3D

1. If the foot of the perpendicular drawn from the point (1, 0, 3) on a line passing through (α, 7, 1) is $\left(\frac{5}{3}, \frac{7}{3}, \frac{17}{3}\right)$, then α is equal to______

2. Let P be a plane passing through the points (2, 1, 0), (4, 1, 1) and (5, 0, 1) and R be any point (2, 1, 6). Then the image of R in the plane P is :

(1) (6, 5, –2) (2) (4, 3, 2)
(3) (3, 4, –2) (4) (6, 5, 2)

3. The mirror image of the point (1,2,3) in a plane is $\left(-\frac{7}{3}, -\frac{4}{3}, -\frac{1}{3}\right)$. Which of the following points lies on this plane ?

(1) (–1, –1, –1) (2) (–1, –1, 1)
(3) (1, 1, 1) (4) (1, –1, 1)

4. The shortest distance between the lines $\frac{x-3}{3}=\frac{y-8}{-1}=\frac{z-3}{1}$ and $\frac{x+3}{-3}=\frac{y+7}{2}=\frac{z-6}{4}$ is

(1) $\frac{7}{2}\sqrt{30}$ (2) $3\sqrt{30}$
(3) 3 (4) $2\sqrt{30}$

5. If the distance between the plane, 23x – 10y – 2z + 48 = 0 and the plane containing the lines $\frac{x+1}{2}=\frac{y-3}{4}=\frac{z+1}{3}$ and $\frac{x+3}{2}=\frac{y+2}{6}=\frac{z-1}{\lambda}(\lambda \in R)$ is equal to $\frac{k}{\sqrt{633}}$, then k is equal to ______.

6. If for some α and β in R, the intersection of the following three places

$x + 4y - 2z = 1$
$x + 7y - 5z = \beta$
$x + 5y + \alpha z = 5$

is a line in R^3, then $\alpha + \beta$ is equal to :

(1) 10 (2) –10
(3) 2 (4) 0

7. The plane passing through the points (1, 2, 1), (2, 1, 2) and parallel to the line, 2x = 3y, z = 1 also passes through the point :

(1) (0, 6, –2) (2) (–2, 0, 1)
(3) (0, –6, 2) (4) (2, 0, –1)

8. A plane passing through the point (3, 1,1) contains two lines whose direction ratios are 1, –2, 2 and 2, 3, –1 respectively. If this plane also passes through the point (α, –3, 5), then α is equal to:

(1) –10 (2) 5
(3) 10 (4) –5

9. The foot of the perpendicular drawn from the point (4, 2, 3) to the line joining the points (1, –2, 3) and (1, 1, 0) lies on the plane :

(1) x + 2y – z = 1 (2) x – 2y + z = 1
(3) x – y – 2z = 1 (4) 2x + y – z = 1

10. The plane which bisects the line joining the points (4, –2, 3) and (2, 4, –1) at right angles also passes through the point :

(1) (4, 0, –1) (2) (4, 0, 1)
(3) (0, 1, –1) (4) (0, –1, 1)

11. If the equation of a plane P, passing through the intesection of the planes, x + 4y – z + 7 = 0 and 3x + y + 5z = 8 is ax + by + 6z = 15 for some a, b $\in$ R, then the distance of the point (3, 2, –1) from the plane P is __________.

12. The distance of the point (1, –2, 3) from the plane x–y+z = 5 measured parallel to the line $\frac{x}{2}=\frac{y}{3}=\frac{z}{-6}$ is:

(1) 7 (2) 1
(3) $\frac{1}{7}$ (4) $\frac{7}{5}$

13. If (a, b, c) is the image of the point (1, 2, –3) in the line, $\frac{x+1}{2}=\frac{y-3}{-2}=\frac{z}{-1}$, then a + b + c is equal to

(1) –1 (2) 2

(3) 3 (4) 1

14. If for some $\alpha \in R$, the lines

$L_1 : \frac{x+1}{2}=\frac{y-2}{-1}=\frac{z-1}{1}$ and

$L_2 : \frac{x+2}{\alpha}=\frac{y+1}{5-\alpha}=\frac{z+1}{1}$ are coplanar, then the line L_2 passes through the point :

(1) (–2, 10, 2) (2) (10, 2, 2)

(3) (10, –2, –2) (4) (2, –10, –2)

15. The shortest distance between the lines $\frac{x-1}{0}=\frac{y+1}{-1}=\frac{z}{1}$ and x + y + z + 1 = 0, 2x – y + z + 3 = 0 is :

(1) $\frac{1}{2}$ (2) 1

(3) $\frac{1}{\sqrt{2}}$ (4) $\frac{1}{\sqrt{3}}$

16. A plane P meets the coordinate axes at A, B and C respectively. The centroid of ΔABC is given to be (1, 1, 2). Then the equation of the line through this centroid and perpendicular to the plane P is:

(1) $\frac{x-1}{1}=\frac{y-1}{2}=\frac{z-2}{2}$

(2) $\frac{x-1}{2}=\frac{y-1}{2}=\frac{z-2}{1}$

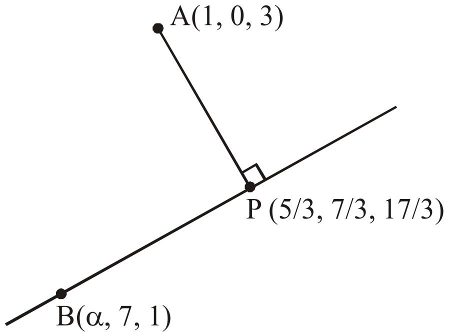
(3) $\frac{x-1}{2}=\frac{y-1}{1}=\frac{z-2}{1}$

(4) $\frac{x-1}{1}=\frac{y-1}{1}=\frac{z-2}{2}$

SOLUTION

1. NTA Ans. (4.00)

Sol. D.R. of BP = $<\frac{5}{3}-\alpha, \frac{7}{3}-7, \frac{17}{3}-1>$



D.R. of AP = $<\frac{5}{3}-1, \frac{7}{3}-0, \frac{17}{3}-3>$

BP $\perp^r$ AP

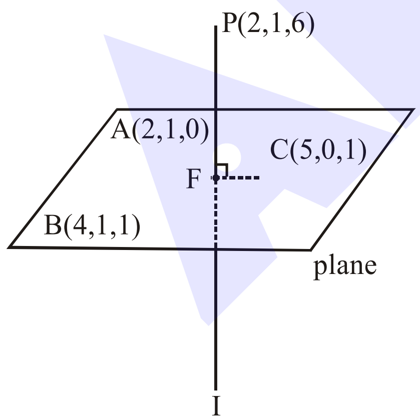
$\Rightarrow \alpha = 4$

2. NTA Ans. (1)

Sol. Plane passing through : (2, 1, 0), (4, 1, 1) and (5, 0, 1)

$$\begin{vmatrix} x-2 & y-1 & z \\ 2 & 0 & 1 \\ 3 & -1 & 1 \end{vmatrix} = 0$$

$\Rightarrow x + y - 2z = 3$



Let I and F are respectively image and foot of perpendicular of point P in the plane.

eq^n of line PI $\frac{x-2}{1} = \frac{y-1}{1} = \frac{z-6}{-2} = \lambda(\text{say})$

Let I $(\lambda + 2, \lambda + 1, -2\lambda + 6)$

$\Rightarrow F\left(2+\frac{\lambda}{2}, 1+\frac{\lambda}{2}, -\lambda+6\right)$

F lies in the plane

$\Rightarrow 2+\frac{\lambda}{2}+1+\frac{\lambda}{2}+2\lambda-12-3=0$

$\Rightarrow \lambda = 4$

$\Rightarrow$ I (6, 5, –2)

3. NTA Ans. (4)

Sol. Point on plane $R\left(\frac{-2}{3}, \frac{1}{3}, \frac{4}{3}\right)$

Normal vector of plane is $\frac{10}{3}\hat{i}+\frac{10}{3}\hat{j}+\frac{10}{3}\hat{k}$

Equation of require plane is $x + y + z = 1$

Hence (1, –1, 1) lies on plane

(4) Option

4. NTA Ans. (2)

Sol. Shortest distance $= \dfrac{\left\|\begin{matrix} 6 & 15 & -3 \\ 3 & -1 & 1 \\ -3 & 2 & 4 \end{matrix}\right\|}{\sqrt{11\times 29-49}} = \frac{270}{\sqrt{270}}$

$= \sqrt{270} = 3\sqrt{30}$

5. NTA Ans. (3)

Sol. If $\lambda = -7$, then planes will be parallel & distance between them will be $\frac{3}{\sqrt{633}} \Rightarrow k = 3$

But if $\lambda \neq -7$, then planes will be intersecting & distance between them will be 0

6. NTA Ans. (1)

Sol. For planes to intersect on a line

$\Rightarrow$ there should be infinite solution of the given system of equations

for infinite solutions

$$\Delta = \begin{vmatrix} 1 & 4 & -2 \\ 1 & 7 & -5 \\ 1 & 5 & \alpha \end{vmatrix} = 0 \Rightarrow 3\alpha + 9 = 0 \Rightarrow \alpha = -3$$

$$\Delta_z = \begin{vmatrix} 1 & 4 & 1 \\ 1 & 7 & \beta \\ 1 & 5 & 5 \end{vmatrix} = 0 \Rightarrow 13 - \beta = 0 \Rightarrow \beta = 13$$

Also for $\alpha = -3$ and $b = 13$ $\Delta_x = \Delta_y = 0$

$\therefore\ \alpha + \beta = -3 + 13 = 10$

7. Official Ans. by NTA (2)

Sol. Two points on the line (L say) $\frac{x}{3} = \frac{y}{2}, z = 1$ are (0, 0, 1) & (3, 2, 1)

So dr's of the line is < 3, 2, 0 >

Line passing through (1, 2, 1), parallel to L and coplanar with given plane is

$\vec{r} = \hat{i} + 2\hat{j} + \hat{k} + t(3\hat{i} + 2\hat{j}), t \in R$ (–2, 0, 1) satisfies the line (for t = –1)

$\Rightarrow$ (–2, 0, 1) lies on given plane.

Answer of the question is (2)

We can check other options by finding eqution of plane

$$\text{Equation plane : } \begin{vmatrix} x-1 & y-2 & z-1 \\ 1+2 & 2-0 & 1-1 \\ 2+2 & 1-0 & 2-1 \end{vmatrix} = 0$$

$\Rightarrow 2(x-1) - 3(y-2) - 5(z-1) = 0$

$\Rightarrow 2x - 3y - 5z + 9 = 0$

8. Official Ans. by NTA (2)

Sol. Hence normal is $\perp^r$ to both the lines so normal vector to the plane is

$$\vec{n} = (\hat{i} - 2\hat{j} + 2\hat{k}) \times (2\hat{i} + 3\hat{j} - \hat{k})$$

$$\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 2 \\ 2 & 3 & -1 \end{vmatrix} = \hat{i}(2-6) - \hat{j}(-1-4) + \hat{k}(3+4)$$

$$\vec{n} = -4\hat{i} + 5\hat{j} + 7\hat{k}$$

Now equation of plane passing through (3,1,1) is

$\Rightarrow -4(x-3) + 5(y-1) + 7(z-1) = 0$

$\Rightarrow -4x + 12 + 5y - 5 + 7z - 7 = 0$

$\Rightarrow -4x + 5y + 7z = 0$ ...(1)

Plane is also passing through $(\alpha, -3, 5)$ so this point satisfies the equation of plane so put in equation (1)

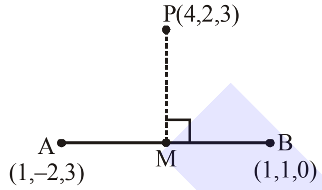
$-4\alpha + 5 \times (-3) + 7 \times (5) = 0$

$\Rightarrow -4\alpha - 15 + 35 = 0$

$\Rightarrow \boxed{\alpha = 5}$

9. Official Ans. by NTA (4)

Sol. Equation of AB = $\vec{r} = (\hat{i} + \hat{j}) + \lambda(3\hat{j} - 3\hat{k})$



Let coordinates of M = $(1, (1 + 3\lambda), -3\lambda)$.

$\overrightarrow{PM} = -3\hat{i} + (3\lambda - 1)\hat{j} - 3(\lambda + 1)\hat{k}$

$\overrightarrow{AB} = 3\hat{j} - 3\hat{k}$

$\because\ \overrightarrow{PM} \perp \overrightarrow{AB} \Rightarrow \overrightarrow{PM} . \overrightarrow{AB} = 0$

$\Rightarrow 3(3\lambda - 1) + 9(\lambda + 1) = 0$

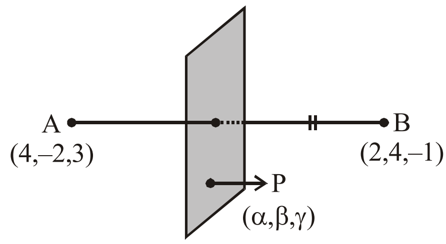
$\Rightarrow \lambda = -\frac{1}{3}$

$\therefore$ M = (1, 0, 1)

Clearly M lies on $2x + y - z = 1$.

10. Official Ans. by NTA (1)

Sol. PA = PB



$\Rightarrow PA^2 = PB^2$

$\Rightarrow (\alpha - 4)^2 + (\beta + 2)^2 + (\gamma - 3)^2$

$= (\alpha - 2)^2 + (\beta - 4)^2 + (\gamma + 1)^2$

$\Rightarrow -4\alpha + 12\beta - 8\gamma = -8$

$\Rightarrow 2x - 6y + 4z = 4$

11. Official Ans. by NTA (3)

Sol. $D_1 = \begin{vmatrix} -7 & 4 & -1 \\ 8 & 1 & 5 \\ 15 & b & 6 \end{vmatrix} = 0 \Rightarrow b = -3$

$D = \begin{vmatrix} 1 & 4 & -1 \\ 3 & 1 & 5 \\ a & b & 6 \end{vmatrix} = 0 \Rightarrow 21a - 8b - 66 = 0$ .... (1)

P : $2x - 3y + 6z = 15$

so required distance $= \frac{21}{7} = 3$

12. Official Ans. by NTA (2)

Sol. equation of line parallel to $\frac{x}{2} = \frac{y}{3} = \frac{z}{-6}$ passes through (1, –2, 3) is

$\frac{x-1}{2} = \frac{y+2}{3} = \frac{z-3}{-6} = r$

$x = 2r + 1$

$y = 3r - 2,$

$z = -6r + 3$

So $2r + 1 - 3r + 2 - 6r + 3 = 5$

$\Rightarrow -7r + 1 = 0$

$r = \frac{1}{7}$

$x = \frac{9}{7},\ y = \frac{-11}{7},,\ z = \frac{15}{7}$

Distance is $= \sqrt{\left(\frac{9}{7} - 1\right)^2 + \left(2 - \frac{11}{7}\right)^2 + \left(3 - \frac{15}{7}\right)^2}$

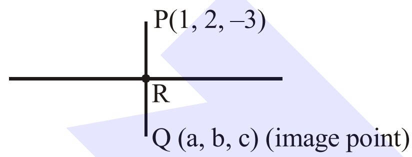
$= \sqrt{\left(\frac{2}{7}\right)^2 + \left(\frac{3}{7}\right)^2 + \left(\frac{6}{7}\right)^2}$

$= \frac{1}{7}\sqrt{4 + 9 + 36}$

$= \frac{1}{7}\sqrt{49} = 1$

13. Official Ans. by NTA (2)

Sol. Line is $\frac{x+1}{2} = \frac{y-3}{-2} = \frac{z}{-1} = \lambda$: Let point R is $(2\lambda - 1, -2\lambda + 3, -\lambda)$



Direction ratio of PQ≡$(2\lambda - 2, -2\lambda + 1, 3 - \lambda)$

PQ is $\perp^r$ to line

$\Rightarrow 2(2\lambda - 2) - 2(-2\lambda + 1) - 1(3 - \lambda) = 0$

$4\lambda - 4 + 4\lambda - 2 - 3 + \lambda = 0$

$9\lambda = 9 \Rightarrow \lambda = 1$

$\Rightarrow$ Point R is (1, 1, –1)

$\frac{a+1}{2} = 1$ | $\frac{b+2}{2} = 1$ | $\frac{c-3}{2} = -1$

$a = 1$ $\quad b = 0$ $\quad c = 1$

$\Rightarrow a + b + c = 2$

14. Official Ans. by NTA (4)

Sol. $L_1 \equiv \frac{x+1}{2} = \frac{y-2}{-1} = \frac{z-1}{1}$

$L_2 \equiv \frac{x+2}{\alpha} = \frac{y+1}{5-\alpha} = \frac{z+1}{1}$

Point A(–1, 2, 1) B(–2, –1, –1)

$\because L_1$ and L_2 are coplanar

$\Rightarrow \begin{vmatrix} 2 & -1 & 1 \\ \alpha & 5-\alpha & 1 \\ 1 & 3 & 2 \end{vmatrix} = 0$

$\alpha = -4$

$L_2 \equiv \frac{x+2}{-4} = \frac{y+1}{9} = \frac{z+1}{1}$

Check options (2, –10, –2) lies on L_2

15. Official Ans. by NTA (4)

Sol. Line of intersection of planes

$x + y + z + 1 = 0$ ...(1)

$2x - y + z + 3 = 0$ ...(2)

eliminate y

$3x + 2z + 4 = 0$

$x = \frac{-2z-4}{3}$ ...(3)

put in equaiton (1)

$z = -3y + 1$ ...(4)

from (3) and (4)

$\frac{3x+4}{-2} = -3y+1 = z$

$$\frac{x-\left(-\frac{4}{3}\right)}{-\frac{2}{3}} = \frac{y-\frac{1}{3}}{-\frac{1}{3}} = \frac{z-0}{1}$$

now shortest distance between skew lines

$\frac{x-1}{0} = \frac{y+1}{-1} = \frac{z}{1}$

$$\frac{x-\left(-\frac{4}{3}\right)}{-\frac{2}{3}} = \frac{y-\left(\frac{1}{3}\right)}{-\frac{1}{3}} = \frac{z-0}{1}$$

$$\text{S.D.} = \left|\frac{(\vec{b}-\vec{a}).(\vec{c}\times\vec{d})}{|\vec{c}\times\vec{d}|}\right|$$

where $\vec{a} = (1,-1,0)$

$\vec{b} = \left(-\frac{4}{3},\frac{1}{3},0\right)$

$\vec{c} = (0,-1,1)$

$\vec{d} = \left(-\frac{2}{3},-\frac{1}{3},1\right)$

$\Rightarrow \text{S.D} = \frac{1}{\sqrt{3}}$

16. Official Ans. by NTA (2)

Sol. $\frac{x}{a}+\frac{y}{b}+\frac{z}{c} = 1$

A ≡ (a, 0, 0), B ≡ (0, b, 0), C ≡ (0, 0, c)

Centroid ≡ $\left(\frac{a}{3}, \frac{b}{3}, \frac{c}{3}\right)$ = (1, 1, 2)

a = 3, b = 3, c = 6

Plane : $\frac{x}{3}+\frac{y}{3}+\frac{z}{6} = 1$

$2x + 2y + z = 6$

line ⊥ to the plane (DR of line = $2\hat{i}+2\hat{j}+\hat{k}$)

$\frac{x-1}{2} = \frac{y-1}{2} = \frac{z-2}{1}$