

$\frac{dy}{dx}=\frac{2\operatorname{cosec}x}{\pi}-y\cot x$

using equation (1)

$\frac{dy}{dx}+y\cot x=\frac{2\operatorname{cosec}x}{\pi}$

$\frac{dy}{dx}+p(x).y=\frac{2\operatorname{cosec}x}{\pi}\quad x\in\left(0,\frac{\pi}{2}\right)$

Compare : $p(x) = \cot x$