BINOMIAL THEOREM

1. The number of ordered pairs (r, k) for which $6 \cdot {}^{35}C_r = (k^2 - 3) \cdot {}^{36}C_{r+1}$, where k is an integer, is :

(1) 3 (2) 2
(3) 4 (4) 6

2. The coefficient of x^7 in the expression $(1 + x)^{10} + x(1 + x)^9 + x^2(1 + x)^8 + \ldots + x^{10}$ is :

(1) 120 (2) 330
(3) 210 (4) 420

3. If the sum of the coefficients of all even powers of x in the product $(1 + x + x^2 + \ldots + x^{2n})(1 - x + x^2 - x^3 + \ldots + x^{2n})$ is 61, then n is equal to ________.

4. If α and β be the coefficients of x^4 and x^2 respectively in the expansion of $\left(x+\sqrt{x^2-1}\right)^6+\left(x-\sqrt{x^2-1}\right)^6$, then

(1) $\alpha + \beta = 60$ (2) $\alpha + \beta = -30$
(3) $\alpha - \beta = -132$ (4) $\alpha - \beta = 60$

5. In the expansion of $\left(\frac{x}{\cos\theta}+\frac{1}{x\sin\theta}\right)^{16}$, if ℓ_1 is the least value of the term independent of x when $\frac{\pi}{8}\le\theta\le\frac{\pi}{4}$ and ℓ_2 is the least value of the term independent of x when $\frac{\pi}{16}\le\theta\le\frac{\pi}{8}$, then the ratio $\ell_2 : \ell_1$ is equal to :

(1) 1 : 8 (2) 1 : 16
(3) 8 : 1 (4) 16 : 1

6. If $C_r \equiv {}^{25}C_r$ and $C_0 + 5.C_1 + 9.C_2 + \ldots + (101).C_{25} = 2^{25}.k$, then k is equal to _____.

7. The coefficient of x^4 is the expansion of $(1 + x + x^2)^{10}$ is ______.

8. Let $\alpha > 0$, $\beta > 0$ be such that $\alpha^3 + \beta^2 = 4$. If the maximum value of the term independent of x in the binomial expansion of $\left(\alpha x^{\frac{1}{9}}+\beta x^{-\frac{1}{6}}\right)^{10}$ is 10k, then k is equal to :

(1) 176 (2) 336
(3) 352 (4) 84

9. For a positive integer n, $\left(1+\frac{1}{x}\right)^n$ is expanded in increasing powers of x. If three consecutive coefficients in this expansion are in the ratio, 2 : 5 : 12, then n is equal to________.

10. If the number of integral terms in the expansion of $(3^{1/2} + 5^{1/8})^n$ is exactly 33, then the least value of n is :

(1) 264 (2) 256
(3) 128 (4) 248

11. If the term independent of x in the expansion of $\left(\frac{3}{2}x^2-\frac{1}{3x}\right)^9$ is k, then 18 k is equal to :

(1) 9 (2) 11
(3) 5 (4) 7

12. The value of $\sum_{r=0}^{20} {}^{50-r}C_6$ is equal to :

(1) ${}^{51}C_7 + {}^{30}C_7$ (2) ${}^{51}C_7 - {}^{30}C_7$
(3) ${}^{50}C_7 - {}^{30}C_7$ (4) ${}^{50}C_6 - {}^{30}C_6$

13. Let $(2x^2 + 3x + 4)^{10} = \sum_{r=0}^{20} a_r x^r$. Then $\frac{a_7}{a_{13}}$ is equal to _________.

14. If for some positive integer n, the coefficients of three consecutive terms in the binomial expansion of $(1+x)^{n+5}$ are in the ratio 5 : 10 : 14, then the largest coefficient in this expansion is :-

(1) 792 (2) 252
(3) 462 (4) 330

15. The natural number m, for which the coefficient of x in the binomial expansion of $\left(x^m + \frac{1}{x^2}\right)^{22}$ is 1540, is ________.

16. The coefficient of x^4 in the expansion of $(1 + x + x^2 + x^3)^6$ in powers of x, is ______.

17. If {p} denotes the fractional part of the number p, then $\left\{\frac{3^{200}}{8}\right\}$, is equal to

(1) $\frac{1}{8}$ (2) $\frac{5}{8}$

(3) $\frac{3}{8}$ (4) $\frac{7}{8}$

18. If the constant term in the binomial expansion of $\left(\sqrt{x} - \frac{k}{x^2}\right)^{10}$ is 405, then |k| equals :

(1) 2 (2) 1

(3) 3 (4) 9

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SOLUTION

1. NTA Ans. (3)

Sol. $6\times{}^{35}C_r = (k^2-3)\,{}^{36}C_{r+1}$

$k^2 - 3 > 0 \Rightarrow k^2 > 3$

$k^2 - 3 = \dfrac{6\times{}^{35}C_r}{{}^{36}C_{r+1}} = \dfrac{r+1}{6}$

Possible values of r for integral values of k, are

r = 5, 35

number of ordered pairs are 4

(5, 2), (5, –2), (35, 3), (35, –3)

2. NTA Ans. (2)

Sol. Coefficient of x^7 is

${}^{10}C_7 + {}^{9}C_6 + {}^{8}C_5 + \ldots + {}^{4}C_1 + {}^{3}C_0$

$\underbrace{{}^{4}C_0 + {}^{4}C_1}_{{}^{5}C_1} + {}^{5}C_2 + \ldots + {}^{10}C_7 = {}^{11}C_7 = 330$

3. NTA Ans. (30)

Sol. Let $(1 + x + x^2 + \ldots + x^{2n})(1 - x + x^2 - x^3 + \ldots + x^{2n})$

$= a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + \ldots + a_{4n}x^{4n}$

So,

$a_0 + a_1 + a_2 + \ldots + a_{4n} = 2n + 1$ …(1)

$a_0 - a_1 + a_2 - a_3 \ldots + a_{4n} = 2n + 1$ …(2)

$\Rightarrow a_0 + a_2 + a_4 + \ldots + a_{4n} = 2n + 1$

$\Rightarrow 2n + 1 = 61 \qquad \Rightarrow n = 30$

4. NTA Ans. (3)

Sol. $2[{}^6C_0x^6 + {}^6C_2x^4(x^2-1) + {}^6C_4x^2(x^2-1)^2 + {}^6C_6(x^2-1)^3]$

$\alpha = -96$ & $\beta = 36$

$\therefore \alpha - \beta = -132$

(3) Option

5. NTA Ans. (4)

Sol. $T_{r+1} = {}^{16}C_r\left(\dfrac{x}{\cos\theta}\right)^{16-r}\left(\dfrac{1}{x\sin\theta}\right)^r$

$= {}^{16}C_r\,(x)^{16-2r} \times \dfrac{1}{(\cos\theta)^{16-r}(\sin\theta)^r}$

For independent of x; $16 - 2r = 0 \Rightarrow r = 8$

$\Rightarrow T_9 = {}^{16}C_8\dfrac{1}{\cos^8\theta\sin^8\theta}$

$= {}^{16}C_8\dfrac{2^8}{(\sin 2\theta)^8}$

for $\theta \in \left[\dfrac{\pi}{8}, \dfrac{\pi}{4}\right]$ ℓ_1 is least for $\theta_1 = \dfrac{\pi}{4}$

for $\theta \in \left[\dfrac{\pi}{16}, \dfrac{\pi}{8}\right]$ ℓ_2 is least for $\theta_2 = \dfrac{\pi}{8}$

$\dfrac{\ell_2}{\ell_1} = \dfrac{(\sin 2\theta_1)^8}{(\sin 2\theta_2)^8} = (\sqrt{2})^8 = \dfrac{16}{1}$

6. NTA Ans. (51)

Sol. $S = 1.{}^{25}C_0 + 5.{}^{25}C_1 + 9.{}^{25}C_2 + \ldots. + (101){}^{25}C_{25}$

$S = 101\,{}^{25}C_{25} + 97\,{}^{25}C_1 + \ldots\ldots\ldots + 1\,{}^{25}C_{25}$

$2S = (102)(2^{25})$

$S = 51\,(2^{25})$

7. NTA Ans. (615.00)

Sol. $(1 + x + x^2)^{10}$

$= {}^{10}C_0 + {}^{10}C_1x(1 + x) + {}^{10}C_2x^2(1 + x)^2$

$+ {}^{10}C_3x^3(1 + x)^3 + {}^{10}C_4x^4(1 + x)^4 + \ldots\ldots.$

Coeff. of $x^4 = {}^{10}C_2 + {}^{10}C_3 \times {}^3C_1 + {}^{10}C_4 = 615.$

8. Official Ans. by NTA (2)

Sol. Let t_{r+1} denotes

r + 1th term of $\left(\alpha x^{\frac{1}{9}} + \beta x^{-\frac{1}{6}}\right)^{10}$

$t_{r+1} = {}^{10}C_r \alpha^{10-r} (x)^{\frac{10-r}{9}} . \beta^r x^{-\frac{r}{6}}$

$= {}^{10}C_r \alpha^{10-r} \beta^r (x)^{\frac{10-r}{9} - \frac{r}{6}}$

If t_{r+1} is independent of x

$\frac{10-r}{9} - \frac{r}{6} = 0 \Rightarrow r = 4$

maximum value of t_5 is 10 K (given)

$\Rightarrow {}^{10}C_4 \alpha^6 \beta^4$ is maximum

By AM $\geq$ GM (for positive numbers)

$$\frac{\frac{\alpha^3}{2} + \frac{\alpha^3}{2} + \frac{\beta^2}{2} + \frac{\beta^2}{2}}{4} \geq \left(\frac{\alpha^6\beta^4}{16}\right)^{\frac{1}{4}}$$

$\Rightarrow \boxed{\alpha^6\beta^4 \leq 16}$

So, 10 K = ${}^{10}C_4 16$

$\Rightarrow$ K = 336

9. Official Ans. by NTA (118)

Sol. ${}^nC_{r-1} : {}^nC_r : {}^nC_{r+1} = 2:5:12$

Now $\frac{{}^nC_{r-1}}{{}^nC_r} = \frac{2}{5}$

$\Rightarrow 7r = 2n + 2$ ...(1)

$\frac{{}^nC_r}{{}^nC_{r+1}} = \frac{5}{12}$

$\Rightarrow 17r = 5n - 12$ ...(2)

On solving (1) & (2)

$\Rightarrow$ n = 118

10. Official Ans. by NTA (2)

Sol. $T_{r+1} = {}^nC_r (3)^{\frac{n-r}{2}} (5)^{\frac{r}{8}}$ $(n \geq r)$

Clearly r should be a multiple of 8.

$\because$ there are exactly 33 integral terms

Possible values of r can be

0, 8, 16, ... , 32 × 8

$\therefore$ least value of n = 256.

11. Official Ans. by NTA (4)

Sol. $T_{r+1} = {}^9C_r \left(\frac{3}{2}x^2\right)^{9-r} \left(-\frac{1}{3x}\right)^r$

$T_{r+1} = {}^9C_r \left(\frac{3}{2}\right)^{9-r} \left(-\frac{1}{3}\right)^r x^{18-3r}$

For independent of x

18 – 3r = 0, r = 6

$\therefore T_7 = {}^9C_6 \left(\frac{3}{2}\right)^3 \left(-\frac{1}{3}\right)^6 = \frac{21}{54} = k$

$\therefore 18k = \frac{21}{54} \times 18 = 7$

12. Official Ans. by NTA (2)

Sol. $\sum_{r=0}^{20} {}^{50-r}C_6 = {}^{50}C_6 + {}^{49}C_6 + {}^{48}C_6 + ... + {}^{30}C_6$

$= {}^{50}C_6 + {}^{49}C_6 + ... + {}^{31}C_6 + \left({}^{30}C_6 + {}^{30}C_7\right) - {}^{30}C_7$

$= {}^{50}C_6 + {}^{49}C_6 + ... + \left({}^{31}C_6 + {}^{31}C_7\right) - {}^{30}C_7$

$= {}^{50}C_6 + {}^{50}C_7 - {}^{30}C_7$

$= {}^{51}C_7 - {}^{30}C_7$

$\boxed{{}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r}$

13. Official Ans. by NTA (8)

Sol. Given $(2x^2 + 3x + 4)^{10} = \sum_{r=0}^{20} a_r x^r$ .... (1)

replace x by $\frac{2}{x}$ in above identity :-

$$\frac{2^{10}\left(2x^2 + 3x + 4\right)^{10}}{x^{20}} = \sum_{r=0}^{20} \frac{a_r 2^r}{x^r}$$

$$\Rightarrow 2^{10} \sum_{r=0}^{20} a_r x^r = \sum_{r=0}^{20} a_r 2^r x^{(20-r)} \text{ (from (i))}$$

now, comparing coefficient of x^7 from both sides

(take r = 7 in L.H.S. & r = 13 in R.H.S.)

$2^{10} a_7 = a_{13} 2^{13} \Rightarrow \frac{a_7}{a_{13}} = 2^3 = 8$

14. Official Ans. by NTA (3)

Sol. Let n + 5 = N

$$^{N}C_{r-1} : {}^{N}C_r : {}^{N}C_{r+1} = 5 : 10 : 14$$

$$\Rightarrow \frac{^{N}C_r}{^{N}C_{r-1}} = \frac{N+1-r}{r} = 2$$

$$\frac{^{N}C_{r+1}}{^{N}C_r} = \frac{N-r}{r+1} = \frac{7}{5}$$

$\Rightarrow$ r = 4, N = 11

$\Rightarrow (1 + x)^{11}$

Largest coefficient = $^{11}C_6 = 462$

15. Official Ans. by NTA (13)

Sol. $T_{r+1} = {}^{22}C_r (x^m)^{22-r}\left(\frac{1}{x^2}\right)^r = {}^{22}C_r x^{22m-mr-2r}$

$= {}^{22}C_r\ x$

$\because\ {}^{22}C_3 = {}^{22}C_{19} = 1540$

$\therefore$ r = 3 or 19

22m – mr – 2r = 1

$$m = \frac{2r+1}{22-5}$$

$$r = 3,\ m = \frac{7}{19} \notin N$$

$$r = 19,\ m = \frac{38+1}{22-19} = \frac{39}{3} = 13$$

m = 13

16. Official Ans. by NTA (120.00)

Sol. $(1 + x + x^2 + x^3)^6 = \left((1+x)(1+x^2)\right)^6$

$= (1 + x)^6 (1 + x^2)^6$

$$= \sum_{r=0}^{6} {}^6C_r x^r \sum_{r=0}^{6} {}^6C_t x^{2t}$$

$$= \sum_{r=0}^{6}\sum_{t=0}^{6} {}^6C_r\, {}^6C_t\, x^{r+2t}$$

For coefficient of $x^4 \Rightarrow r + 2t = 4$

r	t
0	2
2	1
4	0

Coefficient of x^4

$= {}^6C_0\, {}^6C_2 + {}^6C_2\, {}^6C_1 + {}^6C_4\, {}^6C_0$

= 120

17. Official Ans. by NTA (1)

Sol. $$\left\{\frac{3^{200}}{8}\right\} = \left\{\frac{(3^2)^{100}}{8}\right\} = \left\{\frac{(1+8)^{100}}{8}\right\}$$

$$= \left\{\frac{1 + {}^{100}C_1.8 + {}^{100}C_2.8^2 + \ldots + {}^{100}C_{100}8^{100}}{8}\right\}$$

$$= \left\{\frac{1+8m}{8}\right\} = \frac{1}{8}$$

18. Official Ans. by NTA (3)

Sol. $$\left(\sqrt{x} - \frac{k}{x^2}\right)^{10}$$

$$T_{r+1} = {}^{10}C_r \left(\sqrt{x}\right)^{10-r}\left(\frac{-k}{x^2}\right)^r$$

$$T_{r+1} = {}^{10}C_r.x^{\frac{10-r}{2}}.(-k)^r.x^{-2r}$$

$$T_{r+1} = {}^{10}C_r x^{\frac{10-5r}{2}}(-k)^r$$

Constant term : $\frac{10-5r}{2} = 0 \Rightarrow r = 2$

$T_3 = {}^{10}C_2.(-k)^2 = 405$

$$k^2 = \frac{405}{45} = 9$$

$k = \pm 3 \Rightarrow |k| = 3$