

COMPOUND ANGLE

1. Let α and β be two real roots of the equation $(k + 1) \tan^2 x - \sqrt{2} \cdot \lambda \tan x = (1 - k)$, where $k(\neq -1)$ and λ are real numbers. If $\tan^2(\alpha + \beta) = 50$, then a value of λ is ;

- (1) 5 (2) 10 (3) $5\sqrt{2}$ (4) $10\sqrt{2}$

2. If $\frac{\sqrt{2} \sin \alpha}{\sqrt{1 + \cos 2\alpha}} = \frac{1}{7}$ and $\sqrt{\frac{1 - \cos 2\beta}{2}} = \frac{1}{\sqrt{10}}$, $\alpha,$

$\beta \in \left(0, \frac{\pi}{2}\right)$, then $\tan(\alpha + 2\beta)$ is equal to _____.

3. The value of

$\cos^3\left(\frac{\pi}{8}\right) \cdot \cos\left(\frac{3\pi}{8}\right) + \sin^3\left(\frac{\pi}{8}\right) \cdot \sin\left(\frac{3\pi}{8}\right)$ is :

- (1) $\frac{1}{4}$ (2) $\frac{1}{\sqrt{2}}$ (3) $\frac{1}{2\sqrt{2}}$ (4) $\frac{1}{2}$

4. If $L = \sin^2\left(\frac{\pi}{16}\right) - \sin^2\left(\frac{\pi}{8}\right)$ and $M = \cos^2$

$\left(\frac{\pi}{16}\right) - \sin^2\left(\frac{\pi}{8}\right)$, then :

(1) $M = \frac{1}{2\sqrt{2}} + \frac{1}{2} \cos \frac{\pi}{8}$

(2) $L = \frac{1}{4\sqrt{2}} - \frac{1}{4} \cos \frac{\pi}{8}$

(3) $M = \frac{1}{4\sqrt{2}} + \frac{1}{4} \cos \frac{\pi}{8}$

(4) $L = -\frac{1}{2\sqrt{2}} + \frac{1}{2} \cos \frac{\pi}{8}$

SOLUTION

1. NTA Ans. (2)

$$\text{Sol. } \tan\alpha + \tan\beta = \frac{\lambda\sqrt{2}}{k+1}$$

$$\tan\alpha \cdot \tan\beta = \frac{k-1}{k+1}$$

$$\tan(\alpha + \beta) = \frac{\frac{\lambda\sqrt{2}}{k+1}}{1 - \frac{k-1}{k+1}} = \frac{\lambda\sqrt{2}}{2} = \frac{\lambda}{\sqrt{2}}$$

$$\Rightarrow \frac{\lambda^2}{2} = 50 \Rightarrow \lambda = 10 \text{ \& } -10$$

2. NTA Ans. (1)

$$\text{Sol. } \frac{\sqrt{2}\sin\alpha}{\sqrt{2}\cos\alpha} = \frac{1}{7} \Rightarrow \tan\alpha = \frac{1}{7}$$

$$\sin\beta = \frac{1}{\sqrt{10}} \Rightarrow \tan\beta = \frac{1}{3} \Rightarrow \tan 2\beta = \frac{3}{4}$$

$$\tan(\alpha + 2\beta) = \frac{\tan\alpha + \tan 2\beta}{1 - \tan\alpha \tan 2\beta} = 1$$

Ans. 1.00

3. NTA Ans. (3)

$$\text{Sol. } \cos^3 \frac{\pi}{8} \cdot \sin \frac{\pi}{8} + \sin^3 \frac{\pi}{8} \cdot \cos \frac{\pi}{8}$$

$$= \sin \frac{\pi}{8} \cdot \cos \frac{\pi}{8} = \frac{1}{2} \sin \frac{\pi}{4} = \frac{1}{2\sqrt{2}}$$

4. Official Ans. by NTA (1)

$$\text{Sol. } L = \sin^2 \left(\frac{\pi}{16} \right) - \sin^2 \left(\frac{\pi}{8} \right)$$

$$\left(\because \sin^2 \theta = \frac{1 - \cos 2\theta}{2} \right)$$

$$\Rightarrow L = \left(\frac{1 - \cos(\pi/8)}{2} \right) - \left(\frac{1 - \cos(\pi/4)}{2} \right)$$

$$L = \frac{1}{2} \left[\cos \left(\frac{\pi}{4} \right) - \cos \left(\frac{\pi}{8} \right) \right]$$

$$L = \frac{1}{2\sqrt{2}} - \frac{1}{2} \cos \left(\frac{\pi}{8} \right)$$

$$M = \cos^2 \left(\frac{\pi}{16} \right) - \sin^2 \left(\frac{\pi}{8} \right)$$

$$M = \frac{1 + \cos(\pi/8)}{2} - \frac{1 - \cos(\pi/4)}{2}$$

$$M = \frac{1}{2} \cos \left(\frac{\pi}{8} \right) + \frac{1}{2\sqrt{2}}$$