# DEFINITE INTEGRATION

**1.** If $\theta_1$ and $\theta_2$ be respectively the smallest and the largest values of $\theta$ in $(0, 2\pi) - \{\pi\}$ which satisfy the equation, $2\cot^2\theta - \frac{5}{\sin\theta} + 4 = 0$, then $\int_{\theta_1}^{\theta_2} \cos^2 3\theta d\theta$ is equal to :

(1) $\frac{2\pi}{3}$  (2) $\frac{\pi}{3}+\frac{1}{6}$

(3) $\frac{\pi}{9}$  (4) $\frac{\pi}{3}$

**2.** The value of $\alpha$ for which $4\alpha\int_{-1}^{2} e^{-\alpha|x|}dx = 5$, is :

(1) $\log_e\left(\frac{3}{2}\right)$  (2) $\log_e\left(\frac{4}{3}\right)$

(3) $\log_e 2$  (4) $\log_e\sqrt{2}$

**3.** If $f(a + b + 1 - x) = f(x)$, for all x, where a and b are fixed positive real numbers, then $\frac{1}{a+b}\int_a^b x(f(x)+f(x+1))dx$ is equal to :

(1) $\int_{a+1}^{b+1} f(x)dx$  (2) $\int_{a+1}^{b+1} f(x+1)dx$

(3) $\int_{a-1}^{b-1} f(x+1)dx$  (4) $\int_{a-1}^{b-1} f(x)dx$

**4.** If $I = \int_1^2 \frac{dx}{\sqrt{2x^3 - 9x^2 + 12x + 4}}$, then :

(1) $\frac{1}{9} < I^2 < \frac{1}{8}$  (2) $\frac{1}{16} < I^2 < \frac{1}{9}$

(3) $\frac{1}{6} < I^2 < \frac{1}{2}$  (4) $\frac{1}{8} < I^2 < \frac{1}{4}$

**5.** $\lim_{x\to 0}\frac{\int_0^x t\sin(10t)dt}{x}$ is equal to

(1) 0  (2) $-\frac{1}{5}$

(3) $-\frac{1}{10}$  (4) $\frac{1}{10}$

**6.** The value of $\int_0^{2\pi}\frac{x\sin^8 x}{\sin^8 x + \cos^8 x}dx$ is equal to :

(1) $2\pi$  (2) $4\pi$

(3) $2\pi^2$  (4) $\pi^2$

**7.** If for all real triplets (a, b, c), $f(x) = a + bx + cx^2$; then $\int_0^1 f(x)dx$ is equal to :

(1) $\frac{1}{2}\left\{f(1)+3f\left(\frac{1}{2}\right)\right\}$

(2) $2\left\{3f(1)+2f\left(\frac{1}{2}\right)\right\}$

(3) $\frac{1}{6}\left\{f(0)+f(1)+4f\left(\frac{1}{2}\right)\right\}$

(4) $\frac{1}{3}\left\{f(0)+f\left(\frac{1}{2}\right)\right\}$

**8.** The integral $\int_0^2 ||x-1|-x|dx$ is equal to______.

**9.** Let [t] denote the greatest integer less than or equal to t. Then the value of $\int_1^2 |2x-[3x]|dx$ is________.

**10.** $\int_{-\pi}^{\pi} |\pi - |x||dx$ is equal to :

(1) $\pi^2$  (2) $2\pi^2$

(3) $\sqrt{2}\pi^2$  (4) $\frac{\pi^2}{2}$

**11.** If the value of the integral $\int_0^{1/2}\frac{x^2}{(1-x^2)^{3/2}}dx$ is $\frac{k}{6}$, then k is equal to :

(1) $2\sqrt{3}-\pi$  (2) $3\sqrt{2}+\pi$

(3) $3\sqrt{2}-\pi$  (4) $2\sqrt{3}+\pi$

**12.** Let $f(x) = |x - 2|$ and $g(x) = f(f(x))$, $x \in [0, 4]$. Then $\int_0^3 (g(x) - f(x))dx$ is equal to :

(1) $\frac{3}{2}$ (2) 0

(3) $\frac{1}{2}$ (4) 1

**13.** Let $f(x) = \int \frac{\sqrt{x}}{(1+x)^2}dx \ (x \ge 0)$. Then $f(3) - f(1)$ is equal to :

(1) $-\frac{\pi}{6}+\frac{1}{2}+\frac{\sqrt{3}}{4}$ (2) $\frac{\pi}{6}+\frac{1}{2}-\frac{\sqrt{3}}{4}$

(3) $-\frac{\pi}{12}+\frac{1}{2}+\frac{\sqrt{3}}{4}$ (4) $\frac{\pi}{12}+\frac{1}{2}-\frac{\sqrt{3}}{4}$

**14.** $\int_{\pi/6}^{\pi/3} \tan^3 x \cdot \sin^2 3x (2\sec^2 x \cdot \sin^2 3x + 3\tan x \cdot \sin 6x)dx$ is equal to :

(1) $\frac{9}{2}$ (2) $-\frac{1}{9}$

(3) $-\frac{1}{18}$ (4) $\frac{7}{18}$

**15.** Let $\{x\}$ and $[x]$ denote the fractional part of x and the greatest integer $\le$ x respectively of a real number x. If $\int_0^n \{x\}dx, \int_0^n [x]dx$ and $10(n^2 - n)$, $(n \in N, n > 1)$ are three consecutive terms of a G.P., then n is equal to____

**16.** The value of $\int_{-\pi/2}^{\pi/2} \frac{1}{1+e^{\sin x}}dx$ is

(1) $\pi$ (2) $\frac{3\pi}{2}$

(3) $\frac{\pi}{4}$ (4) $\frac{\pi}{2}$

**17.** If $I_1 = \int_0^1 (1 - x^{50})^{100} dx$ and $I_2 = \int_0^1 (1 - x^{50})^{101} dx$ such that $I_2 = \alpha I_1$ then $\alpha$ equals to

(1) $\frac{5050}{5051}$ (2) $\frac{5050}{5049}$

(3) $\frac{5049}{5050}$ (4) $\frac{5051}{5050}$

**18.** The integral $\int_1^2 e^x . x^x (2 + \log_e x)dx$ equal :

(1) e(4e + 1) (2) e(2e – 1)

(3) $4e^2 - 1$ (4) e(4e – 1)

## SOLUTION

**1. NTA Ans. (4)**

**Sol.** $2\cos^2\theta - 5\sin\theta + 4\sin^2\theta = 0$

$3\sin^2\theta - 5\sin\theta + 2 = 0$

$\sin\theta = \frac{1}{2}$, 2 (Rejected)

$$\int_{\theta_1}^{\theta_2} \cos^2 3\theta d\theta = \int_{\pi/6}^{5\pi/6} \frac{1+\cos 6\theta}{2} d\theta$$

$$= \frac{1}{2}\left(\frac{5\pi}{6} - \frac{\pi}{6}\right) = \frac{2\pi}{6} = \frac{\pi}{3}$$

**2. NTA Ans. (3)**

**Sol.** $$4\alpha\left[\int_{-1}^{0} e^{\alpha x}dx + \int_{0}^{2} e^{-\alpha x}dx\right] = 5$$

$$\Rightarrow 4\alpha\left(\left[\frac{e^{\alpha x}}{\alpha}\right]_{-1}^{0} + \left[\frac{e^{-\alpha x}}{-\alpha}\right]_{0}^{2}\right) = 5$$

$\Rightarrow 4e^{-2\alpha} + 4e^{-\alpha} - 3 = 0$

Let $e^{-\alpha} = t$, $4t^2 + 4t - 3 = 0$, $t = \frac{1}{2}, \frac{-3}{2}$

(Rejected)

$e^{-\alpha} = \frac{1}{2}$;  $\alpha = \ell n2$

**3. NTA Ans. (1)**

**ALLEN Ans. (1 OR 3)**

**Note: In this Question, both options (1) as well as (3) are correct, but NTA accepte only option (1).**

**Sol.** $f(x + 1) = f(a + b - x)$

$$I = \frac{1}{(a+b)}\int_a^b x\big(f(x) + f(x+1)\big)dx \quad .....(1)$$

$$I = \frac{1}{(a+b)}\int_a^b (a+b-x)\big(f(x+1) + f(x)\big)dx \quad ...(2)$$

from (1) and (2)

$$2I = \int_a^b \big(f(x) + f(x+1)\big)dx$$

$$2I = \int_a^b f\big(a+b-x\big)dx + \int_a^b f(x+1)dx$$

$$2I = 2\int_a^b f(x+1)dx \Rightarrow I = \int_a^b f(x+1)dx$$

$$= \int_{a+1}^{b+1} f(x)dx$$

**OR**

$$I = \frac{1}{(a+b)}\int_a^b x\big(f(x) + f(x+1)\big)dx \quad ...(1)$$

$$= \frac{1}{(a+b)}\int_a^b (a+b-x)\big(f(a+b-x) + f(a+b+1-x)\big)dx$$

$$I = \frac{1}{(a+b)}\int_a^b (a+b-x)\big(f(x+1) + f(x)\big)dx \quad ...(2)$$

equation (1) + (2)

$$2I = \frac{1}{(a+b)}\int_a^b (a+b)\big(f(x+1) + f(x)\big)dx$$

$$I = \frac{1}{2}\left[\int_a^b f(x+1)dx + \int_a^b f(x)dx\right]$$

$$= \frac{1}{2}\left[\int_a^b f(a+b+1-x)dx + \int_a^b f(x)dx\right]$$

$$= \frac{1}{2}\left[\int_a^b f(x)dx + \int_a^b f(x)dx\right]$$

$$I = \int_a^b f(x)dx$$

Let $x = T + 1$

$$= \int_{a-1}^{b-1} f(T+1)dT$$

$$I = \int_{a-1}^{b-1} f(x+1)dx$$

**4. NTA Ans. (1)**

**Sol.** $f(x)=\dfrac{1}{\sqrt{2x^3-9x^2+12x+4}}$

$f'(x)=\dfrac{-6(x-1)(x-2)}{2\left(2x^3-9x^2+12x+4\right)^{3/2}}$

$\therefore$ $f(x)$ is decreasing in (1,2)

$f(1)=\dfrac{1}{3};\ f(2)=\dfrac{1}{\sqrt{8}}$

$\dfrac{1}{3}<I<\dfrac{1}{\sqrt{8}} \Rightarrow I^2\in\left(\dfrac{1}{9},\dfrac{1}{8}\right)$

(1) Option

**5. NTA Ans. (1)**

**Sol.** Using L.H. Rule

$\lim\limits_{x\to 0}\dfrac{x\sin(10x)}{1}=0$

(1) Option

**6. NTA Ans. (4)**

**Sol.** $I=\displaystyle\int_0^{2\pi}\frac{x\sin^8 x}{\sin^8 x+\cos^8 x}dx$ .......(1)

$=\left[\displaystyle\int_0^{\pi}\frac{x\sin^8 x}{\sin^8 x\cos^8 x}dx+\int_0^{\pi}\frac{(2\pi-x)\sin^8 x}{\sin^8 x+\cos^8 x}dx\right]$

$=2\pi\displaystyle\int_0^{\pi}\frac{\sin^8 x}{\sin^8 x+\cos^8 x}dx$

$I=2\pi\left[\displaystyle\int_0^{\pi/2}\frac{\sin^8 x}{\sin^8 x+\cos^8 x}dx+\int_0^{\pi/2}\frac{\cos^8 xdx}{\sin^8 x+\cos^8 x}dx\right]$

$=2\pi\displaystyle\int_0^{\pi/2}1dx=2\pi.\frac{\pi}{2}=\pi^2$

**7. NTA Ans. (3)**

**Sol.** $f(x) = a + bx + cx^2$

$\displaystyle\int_0^1 f(x)dx=\left[ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right]_0^1$

$=a+\dfrac{b}{2}+\dfrac{c}{3}=\dfrac{1}{6}[6a+3b+c]$

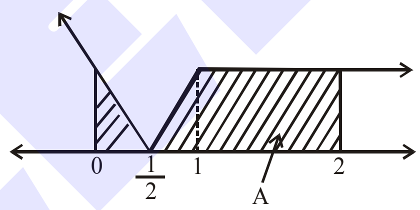
$=\dfrac{1}{6}\left[f(0)+f(1)+4f\left(\dfrac{1}{2}\right)\right]$

**8. Official Ans. by NTA (1.50)**

**Sol.** $\displaystyle\int_0^2 |\,|x-1|-x\,|dx$

Let $f(x)\ ||x-1|-x|$

$=\begin{cases}1, & x\ge 1\\ |1-2x|, & x\le 1\end{cases}$




$A=\dfrac{1}{2}+1=\dfrac{3}{2}$

**OR**

$\displaystyle\int_0^{1/2}(1-2x)dx+\int_{1/2}^{1}(2x-1)+\int_0^2 1dx$

$=\left[x-x^2\right]_0^{\frac{1}{2}}+\left[x^2-x\right]_{1/2}^{1}+[x]_1^2$

$=\boxed{3/2}$

**9. Official Ans. by NTA (1.0)**

**Sol.** $3 < 3x < 6$

Take cases when $3 < 3x < 4$, $4 < 3x < 5$, $5 < 3x < 6$ ;

Now $\displaystyle\int_1^2 |2x-[3x]|dx$

$=\displaystyle\int_1^{4/3}(3-2x)dx+\int_{4/3}^{5/3}(4-2x)\,dx+\int_{5/3}^{2}(5-2x)dx$

$=\dfrac{2}{9}+\dfrac{3}{9}+\dfrac{4}{9}=1$

**10. Official Ans. by NTA (1)**

**Sol.** $\int_{-\pi}^{\pi}\left|\pi-|x|\right|dx = 2\int_{0}^{\pi}|\pi-x|dx$

$= 2\int_{0}^{\pi}(\pi-x)dx$

$= 2\left[\pi x-\frac{x^2}{2}\right]_0^{\pi} = \pi^2$

**11. Official Ans. by NTA (1)**

**Sol.** $\int_0^{1/2}\frac{((x^2-1)+1)}{(1-x^2)^{3/2}}dx$

$\int_0^{1/2}\frac{dx}{(1-x^2)^{3/2}}-\int_0^{1/2}\frac{dx}{\sqrt{1-x^2}}$

$\int_0^{1/2}\frac{x^{-3}}{(x^{-2}-1)^{3/2}}dx-(\sin^{-1}x)_0^{1/2}$

Let $x^{-2}-1 = t^2 \Rightarrow x^{-3}\,dx = -t\,dt$

$\int_{\infty}^{\sqrt{3}}\frac{-t\,dt}{t^3}-\frac{\pi}{6}=\int_{\sqrt{3}}^{\infty}\frac{dt}{t^2}-\frac{\pi}{6}=\frac{1}{\sqrt{3}}-\frac{\pi}{6}=\frac{k}{6}$

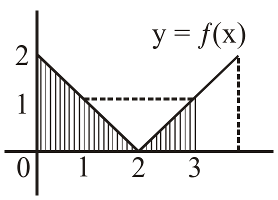
$k = 2\sqrt{3}-\pi$

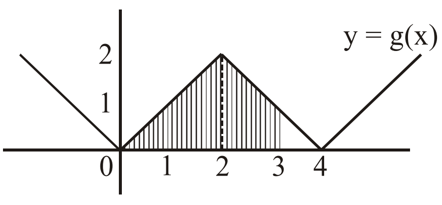
**12. Official Ans. by NTA (4)**

**Sol.** $\int_0^3 g(x)-f(x)=\int_0^3\left||x-2|-2\right|dx-\int_0^3|x-2|dx$

$=\left(\frac{1}{2}\times2\times2+1+\frac{1}{2}\times1\times1\right)-\left(\frac{1}{2}\times2\times2+\frac{1}{2}\times1\times1\right)$

$=\left(2+1+\frac{1}{2}\right)-\left(2+\frac{1}{2}\right) = 1$




**13. Official Ans. by NTA (4)**

**Sol.** $f(x) = \int_1^3\frac{\sqrt{x}\,dx}{(1+x)^2} = \int_1^{\sqrt{3}}\frac{t.2t\,dt}{(1+t^2)^2}$ $\left(\text{put } \sqrt{x}=t\right)$

$=\left(-\frac{t}{1+t^2}\right)_1^{\sqrt{3}}+(\tan^{-1}t)_1^{\sqrt{3}}$ [Appling by parts]

$=-\left(\frac{\sqrt{3}}{4}-\frac{1}{2}\right)+\frac{\pi}{3}-\frac{\pi}{4}$

$=\frac{1}{2}-\frac{\sqrt{3}}{4}+\frac{\pi}{12}$

**14. Official Ans. by NTA (3)**

**Sol.** $I = \int_{\pi/6}^{\pi/3}((2\tan^3x\cdot\sec^2x\cdot\sin^4 3x)+(3\tan^4x\ \cdot\sin^3 3x$

$\cdot\cos 3x))dx$

$\Rightarrow I = \frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}d((\sin 3x)^4(\tan x)^4)$

$\Rightarrow I = ((\sin 3x)^4(\tan x)^4)_{\pi/6}^{\pi/3}$

$\Rightarrow I = -\frac{1}{18}$

**15. Official Ans. by NTA (21)**

**Sol.** $\int_0^n\{x\}dx = n\int_0^1\{x\}dx = n\int_0^1 x\,dx = \frac{n}{2}$

$\int_0^n[x]dx = \int_0^n(x-\{x\})dx = \frac{n^2}{2}-\frac{n}{2}$

$\Rightarrow \left(\frac{n^2-n}{2}\right)^2 = \frac{n}{2}\cdot10\cdot n(n-1)$ (where $n > 1$)

$\Rightarrow \frac{n-1}{4} = 5 \Rightarrow n = 21$

**16. Official Ans. by NTA (4)**

**Sol.** $I = \int_{-\pi/2}^{\pi/2} \frac{1}{1+e^{\sin x}} dx$ ....(1)

Apply King property

$I = \int_{-\pi/2}^{\pi/2} \frac{1}{1+e^{-\sin x}} dx = \int_{-\pi/2}^{\pi/2} \frac{e^{\sin x}}{1+e^{\sin x}} dx$ ...(2)

Add (1) & (2)

$2I = \int_{-\pi/2}^{\pi/2} dx = \pi$

$I = \frac{\pi}{2}$

**17. Official Ans. by NTA (1)**

**Sol.** $I_1 = \int_0^1 (1-x^{50})^{100} dx$ and $I_2 = \int_0^1 (1-x^{50})^{101} dx$

and $I_1 = \lambda I_2$

$I_2 = \int_0^1 (1-x^{50})^{101} dx$

$I_2 = \int_0^1 (1-x^{50})(1-x^{50})^{100} dx$

$I_2 = \int_0^1 (1-x^{50}) dx - \int_0^1 x^{50}.(1-x^{50})^{100} dx$

$I_2 = I_1 - \int_0^1 \underbrace{x}_{I}.\underbrace{x^{49}.(1-x^{50})^{100} dx}_{II}$

Now apply IBP

$I_2 = I_1 - \left[x\int x^{49}.(1-x^{50})^{100} dx - \int \frac{d(x)}{dx}.\int \frac{d(x)}{dx}.\int x^{49}.(1-x^{50})^{100} dx\right]$

Let $(1 - x^{50}) = t$

$-50x^{49}dx = dt$

$I_2 = I_1 - \left[x.\left(-\frac{1}{50}\right)\frac{(1-x^{50})^{101}}{101}\Bigg|_{x=0}^{x=1} - \int_0^1 \left(-\frac{1}{50}\right)\frac{(1-x^{50})^{101}}{101} dx\right]$

$I_2 = I_1 - 0 - \frac{1}{50}.\frac{1}{101}.I_2 = I_1 - \frac{1}{5050} I_2$

$I_2 + \frac{1}{5050} I_2 = I_1 \Rightarrow \frac{5051}{5050} I_2 = I_1$

$\therefore \alpha = \frac{5050}{5051}$

$I_2 = \frac{5050}{5051} I_1$

$\because I_2 = \alpha.I_1$

**18. Official Ans. by NTA (4)**

**Sol.** $\int_1^2 e^x.x^x (2 + \log_e x) dx$

$\int_1^2 e^x (2x^x + x^x \log_e x) dx$

$\int_1^2 e^x \left(\underbrace{x^x}_{f(x)} + \underbrace{x^x(1+\log_e x)}_{f'(x)}\right) dx$

$(e^x.x^x)_1^2 = 4e^2 - e$