CIRCLE

1. Let the tangents drawn from the origin to the circle, $x^2 + y^2 - 8x - 4y + 16 = 0$ touch it at the points A and B. The $(AB)^2$ is equal to :

(1) $\frac{52}{5}$ (2) $\frac{32}{5}$

(3) $\frac{56}{5}$ (4) $\frac{64}{5}$

2. If a line, $y = mx + c$ is a tangent to the circle, $(x - 3)^2 + y^2 = 1$ and it is perpendicular to a line L_1, where L_1 is the tangent to the circle, $x^2 + y^2 = 1$ at the point $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$, then

(1) $c^2 - 6c + 7 = 0$ (2) $c^2 + 6c + 7 = 0$

(3) $c^2 + 7c + 6 = 0$ (4) $c^2 - 7c + 6 = 0$

3. If the curves, $x^2 - 6x + y^2 + 8 = 0$ and $x^2 - 8y + y^2 + 16 - k = 0$, $(k > 0)$ touch each other at a point, then the largest value of k is _____.

4. The number of integral values of k for which the line, $3x + 4y = k$ intersects the circle, $x^2 + y^2 - 2x - 4y + 4 = 0$ at two distinct points is _____.

5. The diameter of the circle, whose centre lies on the line $x + y = 2$ in the first quadrant and which touches both the lines $x = 3$ and $y = 2$, is _____ .

6. The circle passing through the intersection of the circles, $x^2 + y^2 - 6x = 0$ and $x^2 + y^2 - 4y = 0$, having its centre on the line, $2x - 3y + 12 = 0$, also passes through the point :

(1) (1, –3) (2) (–1, 3)

(3) (–3, 1) (4) (–3, 6)

7. Let PQ be a diameter of the circle $x^2+y^2=9$. If α and β are the lengths of the perpendiculars from P and Q on the straight line, $x + y = 2$ respectively, then the maximum value of $\alpha\beta$ is ____

8. If the length of the chord of the circle, $x^2 + y^2 = r^2$ $(r > 0)$ along the line, $y - 2x = 3$ is r, then r^2 is equal to:

(1) $\frac{9}{5}$ (2) $\frac{12}{5}$

(3) 12 (4) $\frac{24}{5}$

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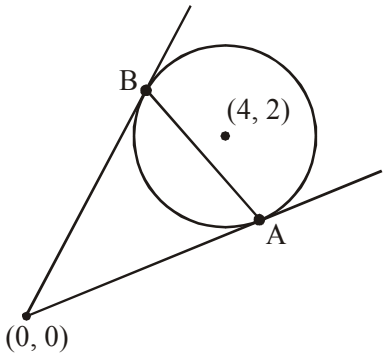
SOLUTION

1. NTA Ans. (4)

Sol. $R = \sqrt{16+4-16} = 2$

$L = \sqrt{S_1} = 4$



AB(Chord of contact) = $\dfrac{2LR}{\sqrt{L^2+R^2}} = \dfrac{8}{\sqrt{5}}$

$(AB)^2 = \dfrac{64}{5}$

2. NTA Ans. (2)

Sol. Slope of tangent to $x^2 + y^2 = 1$ at $P\left(\dfrac{1}{\sqrt{2}}, \dfrac{1}{\sqrt{2}}\right)$

$2x + 2yy' = 0 \Rightarrow m_T\big|_P = -1$

$y = mx + c$ is tangent to $(x-3)^2 + y^2 = 1$

$y = x + c$ is tangent to $(x-3)^2 + y^2 = 1$

$\left|\dfrac{c+3}{\sqrt{2}}\right| = 1 \Rightarrow c^2 + 6c + 7 = 0$

(2) Option

3. NTA Ans. (36)

Sol. Common tangent is $S_1 - S_2 = 0$

$\Rightarrow -6x + 8y - 8 + k = 0$

Use p = r for Ist circle

$\Rightarrow \dfrac{|-18-8+k|}{10} = 1$

$\Rightarrow k = 36$ or $16 \quad \Rightarrow k_{max} = 36$

4. Official Ans. by NTA (9.00)

Sol. Circle $x^2 + y^2 - 2x - 4y + 4 = 0$

$\Rightarrow (x-1)^2 + (y-2)^2 = 1$

Centre : (1, 2) radius = 1

line $3x + 4y - k = 0$ intersects the circle at two distinct points.

$\Rightarrow$ distance of centre from the line < radius

$\Rightarrow \left|\dfrac{3\times1+4\times2-k}{\sqrt{3^2+4^2}}\right| < 1$

$\Rightarrow |11 - k| < 5$

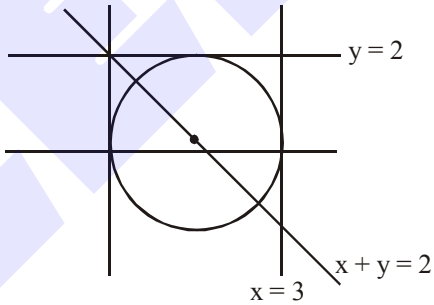
$\Rightarrow 6 < k < 16$

$\Rightarrow k \in \{7, 8, 9, ... 15\}$ since $k \in I$

Number of K is $\boxed{9}$

5. Official Ans. by NTA (3)

Sol.



$\because$ center lies on $x + y = 2$ and in 1st quadrant

center = $(\alpha, 2-\alpha)$

where $\alpha > 0$ and $2 - \alpha > 0 \Rightarrow 0 < \alpha < 2$

$\because$ circle touches $x = 3$ and $y = 2$

$\Rightarrow |3 - \alpha| = |2 - (2 - \alpha)|$ = radius

$\Rightarrow |3 - \alpha| = |\alpha| \Rightarrow \alpha = \dfrac{3}{2}$

$\therefore$ radius = α

$\Rightarrow$ Diameter = $2\alpha = 3$.

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6. Official Ans. by NTA (4)

Sol. Let S be the circle pasing through point of intersection of S_1 & S_2

$\therefore\ S = S_1 + \lambda S_2 = 0$

$\Rightarrow S : (x^2 + y^2 - 6x) + \lambda\,(x^2 + y^2 - 4y) = 0$

$$\Rightarrow S : x^2 + y^2 - \left(\frac{6}{1+\lambda}\right)x - \left(\frac{4\lambda}{1+\lambda}\right)y = 0 \ldots(1)$$

Centre $\left(\frac{3}{1+\lambda}, \frac{2\lambda}{1+\lambda}\right)$ lies on

$2x - 3y + 12 = 0 \Rightarrow \lambda = -3$

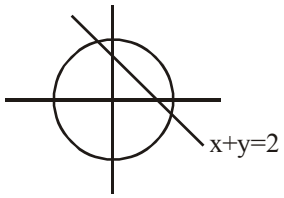
put in (1) $\Rightarrow S : x^2 + y^2 + 3x - 6y = 0$

Now check options point (–3, 6)

lies on S.

7. Official Ans. by NTA (7)

Sol. Let P $(3\cos\theta, 3\sin\theta)$



Q $(-3\cos\theta, -3\sin\theta)$

$$\Rightarrow \alpha\beta = \frac{|(3\cos\theta + 3\sin\theta)^2 - 4|}{2}$$

$$\Rightarrow \alpha\beta = \frac{5 + 9\sin 2\theta}{2} \le 7$$

8. Official Ans. by NTA (2)

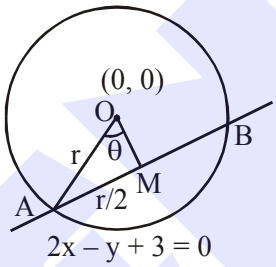
Sol. Let chord

AB = r

$\because\ \Delta AOM$ is right angled triangle

$\therefore\ OM = \frac{r\sqrt{3}}{2}$ = perpendicular distance of line AB from (0,0)

$$\frac{r\sqrt{3}}{2} = \left|\frac{3}{\sqrt{5}}\right|$$



$$r^2 = \frac{12}{5}$$

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