# DIFFERENTIABILITY

**1.** Let S be the set of points where the function, $f(x) = |2-|x-3||$, $x\in R$, is not differentiable.

Then $\sum_{x\in S} f(f(x))$ is equal to ________.

**2.** If a function f(x) defined by

$$f(x)=\begin{cases} ae^{x}+be^{-x}, & -1\le x<1 \\ cx^{2}, & 1\le x\le 3 \\ ax^{2}+2cx, & 3<x\le 4 \end{cases}$$ be continuous

for some a, b, c $\in$ R and $f'(0) + f'(2) = e$, then the value of of a is :

(1) $\frac{e}{e^2-3e-13}$ (2) $\frac{e}{e^2+3e+13}$

(3) $\frac{1}{e^2-3e+13}$ (4) $\frac{e}{e^2-3e+13}$

**3.** Suppose a differentiable function f(x) satisfies the identity $f(x + y) = f(x) + f(y) + xy^2 + x^2y$, for all real x and y. If $\lim_{x\to 0}\frac{f(x)}{x}=1$, then $f'(3)$ is equal to ________.

**4.** The function $f(x) = \begin{cases} \frac{\pi}{4}+\tan^{-1}x, & |x|\le 1 \\ \frac{1}{2}(|x|-1), & |x|>1 \end{cases}$ is :

(1) continuous on R–{1} and differentiable on R – {–1, 1}.

(2) both continuous and differentiable on R – {–1}.

(3) continuous on R – {–1} and differentiable on R – {–1, 1}.

(4) both continuous and differentiable on R –{1}

**5.** If the function $f(x)=\begin{cases} k_1(x-\pi)^2-1, & x\le\pi \\ k_2\cos x, & x>\pi \end{cases}$ is twice differentiable, then the ordered pair $(k_1, k_2)$ is equal to :

(1) $\left(\frac{1}{2},1\right)$ (2) (1, 1)

(3) $\left(\frac{1}{2},-1\right)$ (4) (1, 0)

**6.** Let $f : R \to R$ be defined as

$$f(x)=\begin{cases} x^5\sin\left(\frac{1}{x}\right)+5x^2, & x<0 \\ 0, & x=0 \\ x^5\cos\left(\frac{1}{x}\right)+\lambda x^2, & x>0 \end{cases}$$

The value of $\lambda$ for which $f''(0)$ exists, is _.

**7.** Let $f : R \to R$ be a function defined by $f(x) = \max\{x, x^2\}$. Let S denote the set of all points in R, where $f$ is not differentiable. Then :

(1) {0, 1} (2) {0}

(3) $\phi$(an empty set) (4) {1}

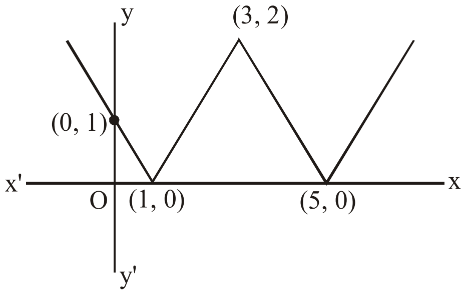
## SOLUTION

**1. NTA Ans. (3)**

**Sol.** $f(x) = |2 - |x - 3||$

f is not differentiable at

$x = 1, 3, 5$




$\Rightarrow \sum_{x \in s} f(f(x)) = f(f(1)) + f(f(3)) + f(f(5))$

$= f(0) + f(2) + f(0)$

$= 1 + 1 + 1 = 3$

**2. Official Ans. by NTA (4)**

**Sol.** $f(x) = \begin{cases} ae^x + be^{-x}, & -1 \le x < 1 \\ cx^2, & 1 \le x \le 3 \\ ax^2 + 2cx, & 3 < x \le 4 \end{cases}$

For continuity at x = 1

$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x)$

$\Rightarrow \boxed{ae + be^{-1} = c} \Rightarrow \boxed{b = ce - ae^2}$

...(1)

For continuity at x = 3

$\lim_{x \to 3^-} f(x) = \lim_{x \to 3^+} f(x)$

$\Rightarrow 9c = 9a + 6c$

$\Rightarrow c = 3a$ ...(2)

$f'(0) + f'(2) = e$

$(ae^x - be^x)_{x = 0} + (2cx)_{x = 2} = e$

$\Rightarrow \boxed{a - b + 4c = e}$ ...(3)

From (1), (2) & (3)

$a - 3ae + ae^2 + 12a = e$

$\Rightarrow a(e^2 + 13 - 3e) = e$

$\Rightarrow a = \dfrac{e}{e^2 - 3e + 13}$

**3. Official Ans. by NTA (10)**

**Sol.** Since, $\lim_{x \to 0} \dfrac{f(x)}{x}$ exist $\Rightarrow f(0) = 0$

Now, $f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

$= \lim_{h \to 0} \dfrac{f(h) + xh^2 + x^2h}{h}$ (take y = h)

$= \lim_{h \to 0} \dfrac{f(h)}{h} + \lim_{h \to 0}(xh) + x^2$

$\Rightarrow f'(x) = 1 + 0 + x^2$

$\Rightarrow f'(3) = 10$

**4. Official Ans. by NTA (1)**

**Sol.** $f(x) = \begin{cases} \dfrac{\pi}{4} + \tan^{-1} x, & x \in (-\infty, -1] \cup [1, \infty) \\ -\dfrac{(x+1)}{2}, & x \in (-1, 0] \\ \dfrac{x-1}{2}, & x \in (0, 1) \end{cases}$

for continuity at x = –1

L.H.L. = $\dfrac{\pi}{4} - \dfrac{\pi}{4} = 0$

R.H.L. = 0

so, continuous at x = –1

for continuity at x = 1

L.H.L. = 0

R.H.L. = $\dfrac{\pi}{4} + \dfrac{\pi}{4} = \dfrac{\pi}{2}$

so, not continuous at x = 1

For differentiability at x = –1

L.H.D. = $\dfrac{1}{1+1} = \dfrac{1}{2}$

R.H.D. = $-\dfrac{1}{2}$

so, non differentiable at x = –1

**5. Official Ans. by NTA (1)**

**Sol.** f(x) is continuous and differentiable

$f(\pi^-) = f(\pi) = f(\pi^+)$

$-1 = -k_2$

$\boxed{k_2 = 1}$

$$f'(x) = \begin{cases} 2k_1(x-\pi)\ ; & x \le \pi \\ -k_2 \sin x\ ; & x > \pi \end{cases}$$

$f'(\pi^-) = f'(\pi^+)$

$0 = 0$

so, differentiable at $x = 0$

$$f''(x) = \begin{cases} 2k_1\ ; & x \le \pi \\ -k_2 \cos x\ ; & x > \pi \end{cases}$$

$f''(\pi^-) = f''(\pi^+)$

$2k_1 = k_2$

$\boxed{k_1 = \frac{1}{2}}$

$$(k_1, k_2) = \left(\frac{1}{2}, 1\right)$$

**6. Official Ans. by NTA (5.00)**

**Sol.** $f(x) = x^5.\sin\frac{1}{x} + 5x^2 \quad$ if $x < 0$

$f(x) = 0 \quad$ if $x = 0$

$f(x) = x^5.\cos\frac{1}{x} + \lambda x^2 \quad$ if $x > 0$

LHD of $f'(x)$ at $x = 0$ is 10

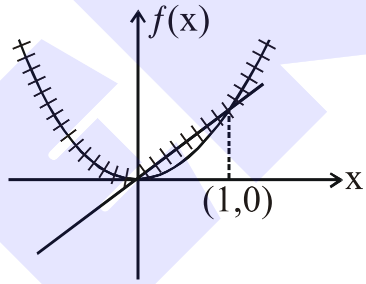
RHD of $f'(x)$ at $x = 0$ is $2\lambda$

if $f''(0)$ exists then

$2\lambda = 10 \Rightarrow \lambda = 5$

**7. Official Ans. by NTA (1)**

**Sol.** $f(x) = \max(x, x^2)$




Non-differentiable at $x = 0, 1$

$S = \{0, 1\}$