CONTINUITY

1. If the function f defined on $\left(-\frac{1}{3},\frac{1}{3}\right)$ by

$$f(x)=\begin{cases}\frac{1}{x}\log_e\left(\frac{1+3x}{1-2x}\right) & ,\text{when } x\neq 0\\ k & ,\text{when } x=0\end{cases}$$ is

continuous, then k is equal to_______

2. Let [t] denote the greatest integer ≤ t and $\lim_{x\to 0} x\left[\frac{4}{x}\right]=A$. Then the function, $f(x) = [x^2]\sin(\pi x)$ is discontinuous, when x is equal to :

(1) $\sqrt{A+5}$ (2) $\sqrt{A+1}$

(3) $\sqrt{A}$ (4) $\sqrt{A+21}$

3. If $f(x)=\begin{cases}\frac{\sin(a+2)x+\sin x}{x} & ; x<0\\ b & ; x=0\\ \frac{(x+3x^2)^{\frac{1}{3}}-x^{\frac{1}{3}}}{x^{\frac{4}{3}}} & ; x>0\end{cases}$

is continuous at x = 0, then a + 2b is equal to :

(1) –1 (2) 1

(3) –2 (4) 0

4. Let $f(x) = x\cdot\left[\frac{x}{2}\right]$, for –10 < x < 10, where [t] denotes the greatest integer function. Then the number of points of discontinuity of f is equal to ________.

SOLUTION

1. NTA Ans. (5.00)

Sol. $k = \lim_{x\to 0}\left(\frac{\ell n(1+3x)}{x} - \frac{\ell n(1-2x)}{x}\right)$

$k = 3 + 2 = 5$

2. NTA Ans. (2)

Sol. $A = \lim_{x\to 0} x\left[\frac{4}{x}\right] = \lim_{x\to 0} x\left(\frac{4}{x}\right) - x\left\{\frac{4}{x}\right\} = 4$

$f(x) = [x^2]\sin(\pi x)$ will be discontinuous at nonintegers

$\therefore\ x = \sqrt{A+1}$ i.e. $\sqrt{5}$

3. NTA Ans. (4)

Sol. $\lim_{x\to 0^-} f(x) = \lim_{x\to 0}\left(\frac{\sin(a+2)x}{x} + \frac{\sin x}{x}\right) = a+3$

$\lim_{x\to 0^-} f(x) = \lim_{x\to 0}\frac{(x+3x^2)^{1/3} - x^{1/3}}{x^{4/3}}$

$= \lim_{x\to 0}\frac{(1+3x)^{1/3} - 1}{x} = 1$

$f(0) = b$

for continuity at $x = 0$

$\lim_{x\to 0^-} f(x) = f(0) = \lim_{x\to 0^+} f(x)$

$\Rightarrow a + 3 = b = 1$

$\therefore\ a = -2,\ b = 1$

$\therefore\ a + 2b = 0$

4. Official Ans. by NTA (8)

Sol. $x \in (-10, 10)$

$\frac{x}{2} \in (-5, 5) \rightarrow 9$ integers

check continuity at $x = 0$

$\left.\begin{array}{l} f(0) = 0 \\ f(0^+) = 0 \\ f(0^-) = 0 \end{array}\right\}$ continuous at $x = 0$

function will be distcontinuous when

$\frac{x}{2} = \pm 4,\ \pm 3,\ \pm 2,\ \pm 1$

8 points of discontinuity