ALLEN CAREER INSTITUTE KOTA (RAJASTHAN)
path to success

Chapter 03 Contents

JEE (MAIN) TOPICWISE SOLUTION OF TEST PAPERS JANUARY & SEPTEMBER 2020

MATHEMATICS

JANUARY AND SEPTEMBER 2020 ATTEMPT (MATHEMATICS)

LOGARITHM

1. Official Ans. by NTA (4)

Sol. $(0.16)^{\log_{2.5}\left(\frac{1}{3}+\frac{1}{3^2}+\ldots\ldots\ldots\text{to }\infty\right)}$

$=\left(\frac{4}{25}\right)^{\log_{\left(\frac{5}{2}\right)}\left(\frac{1}{2}\right)}$

$=\left(\frac{1}{2}\right)^{\log_{\left(\frac{5}{2}\right)}\left(\frac{4}{25}\right)}=\left(\frac{1}{2}\right)^{-2}=4$

COMPOUND ANGLE

1. NTA Ans. (2)

Sol. $\tan\alpha + \tan\beta = \frac{\lambda\sqrt{2}}{k+1}$

$\tan\alpha.\ \tan\beta = \frac{k-1}{k+1}$

$\tan(\alpha+\beta) = \frac{\frac{\lambda\sqrt{2}}{k+1}}{1-\frac{k-1}{k+1}}=\frac{\lambda\sqrt{2}}{2}=\frac{\lambda}{\sqrt{2}}$

$\Rightarrow \frac{\lambda^2}{2}=50 \Rightarrow \lambda = 10 \ \& -10$

2. NTA Ans. (1)

Sol. $\frac{\sqrt{2}\sin\alpha}{\sqrt{2}\cos\alpha}=\frac{1}{7} \Rightarrow \tan\alpha=\frac{1}{7}$

$\sin\beta=\frac{1}{\sqrt{10}} \Rightarrow \tan\beta=\frac{1}{3} \Rightarrow \tan 2\beta=\frac{3}{4}$

$\tan(\alpha+2\beta)=\frac{\tan\alpha+\tan 2\beta}{1-\tan\alpha\tan 2\beta}=1$

Ans. 1.00

3. NTA Ans. (3)

Sol. $\cos^3\frac{\pi}{8}\cdot\sin\frac{\pi}{8}+\sin^3\frac{\pi}{8}\cdot\cos\frac{\pi}{8}$

$=\sin\frac{\pi}{8}.\cos\frac{\pi}{8}=\frac{1}{2}\sin\frac{\pi}{4}=\frac{1}{2\sqrt{2}}$

4. Official Ans. by NTA (1)

Sol. $L = \sin^2\left(\frac{\pi}{16}\right) - \sin^2\left(\frac{\pi}{8}\right)$

$\left(\because \sin^2\theta=\frac{1-\cos 2\theta}{2}\right)$

$\Rightarrow L = \left(\frac{1-\cos(\pi/8)}{2}\right)-\left(\frac{1-\cos(\pi/4)}{2}\right)$

$L = \frac{1}{2}\left[\cos\left(\frac{\pi}{4}\right)-\cos\left(\frac{\pi}{8}\right)\right]$

$L = \frac{1}{2\sqrt{2}} - \frac{1}{2}\cos\left(\frac{\pi}{8}\right)$

$M = \cos^2\left(\frac{\pi}{16}\right) - \sin^2\left(\frac{\pi}{8}\right)$

$M = \frac{1+\cos(\pi/8)}{2}-\frac{1-\cos(\pi/4)}{2}$

$M = \frac{1}{2}\cos\left(\frac{\pi}{8}\right)+\frac{1}{2\sqrt{2}}$

QUADRATIC EQUATION

1. NTA Ans (4)

Sol. $\alpha+\beta=1,\ \alpha\beta=-1$

$P_k = \alpha^k+\beta^k$

$\alpha^2-\alpha-1=0$

$\Rightarrow \alpha^k-\alpha^{k-1}-\alpha^{k-2}=0$

$\& \ \beta^k-\beta^{k-1}-\beta^{k-2}=0$

$\Rightarrow P_k = P_{k-1}+P_{k-2}$

$P_1=\alpha+\beta=1$

$P_2=(\alpha+\beta)^2-2\alpha\beta=1+2=3$

$P_3=4$

$P_4=7$

$P_5=11$

2. NTA Ans.(4)

Sol. Let $3^x = t$; $t > 0$

$t(t-1) + 2 = |t-1| + |t-2|$

$t^2 - t + 2 = |t-1| + |t-2|$

Case-I : $t < 1$

$t^2 - t + 2 = 1 - t + 2 - t$

$t^2 + 2 = 3 - t$

$t^2 + t - 1 = 0$

$$t = \frac{-1 \pm \sqrt{5}}{2}$$

$t = \frac{\sqrt{5}-1}{2}$ is only acceptable

Case-II : $1 \le t < 2$

$t^2 - t + 2 = t - 1 + 2 - t$

$t^2 - t + 1 = 0$

$D < 0$ no real solution

Case-III : $t \ge 2$

$t^2 - t + 2 = t - 1 + t - 2$

$t^2 - 3t \;\; 5 = 0 \Rightarrow D < 0$ no real solution

(4) Option

3. NTA Ans. (8.00)

Sol. $D \ge 0 \Rightarrow (a-10)^2 - 4 \times 2 \times \left(\frac{33}{2} - 2a\right) \ge 0$

$\Rightarrow a^2 - 4a - 32 \ge 0$

$\Rightarrow a \in (-\infty, 4] \cup [8, \infty)$

4. NTA Ans. (2)

Sol. $ax^2 - 2bx + 5 = 0 \begin{cases} \alpha \\ \alpha \end{cases}$

$\Rightarrow \alpha = \frac{b}{a};\ \alpha^2 = \frac{5}{a} \Rightarrow b^2 = 5a$

$x^2 - 2bx - 10 = 0 \begin{cases} \alpha \\ \beta \end{cases} \Rightarrow \alpha^2 - 2b\alpha - 10 = 0$

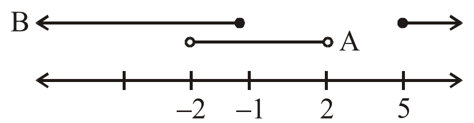
$\Rightarrow a = \frac{1}{4} \Rightarrow \alpha^2 = 20;\ \alpha\beta = -10 \Rightarrow \beta^2 = 5$

$\Rightarrow \alpha^2 + \beta^2 = 25$

5. NTA Ans. (3)

Sol. $A : x \in (-2, 2)$; $B : x \in (-\infty, -1] \cup [5, \infty)$

$\Rightarrow B - A = R - (-2, 5)$



6. NTA Ans. (4)

Sol. $e^{4x} + e^{3x} - 4e^x + e^x + 1 = 0$

Divide by e^{2x}

$$\Rightarrow e^{2x} + e^x - 4 + \frac{1}{e^x} + \frac{1}{e^{2x}} = 0$$

$$\Rightarrow \left(e^{2x} + \frac{1}{e^{2x}}\right) + \left(e^x + \frac{1}{e^x}\right) - 4 = 0$$

$$\Rightarrow \left(e^x + \frac{1}{e^x}\right)^2 - 2 + \left(e^x + \frac{1}{e^x}\right) - 4 = 0$$

Let $e^x + \frac{1}{e^x} = t \Rightarrow (e^x - 1)^2 = 0 \Rightarrow x = 0.$

$\therefore$ Number of real roots = 1

7. Official Ans. by NTA (1)

Sol. α and β are roots of $5x^2 + 6x - 2 = 0$

$\Rightarrow 5\alpha^2 + 6\alpha - 2 = 0$

$\Rightarrow 5\alpha^{n+2} + 6\alpha^{n+1} - 2\alpha^n = 0$ ...(1)

(By multiplying α^n)

Similarly $5\beta^{n+2} + 6\beta^{n+1} - 2\beta^n = 0$ ...(2)

By adding (1) & (2)

$5S_{n+2} + 6S_{n+1} - 2S_n = 0$

For n = 4

$$\boxed{5S_6 + 6S_5 = 2S_4}$$

8. Official Ans. by NTA (3)

Sol. $f(x) = a(x-3)(x-\alpha)$

$f(2) = a(\alpha - 2)$

$f(-1) = 4a(1 + \alpha)$

$f(-1) + f(2) = 0 \Rightarrow a(\alpha - 2 + 4 + 4\alpha) = 0$

$a \ne 0 \Rightarrow 5\alpha = -2$

$\alpha = -\frac{2}{5} = -0.4$

$\alpha \in (-1, 0)$

9. Official Ans. by NTA (3)

Sol. α, β are roots of $x^2 + px + 2 = 0$

$\Rightarrow \alpha^2 + p\alpha + 2 = 0$ & $\beta^2 + p\beta + 2 = 0$

$\Rightarrow \frac{1}{\alpha}, \frac{1}{\beta}$ are roots of $2x^2 + px + 1 = 0$

But $\frac{1}{\alpha}, \frac{1}{\beta}$ are roots of $2x^2 + 2qx + 1 = 0$

$\Rightarrow p = 2q$

Also $\alpha + \beta = -p \quad \alpha\beta = 2$

$$\left(\alpha - \frac{1}{\alpha}\right)\left(\beta - \frac{1}{\beta}\right)\left(\alpha + \frac{1}{\beta}\right)\left(\beta + \frac{1}{\alpha}\right)$$

$$= \left(\frac{\alpha^2 - 1}{\alpha}\right)\left(\frac{\beta^2 - 1}{\beta}\right)\left(\frac{\alpha\beta + 1}{\beta}\right)\left(\frac{\alpha\beta + 1}{\alpha}\right)$$

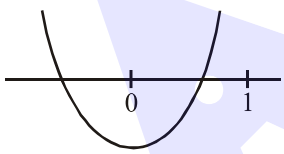
$$= \frac{(-p\alpha - 3)\,(-p\beta - 3)\,(\alpha\beta + 1)^2}{(\alpha\beta)^2}$$

$$= \frac{9}{4}(p\alpha\beta + 3p(\alpha + \beta) + 9)$$

$$= \frac{9}{4}(9 - p^2) = \frac{9}{4}(9 - 4q^2)$$

10. Official Ans. by NTA (2)

Sol. If exactly one root in (0, 1) then



$\Rightarrow f(0).f(1) < 0$

$\Rightarrow 2(\lambda^2 - 4\lambda + 3) < 0$

$\Rightarrow 1 < \lambda < 3$

Now for $\lambda = 1$, $2x^2 - 4x + 2 = 0$

$(x - 1)^2 = 0$, $x = 1, 1$

So both roots doesn't lie between (0, 1)

$\therefore \lambda \neq 1$

Again for $\lambda = 3$

$10x^2 - 12x + 2 = 0$

$\Rightarrow x = 1, \frac{1}{5}$

so if one root is 1 then second root lie between (0, 1)

so $\lambda = 3$ is correct.

$\therefore \lambda \in (1, 3]$.

11. Official Ans. by NTA (3)

Sol. $x^2 - 3x + p = 0 \begin{cases} \alpha \\ \beta \end{cases}$

$\alpha, \beta, \gamma, \delta$ in G.P.

$\alpha + \alpha r = 3 \quad ....(1)$

$x^2 - 6x + q = 0 \begin{cases} \gamma \\ \delta \end{cases}$

$\alpha r^2 + \alpha r^3 = 6 \quad ...(2)$

$(2) \div (1)$

$r^2 = 2$

So, $\frac{2q + p}{2q - p} = \frac{2r^5 + r}{2r^5 - r} = \frac{2r^4 + 1}{2r^4 - 1} = \frac{9}{7}$

12. Official Ans. by NTA (4)

Sol. $\alpha + \beta = 1, \ \alpha\beta = 2\lambda$

$\alpha + \beta = \frac{10}{3}, \ \alpha\gamma = \frac{27\lambda}{3} = 9\lambda$

$\gamma - \beta = \frac{7}{3},$

$\frac{\gamma}{\beta} = \frac{9}{2} \Rightarrow \gamma = \frac{9}{2}\beta = \frac{9}{2} \times \frac{2}{3} \Rightarrow \gamma = 3$

$\frac{9}{2}\beta - \beta = \frac{7}{3}$

$\frac{9}{2}\beta = \frac{7}{3} \Rightarrow \beta = \frac{2}{3}$

$\alpha = 1 - \frac{2}{3} = \frac{1}{3}$

$2\lambda = \frac{2}{9} \Rightarrow \lambda = \frac{1}{9}$

$$\frac{\beta\gamma}{\lambda} = \frac{\frac{2}{3} \times 3}{\frac{1}{9}} = 18$$

13. Official Ans. by NTA (2)

Sol. $9x^2 - 18|x| + 5 = 0$

$9|x|^2 - 15|x| - 3|x| + 5 = 0$ ($\because x^2 = |x|^2$)

$3|x|(3|x| - 5) - (3|x| - 5) = 0$

$$|x| = \frac{1}{3}, \frac{5}{3}$$

$$x = \pm\frac{1}{3}, \pm\frac{5}{3}$$

Product of roots $= \dfrac{25}{81}$

14. Official Ans. by NTA (1)

Sol. $7x^2 - 3x - 2 = 0$

$$\alpha + \beta = \frac{3}{7} \quad \alpha\beta = \frac{-2}{7}$$

$$\frac{\alpha}{1-\alpha^2} + \frac{\beta}{1-\beta^2} = \frac{\alpha+\beta-\alpha\beta(\alpha+\beta)}{1-\alpha^2-\beta^2+\alpha^2\beta^2}$$

$$= \frac{\frac{3}{7}+\frac{2}{7}\left(\frac{3}{7}\right)}{1-(\alpha+\beta)^2+2\alpha\beta+\alpha^2\beta^2} = \frac{27}{16}$$

15. Official Ans. by NTA (4)

Sol. $x^2 - 64x + 256 = 0$

$\alpha + \beta = 64, \alpha\beta = 256$

$$\left(\frac{\alpha^3}{\beta^5}\right)^{1/8} + \left(\frac{\beta^3}{\alpha^5}\right)^{1/8} = \frac{\alpha^{3/8}}{\beta^{5/8}} + \frac{\beta^{3/8}}{\alpha^{5/8}}$$

$$= \frac{\alpha+\beta}{(\alpha\beta)^{5/8}} = \frac{64}{(256)^{5/8}} = 2$$

16. Official Ans. by NTA (3)

Sol. α and β are the roots of the equation

$4x^2 + 2x - 1 = 0$

$$4\alpha^2 + 2\alpha = 1 \Rightarrow \frac{1}{2} = 2\alpha^2 + \alpha \quad ...(1)$$

$$\beta = \frac{-1}{2} - \alpha$$

using equation (1)

$\beta = -(2\alpha^2 + \alpha) - \alpha$

$\beta = -2\alpha^2 - 2\alpha$

$\beta = -2\alpha(\alpha + 1)$

SEQUENCE & PROGRESSION

1. NTA Ans. (1)

Sol. Sum of the 40 terms of

$3 + 4 + 8 + 9 + 13 + 14 + 18 + 19\ldots$

$= (3 + 8 + 13 + \ldots$upto 20 term)

$+ [4 + 9 + 15 + \ldots$ upto 20 terms]

$= 10 [\{6 + 19 \times 5\} + \{8 + 19 \times 5\}]$

$= 10 \times 204 = 20 \times 102$

2. NTA Ans. (1)

Sol. $a_1 + a_2 = 4$

$r^2a_1 + r^2a_2 = 16$

$\Rightarrow r^2 = 4 \Rightarrow r = -2$ as $a_1 < 0$

and $a_1 + a_2 = 4$

$a_1 + a_1(-2) = 4 \Rightarrow a_1 = -4$

$$4\lambda = (-4)\left(\frac{(-2)^9-1}{-2-1}\right) = (-4)\times\frac{513}{3}$$

$\Rightarrow \lambda = -171$

3. NTA Ans. (3)

Sol. Let the A.P is

$a - 2d, a - d, a, a + d, a + 2d$

$\because$ sum $= 25 \Rightarrow a = 5$

Product = 2520

$(25 - 4d^2)(25 - d^2) = 504$

$4d^4 - 125d^2 + 121 = 0$

$$\Rightarrow d^2 = 1, \frac{121}{4}$$

$$\Rightarrow d = \pm 1, \pm\frac{11}{2}$$

$d = \pm 1$ is rejected because none of the term can be $\dfrac{-1}{2}$.

$$\Rightarrow d = \pm\frac{11}{2}$$

$$\Rightarrow \text{AP will be } -6, -\frac{1}{2}, 5, \frac{21}{2}, 16$$

Largest term is 16.

4. NTA Ans. (3)

Sol. $1 + 49 + 49^2 + \ldots + 49^{12}$

$= (49)^{126} - 1 = (49^{63} + 1)\dfrac{(49^{63}-1)}{(48)}$

So greatest value of k = 63

5. NTA Ans. (2)

Sol. $T_{10} = \dfrac{1}{20} = a + 9d$ ...(i)

$T_{20} = \dfrac{1}{10} = a + 19d$ ...(ii)

$a = \dfrac{1}{200} = d$

Hence, $S_{200} = \dfrac{200}{2}\left[\dfrac{2}{200} + \dfrac{199}{200}\right] = \dfrac{201}{2}$

(2) Option

6. NTA Ans. (504)

Sol. $\dfrac{1}{4}\left(\sum_{n=1}^{7} 2n^3 + \sum_{n=1}^{7} 3n^2 + \sum_{n=1}^{7} n\right)$

$= \dfrac{1}{4}\left(2\left(\dfrac{7\times 8}{2}\right)^2 + 3\left(\dfrac{7\times 8\times 15}{6}\right) + \dfrac{7\times 8}{2}\right)$

= 504

Ans. 504.00

7. NTA Ans. (1540.00)

Sol. $\sum_{k=1}^{20} \dfrac{k(k+1)}{2} = \dfrac{1}{2}\sum_{k=1}^{20} \dfrac{k(k+1)(k+2)-(k-1)k(k+1)}{3}$

$= \dfrac{1}{6}\times 20\times 21\times 22 = 1540.00$

8. NTA Ans. (3)

Sol. $x = \sum_{n=0}^{\infty} (-1)^n \tan^{2n}\theta = 1 - \tan^2\theta + \tan^4\theta + \ldots.$

$\Rightarrow x = \cos^2\theta$

$y = \sum_{n=0}^{\infty} \cos^{2n}\theta \Rightarrow y = 1 + \cos^2\theta + \cos^4\theta + \ldots..$

$\Rightarrow y = \dfrac{1}{\sin^2\theta} \Rightarrow y = \dfrac{1}{1-x}$

$\Rightarrow y(1 - x) = 1$

9. NTA Ans. (4)

Sol. $\sum_{n=1}^{100} a_{2n+1} = 200 \Rightarrow a_3 + a_5 + a_7 + \ldots. + a_{201} = 200$

$\Rightarrow ar^2\dfrac{(r^{200}-1)}{(r^2-1)} = 200$

$\sum_{n=1}^{100} a_{2n} = 100 \Rightarrow a_2 + a_4 + a_6 + \ldots + a_{200} = 100$

$\Rightarrow \dfrac{ar(r^{200}-1)}{(r^2-1)} = 100$

On dividing r = 2

on adding $a_2 + a_3 + a_4 + a_5 + \ldots + a_{200} + a_{201} = 300$

$\Rightarrow r(a_1 + a_2 + a_3 + \ldots. + a_{200}) = 300$

$\Rightarrow \sum_{n=1}^{200} a_n = 150$

10. NTA Ans. (14)

Sol. Common term are : 23, 51, 79, ... T_n

$T_n \le 407 \Rightarrow 23 + (n - 1)28 \le 407$

$\Rightarrow n \le 14.71$

n = 14

11. NTA Ans. (1)

Sol. $2^{\frac{1}{4}}\cdot 4^{\frac{1}{16}}\cdot 8^{\frac{1}{48}}\cdot 16^{\frac{1}{128}}\cdot \ldots.\infty$

$= 2^{\frac{1}{4}}\cdot 2^{\frac{2}{16}}\cdot 2^{\frac{3}{48}}\cdot 2^{\frac{4}{128}}\cdot \ldots.\infty$

$= 2^{\frac{1}{4}}\cdot 2^{\frac{1}{8}}\cdot 2^{\frac{1}{16}}\cdot 2^{\frac{1}{32}}\cdot \ldots.\infty$

$= 2^{\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}+\ldots..\infty} = (2)^{\left(\frac{1/4}{1-1/2}\right)} = 2^{1/2}$

12. Official Ans. by NTA (1)

Sol. $|x| < 1, |y| < 1, x \neq y$

$(x + y) + (x^2 + xy + y^2) + (x^3 + x^2y + xy^2 + y^3) + ...$

By multiplying and dividing $x - y$:

$$\frac{(x^2 - y^2) + (x^3 - y^3) + (x^4 - y^4) + ...}{x - y}$$

$$= \frac{(x^2 + x^3 + x^4 + ...) - (y^2 + y^3 + y^4 + ...)}{x - y}$$

$$= \frac{\frac{x^2}{1-x} - \frac{y^2}{1-y}}{x - y}$$

$$= \frac{(x^2 - y^2) - xy(x - y)}{(1 - x)(1 - y)(x - y)}$$

$$= \boxed{\frac{x + y - xy}{(1 - x)(1 - y)}}$$

13. Official Ans. by NTA (4)

Sol. Let three terms of G.P. are $\frac{a}{r}$, a, ar

product = 27

$\Rightarrow a^3 = 27 \Rightarrow a = 3$

$$S = \frac{3}{r} + 3r + 3$$

For $r > 0$

$$\frac{\frac{3}{r} + 3r}{2} \geq \sqrt{3^2} \quad \text{(By AM} \geq \text{GM)}$$

$$\Rightarrow \frac{3}{r} + 3r \geq 6 \qquad ...(1)$$

For $r < 0$ $\quad \frac{3}{r} + 3r \leq -6 \qquad ...(2)$

From (1) & (2)

$S \in (-\infty - 3] \cup [9, \infty]$

14. Official Ans. by NTA (2)

Sol. $a_1 + a_2 + a_3 + ...+ a_{11} = 0$

$\Rightarrow (a_1 + a_{11}) \times \frac{11}{2} = 0$

$\Rightarrow a_1 + a_{11} = 0$

$\Rightarrow a_1 + a_1 + 10d = 0$

where d is common difference

$\Rightarrow \boxed{a_1 = -5d}$

$a_1 + a_3 + a_5 + ...+ a_{23}$

$$= (a_1 + a_{23}) \times \frac{12}{2} = (a_1 + a_1 + 22d) \times 6$$

$$= \left(2a_1 + 22\left(\frac{-a_1}{5}\right)\right) \times 6$$

$$= -\frac{72}{5} a_1 \Rightarrow K = \frac{-72}{5}$$

15. Official Ans. by NTA (3)

Sol. $S = [x + ka + 0] + [x^2 + ka + 2a] + [x^3 + ka + 4a] + [x^4 + ka + 6a] + ...9$ terms

$\Rightarrow S = (x + x^2 + x^3 + x^4 + ...9 \text{ terms}) + (ka + ka + ka + ka + ...9 \text{ terms}) + (0 + 2a + 4a + 6a + ...9 \text{ terms})$

$$\Rightarrow S = x\left[\frac{x^9 - 1}{x - 1}\right] + 9ka + 72a$$

$$\Rightarrow S = \frac{(x^{10} - x) + (9k + 72)a(x - 1)}{(x - 1)}$$

Compare with given sum, then we get, $(9k + 72) = 45$

$\Rightarrow \boxed{k = -3}$

16. Official Ans. by NTA (4)

Sol. Sum of 1st 25 terms = sum of its next 15 terms

$\Rightarrow (T_1 + ... + T_{25}) = (T_{26} + ... + T_{40})$

$\Rightarrow (T_1 + ... + T_{40}) = 2(T_1 + ... + T_{25})$

$$\Rightarrow \frac{40}{2}[2 \times 3 + (39d)] = 2 \times \frac{25}{2}[2 \times 2 + 24d]$$

$$\Rightarrow d = \frac{1}{6}$$

17. Official Ans. by NTA (3)

Sol. $S = \frac{100}{5} + \frac{98}{5} + \frac{96}{5} + \frac{94}{5} + ...n$

$S_n = \frac{n}{2}\left(2 \times \frac{100}{5} + (n-1)\left(-\frac{2}{5}\right)\right) = 188$

$n(100 - n + 1) = 488 \times 5$

$n^2 - 101n + 488 \times 5 = 0$

$n = 61, 40$

$T_n = a + (n - 1)d = \frac{100}{5} - \frac{2}{5} \times 60$

$= 20 - 24 = -4$

18. Official Ans. by NTA (39)

Sol. $3, A_1, A_2 .... A_m, 243$

$d = \frac{243 - 3}{m+1} = \frac{240}{m+1}$

Now $3, G_1, G_2, G_3, 243$

$r = \left(\frac{243}{3}\right)^{\frac{1}{3+1}} = 3$

$\therefore A_4 = G_2$

$\Rightarrow a + 4d = ar^2$

$3 + 4\left(\frac{240}{m+1}\right) = 3(3)^2$

$m = 39$

19. Official Ans. by NTA (2)

Sol. $1 + (1 - 2^2.1) + (1 - 4^2.3) + ... + (1 - 20^2.19)$

$= \alpha - 220\,\beta$

$= 11 - (2^2.1 + 4^2.3 + ... + 20^2.19)$

$= 11 - 2^2.\sum_{r=1}^{10} r^2(2r-1) = 11 - 4\left(\frac{110^2}{2} - 35 \times 11\right)$

$= 11 - 220(103)$

$\Rightarrow \alpha = 11, \beta = 103$

20. Official Ans. by NTA (3)

Sol. $a_n = a_1 + (n - 1)d$

$\Rightarrow 300 = 1 + (n - 1)\,d$

$\Rightarrow (n - 1)d = 299 = 13 \times 23$

since, $n \in [15, 50]$

$\therefore n = 24$ and $d = 13$

$a_{n-4} = a_{20} = 1 + 19 \times 13 = 248$

$\Rightarrow a_{n-4} = 248$

$S_{n-4} = \frac{20}{2}\{1 + 248\} = 2490$

21. Official Ans. by NTA (1)

Sol. Usnign AM $\geq$ GM

$\Rightarrow \frac{2^{\sin x} + 2^{\cos x}}{2} \geq \sqrt{2^{\sin x} \cdot 2^{\cos x}}$

$\Rightarrow 2^{\sin x} + 2^{\cos x} \geq 2^{1+\left(\frac{\sin x + \cos x}{2}\right)}$

$\Rightarrow \min\left(2^{\sin x} + 2^{\cos x}\right) = 2^{1-\frac{1}{\sqrt{2}}}$

22. Official Ans. by NTA (1)

Sol. Given that

$3^{4 - \sin 2\alpha} + 3^{2 \sin 2\alpha - 1} = 28$

Let $3^{2 \sin 2\alpha} = t$

$\frac{81}{t} + \frac{t}{3} = 28$

$t = 81, 3$

$3^{2 \sin 2\alpha} = 3^1, 3^4$

$2\sin 2\alpha = 1, 4$

$\sin 2\alpha = \frac{1}{2}$, 2 (rejected)

First term $a = 3^{2 \sin 2\alpha - 1}$

$a = 1$

Second term = 14

$\therefore$ common difference $d = 13$

$T_6 = a + 5d$

$T_6 = 1 + 5 \times 13$

$T_6 = 66$

23. Official Ans. by NTA (3)

Sol. $a = 2^{10}$; $r = \frac{3}{2}$; $n = 11$ (G.P.)

$$S' = (2^{10})\frac{\left(\left(\frac{3}{2}\right)^{11}-1\right)}{\frac{3}{2}-1} = 2^{11}\left(\frac{3^{11}}{2^{11}}-1\right)$$

$S' = 3^{11} - 2^{11} = S - 2^{11}$ (Given)

$\therefore\ S = 3^{11}$

24. Official Ans. by NTA (4)

Sol. $460 = \log_7 x \cdot (2 + 3 + 4 + ... + 20 + 21)$

$\Rightarrow 460 = \log_7 x \cdot \left(\frac{21\times 22}{2}-1\right)$

$\Rightarrow 460 = 230 \cdot \log_7 x$

$\Rightarrow \log_7 x = 2 \Rightarrow x = 49$

25. Official Ans. by NTA (2)

Sol. Let first term = a > 0

Common ratio = r > 0

$ar + ar^2 + ar^3 = 3$ ....(i)

$ar^5 + ar^6 + ar^7 = 243$ ....(ii)

$r^4(ar + ar^2 + ar^3) = 243$

$r^4(3) = 243 \Rightarrow r = 3$ as $r > 0$

from (1)

$3a + 9a + 27a = 3$

$a = \frac{1}{13}$

$S_{50} = \frac{a(r^{50}-1)}{(r-1)} = \frac{1}{26}(3^{50} - 1)$

26. Official Ans. by NTA (2)

Sol. $f(x + y) = f(x).\ f(y)$

$\sum_{x=1}^{\infty} f(x) = 2$ where $x,y \in N$

$f(1) + f(2) + f(3) + ... \infty = 2$ ...(1) (Given)

Now for $f(2)$ put $x = y = 1$

$f(2) = f(1 + 1) = f(1).\ f(1) = (f(1))^2$

$f(3) = f(2 + 1) = f(2).\ f(1) = (f(1))^3$

Now put these values in equation (1)

$f(1) + (f(1))^2 + [f(1)^2 + ...\infty = 2]$

$\frac{f(1)}{1-f(1)} = 2$

$\Rightarrow f(1) = \frac{2}{3}$

Now $f(2) = \left(\frac{2}{3}\right)^2$

$f(4) = \left(\frac{2}{3}\right)^4$

then the value of $\frac{f(4)}{f(2)} = \frac{\left(\frac{2}{3}\right)^4}{\left(\frac{2}{3}\right)^2} = \frac{4}{9}$

27. Official Ans. by NTA (3)

Sol. $(a^2 + b^2 + c^2)p^2 + 2(ab + bc + cd)p + b^2 + c^2 + d^2 = 0$

$\Rightarrow (a^2p^2 + 2abp + b^2) + (b^2p^2 + 2bcp + c^2) + (c^2p^2 + 2cdp + d^2) = 0$

$\Rightarrow (ab + b)^2 + (bp + c)^2 + (cp + d)^2 = 0$

This is possible only when

$ap + b = 0$ and $bp + c = 0$ and $cp + d = 0$

$p = -\frac{b}{a} = -\frac{c}{b} = -\frac{d}{c}$

or $\frac{b}{a} = \frac{c}{b} = \frac{d}{c}$

$\therefore$ a,b,c,d are in G.P.

28. Official Ans. by NTA (2)

Sol. $a_1, a_2, ..., a_n \rightarrow (CD = d)$

$b_1, b_2, ..., b_m \rightarrow (CD = d + 2)$

$a_{40} = a + 39d = -159$

...(1)

$a_{100} = a + 99d = -399$

...(2)

Subtract : $60d = -240 \Rightarrow d = -4$

using equation (1)

$a + 39(-4) = -159$

$a = 156 - 159 = -3$

$a_{70} = a + 69d = -3 + 69(-4) = -279 = b_{100}$

$b_{100} = -279$

$b_1 + 99(d + 2) = -279$

$b_1 - 198 = -279 \Rightarrow b_1 = -81$

TRIGONOMETRIC EQUATION

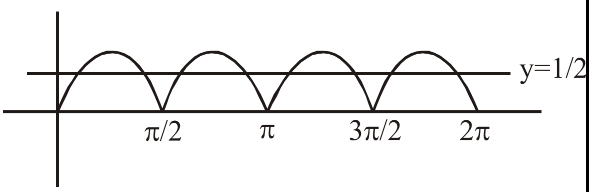
1. NTA Ans. (8.00)

Sol. $\log_{1/2}|\sin x| = 2 - \log_{1/2}|\cos x|;\ x \in [0, 2\pi]$

$\Rightarrow \log_{1/2}|\sin x| + \log_{1/2}|\cos x| = 2$

$\Rightarrow \log_{1/2}(|\sin x\cos x|) = 2$

$\Rightarrow |\sin x\cos x| = \frac{1}{4} \ \Rightarrow |\sin 2x| = \frac{1}{2}$



$\Rightarrow$ 8 solutions

2. Official Ans. by NTA (4)

Sol. $\lambda = -(\sin^4\theta + \cos^4\theta)$

$\lambda = -(\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta\cos^2\theta$

$$\lambda = \frac{\sin^2 2\theta}{2} - 1$$

$$\frac{\sin^2 2\theta}{2} \in \left[0, \frac{1}{2}\right]$$

$$\lambda \in \left[-1, -\frac{1}{2}\right]$$

3. Official Ans. by NTA (1)

Sol. $\cos\phi = \frac{\bar{p}.\bar{q}}{|\bar{p}||\bar{q}|} = \frac{ab+bc+ca}{a^2+b^2+c^2} = \frac{\Sigma ab}{1}$

$$= abc\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$$

$$= \frac{abc}{\lambda}\left(\cos\theta + \cos\left(\theta+\frac{2\pi}{3}\right) + \cos\left(\theta+\frac{4\pi}{3}\right)\right)$$

$$= \frac{abc}{\lambda}\left(\cos + 2\cos(\theta+\pi)\cos\frac{\pi}{3}\right)$$

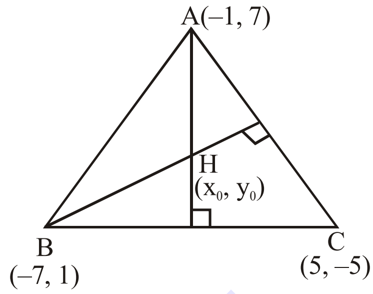
$$= \frac{abc}{\lambda}(\cos\theta - \cos\theta) = 0$$

$$\phi = \frac{\pi}{2}$$

SOLUTION OF TRIANGLE

1. Official Ans. by NTA (3)

Sol. Let orthocentre is $H(x_0, y_0)$



$m_{AH}.m_{BC} = -1$

$$\Rightarrow \left(\frac{y_0-7}{x_0+1}\right)\left(\frac{1+5}{-7-5}\right) = -1$$

$\Rightarrow 2x_0 - y_0 + 9 = 0$

.... (1)

and $m_{BH}.m_{AC} = -1$

$$\Rightarrow \left(\frac{y_0-1}{x_0+7}\right)\left(\frac{7-(-5)}{-1-5}\right) = -1$$

$\Rightarrow x_0 - 2y_0 + 9 = 0$ .... (2)

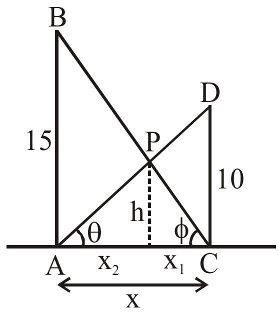
Solving equation (1) and (2) we get

$(x_0, y_0) \equiv (-3, 3)$

HEIGHT & DISTANCE

1. Official Ans. by NTA (4)

Sol.



$$\tan\theta = \frac{10}{x} = \frac{h}{x_2} \Rightarrow x_2 = \frac{hx}{10}$$

$$\tan\phi = \frac{15}{x} = \frac{h}{x_1} \Rightarrow x_1 = \frac{hx}{15}$$

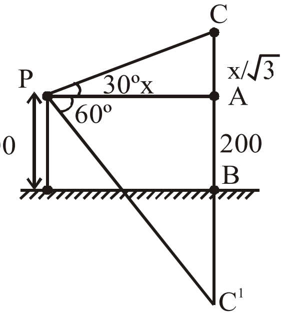
Now, $x_1 + x_2 = x = \frac{hx}{15} + \frac{hx}{10}$

$\Rightarrow 1 = \frac{h}{10} + \frac{h}{15} \ \Rightarrow h = 6$

2. Official Ans. by NTA (1)

Sol. Let PA = x

For ΔAPC



$$AC = \frac{PA}{\sqrt{3}} = \frac{x}{\sqrt{3}}$$

$AC^1 = AB + BC^1$

$AC^1 = AB + BC$

$$AC^1 = 400 + \frac{x}{\sqrt{3}}$$

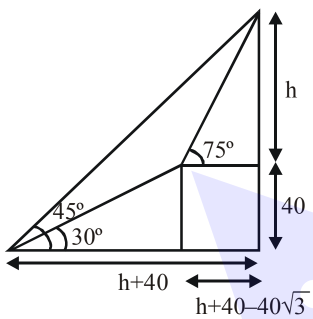
From ΔC^1PA : $AC^1 = \sqrt{3}\,PA$

$$\Rightarrow \left(400 + \frac{x}{\sqrt{3}}\right) = \sqrt{3}x \Rightarrow x = (200)(\sqrt{3})$$

from Δ APC : $PC = \dfrac{2x}{\sqrt{3}} \Rightarrow PC = 400$

3. Official Ans. by NTA (80.00)

Sol.



$$\tan 75° = \frac{h}{h + 40 - 40\sqrt{3}}$$

$$\frac{2+\sqrt{3}}{1} = \frac{h}{h + 40 - 40\sqrt{3}}$$

$$\Rightarrow 2h + 80 - 80\sqrt{3} + \sqrt{3}h + 40\sqrt{3} - 120 = h$$

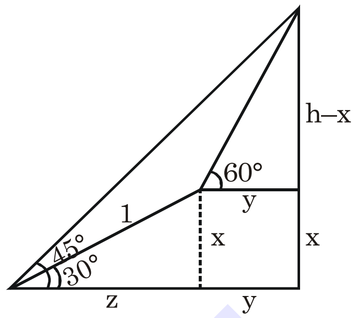
$$\Rightarrow h\left(\sqrt{3}+1\right) = 40 + 40\sqrt{3}$$

$$\Rightarrow h = 40$$

$\therefore$ Height of hill = 40 + 40 = 80m

4. Official Ans. by NTA (1)

Sol. $\sin 30° = x \Rightarrow x = \dfrac{1}{2}$



$$\cos 30° = z \Rightarrow z = \frac{\sqrt{3}}{2}$$

$$\tan 45° = \frac{h}{y+z} \Rightarrow h = y + z$$

$$\tan 60° = \frac{h-x}{y} \Rightarrow \tan 60° = \frac{h-x}{h-z}$$

$$\sqrt{3}(h-z) = h - x$$

$$\left(\sqrt{3}-1\right)h = \sqrt{3}\,z - x$$

$$\Rightarrow \left(\sqrt{3}-1\right)h = \frac{3}{2} - \frac{1}{2}$$

$$\Rightarrow \left(\sqrt{3}-1\right)h = 1$$

$$h = \frac{1}{\sqrt{3}-1}$$

DETERMINANT

1. NTA Ans. (13.00)

Sol. System has intfinitely many solution

$$\Rightarrow \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 3 & 2 & \lambda \end{vmatrix} = 0$$

$\Rightarrow \lambda = 1$

$$D_1 = \begin{vmatrix} 6 & 1 & 1 \\ 10 & 2 & 3 \\ \mu & 2 & 1 \end{vmatrix} = 0$$

$\mu = 14$

$\mu - \lambda^2 = 13$

2. NTA Ans. (4)

Sol. For non-zero solution

$$\begin{vmatrix} 2 & 2a & a \\ 2 & 3b & b \\ 2 & 4c & c \end{vmatrix} = 0, \Rightarrow \begin{vmatrix} 1 & 2a & a \\ 0 & 3b-2a & b-a \\ 0 & 4c-2a & c-a \end{vmatrix} = 0$$

$\Rightarrow (3b - 2a)(c - a) - (b - a)(4c - 2a) = 0$

$\Rightarrow 2ac = bc + ab$

$\Rightarrow \frac{2}{b} = \frac{1}{a} + \frac{1}{c}$ Hence $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P.

3. NTA Ans. (4)

Sol. $D = \begin{vmatrix} \lambda & 3 & 2 \\ 2\lambda & 3 & 5 \\ 4 & \lambda & 6 \end{vmatrix} = (\lambda + 8)(2 - \lambda)$

for $\lambda = 2$; $D_1 \neq 0$

Hence, no solution for $\lambda = 2$

(4) Option

4. NTA Ans. (4)

Sol. $2 \times (ii) - 2 \times (i) - (iii)$: -

$0 = 2\mu - 2 - \delta$

$\Rightarrow \delta = 2(\mu - 1)$

5. NTA Ans. (3)

Sol. $R_1 \to R_1 + R_3 - 2R_2$

$$f(x) = \begin{vmatrix} a+c-2b & 0 & 0 \\ x+b & x+3 & x+2 \\ x+c & x+4 & x+3 \end{vmatrix}$$

$= (a + c - 2b)((x + 3)^2 - (x + 2)(x + 4))$

$= x^2 + 6x + 9 - x^2 - 6x - 8 = 1$

$\Rightarrow f(x) = 1 \Rightarrow f(50) = 1$

6. NTA Ans. (1)

Sol. $7x + 6y - 2z = 0$ .... (1)

$3x + 4y + 2z = 0$ .... (2)

$x - 2y - 6z = 0$ .... (3)

$$\Delta = \begin{vmatrix} 7 & 6 & -2 \\ 3 & 4 & 2 \\ 1 & -2 & -6 \end{vmatrix} = 0 \Rightarrow \text{infinite solutions}$$

Now (1) + (2) $\Rightarrow y = -x$ put in (1), (2) & (3)

all will lead to $x = 2z$

7. Official Ans. by NTA (3)

Sol. $2x - y + 2z = 2$

$x - 2y + \lambda z = -4$

$x + \lambda y + z = 4$

For no solution :

$$D = \begin{vmatrix} 2 & -1 & 2 \\ 1 & -2 & \lambda \\ 1 & \lambda & 1 \end{vmatrix} = 0$$

$\Rightarrow 2(-2 - \lambda^2) + 1(1 - \lambda) + 2(\lambda + 2) = 0$

$\Rightarrow -2\lambda^2 + \lambda + 1 = 0$

$\Rightarrow \lambda = 1, -\frac{1}{2}$

$$D_x = \begin{vmatrix} 2 & -1 & 2 \\ -4 & 2 & \lambda \\ 4 & \lambda & 1 \end{vmatrix} = 2\begin{vmatrix} 1 & -1 & 2 \\ -2 & -2 & \lambda \\ \lambda & \lambda & 1 \end{vmatrix}$$

$= 2(1 + \lambda)$

whichis not equal to zero for

$\lambda = 1, -\frac{1}{2}$

8. Official Ans. by NTA (8)

Sol. $\Delta = \begin{vmatrix} 1 & -2 & 5 \\ -2 & 4 & 1 \\ -7 & 14 & 9 \end{vmatrix} = 0$

Let $x = k$

$\Rightarrow$ Put in (1) & (2)

$k - 2y + 5z = 0$

$-2k + 4y + z = 0$

$z = 0, \ y = \frac{k}{2}$

$\therefore$ x, y, z are integer

$\Rightarrow$ k is even integer

Now $x = k$, $y = \frac{k}{2}$, $z = 0$ put in condition

$15 \leq k^2 + \left(\frac{k}{2}\right)^2 + 0 \leq 150$

$12 \leq k^2 \leq 120$

$\Rightarrow k = \pm 4, \pm 6, \pm 8, \pm 10$

$\Rightarrow$ Number of element in S = 8.

9. Official Ans. by NTA (3)

Sol. $\Delta = \begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ 2x-3 & 3x-4 & 4x-5 \\ 3x-5 & 5x-8 & 10x-17 \end{vmatrix} = Ax^3 + Bx^2 +$

Cx + D.

$R_2 \to R_2 - R_1$ $\quad R_3 \to R_3 - R_2$

$$\Delta = \begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ x-1 & x-1 & x-1 \\ x-2 & 2(x-2) & 6(x-2) \end{vmatrix}$$

$$= (x-1)(x-2)\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ 1 & 1 & 1 \\ 1 & 2 & 6 \end{vmatrix}$$

$= -3(x-1)^2(x-2) = -3x^3 + 12x^2 - 15x + 6$

$\therefore$ B + C = 12 – 15 = –3

10. Official Ans. by NTA (5)

Sol. $D = \begin{vmatrix} 1 & -2 & 3 \\ 2 & 1 & 1 \\ 1 & -7 & a \end{vmatrix} = 0 \Rightarrow a = 8$

also, $D_1 = \begin{vmatrix} 9 & -2 & 3 \\ b & 1 & 1 \\ 24 & -7 & 8 \end{vmatrix} = 0 \Rightarrow b = 3$

hence, a – b = 8 – 3 = 5

11. Official Ans. by NTA (3)

Sol. For infinite solutions

$\Delta = \Delta_x = \Delta_y = \Delta_z = 0$

Now $\Delta = 0 \Rightarrow \begin{vmatrix} 1 & 1 & 1 \\ 2 & 4 & -1 \\ 3 & 2 & \lambda \end{vmatrix} = 0$

$\Rightarrow \lambda = \frac{9}{2}$

$\Delta_{x=0} \Rightarrow \begin{vmatrix} 2 & 1 & 1 \\ 6 & 4 & -1 \\ \mu & 2 & -\frac{9}{2} \end{vmatrix} = 0$

$\Rightarrow \mu = 5$

For $\lambda = \frac{9}{2}$ & $\mu = 5$, $\Delta_y = \Delta_z = 0$

Now check option $2\lambda + \mu = 14$

12. Official Ans. by NTA (4)

Sol. $C_3 \to C_3 - (C_1 - C_2)$

$$f(\theta) = \begin{vmatrix} -\sin^2\theta & -1-\sin^2\theta & 0 \\ -\cos^2\theta & -1-\cos^2\theta & 0 \\ 12 & 10 & -4 \end{vmatrix}$$

$= -4[(1 + \cos^2\theta)\sin^2\theta - \cos^2\theta(1 + \sin^2\theta)]$

$= -4[\sin^2\theta + \sin^2\theta\cos^2\theta - \cos^2\theta - \cos^2\theta\sin^2\theta]$

$f(\theta) = 4\cos 2\theta$

$\theta \in \left[\frac{\pi}{4}, \frac{\pi}{2}\right]$

$2\theta \in \left[\frac{\pi}{2}, \pi\right]$

$f(\theta) \in [-4, 0]$

(m, M) = (–4, 0)

13. Official Ans. by NTA (1)

Sol. $D = \begin{vmatrix} 2 & -4 & \lambda \\ 1 & -6 & 1 \\ \lambda & -10 & 4 \end{vmatrix}$

$= 2(3\lambda + 2)(\lambda - 3)$

$D_1 = -2(\lambda - 3)$

$D_2 = -2(\lambda + 1)(\lambda - 3)$

$D_3 = -2(\lambda - 3)$

When $\boxed{\lambda = 3}$, then

$D = D_1 = D_2 = D_3 = 0$

$\Rightarrow$ Infinite many solution

when $\boxed{\lambda = -\frac{2}{3}}$ then D_1, D_2, D_3 none of them is zero so equations are inconsistant

$\boxed{\therefore \lambda = -\frac{2}{3}}$

14. Official Ans. by NTA (2)

Sol. $x + y + 3z = 0$ .....(i)

$x + 3y + k^2z = 0$ .....(ii)

$3x + y + 3z = 0$ .....(iii)

$$\begin{vmatrix} 1 & 1 & 3 \\ 1 & 3 & k^2 \\ 3 & 1 & 3 \end{vmatrix} = 0$$

$\Rightarrow 9 + 3 + 3k^2 - 27 - k^2 - 3 = 0$

$\Rightarrow k^2 = 9$

(i) – (iii) $\Rightarrow -2x = 0 \Rightarrow x = 0$

Now from (i) $\Rightarrow y + 3z = 0$

$\Rightarrow \frac{y}{z} = -3$

$x + \frac{y}{z} = -3$

15. Official Ans. by NTA (2)

Sol. $a + x = b + y = c + z + 1$

$$\begin{vmatrix} x & a+y & x+a \\ y & b+y & y+b \\ z & c+y & z+c \end{vmatrix} \quad C_3 \to C_3 - C_1$$

$$\begin{vmatrix} x & a+y & a \\ y & b+y & b \\ z & c+y & c \end{vmatrix} \quad C_2 \to C_2 - C_3$$

$$\begin{vmatrix} x & y & a \\ y & y & b \\ z & y & c \end{vmatrix} \quad R_3 \to R_3 - R_1, R_2 \to R_2 - R_1$$

$$\begin{vmatrix} x & y & a \\ y-x & 0 & b-a \\ z-x & 0 & c-a \end{vmatrix}$$

$= (-y)[(y - x)(c - a) - (b - a)(z - x)]$

$= (-y)[(a - b)(c - a) + (a - b)(a - c - 1)]$

$= (-y)[(a - b)(c - a) + (a - b)(a - c) + b - a)$

$= -y(b - a) = y(a - b)$

16. Official Ans. by NTA (4)

Sol. For infinite many solutions

$D = D_1 = D_2 = D_3 = 0$

$$\text{Now } D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & \lambda \end{vmatrix} = 0$$

$1.(2\lambda - 9) - 1.(\lambda - 3) + 1.(3 - 2) = 0$

$\therefore \lambda = 5$

$$\text{Now } D_1 = \begin{vmatrix} 2 & 1 & 1 \\ 5 & 2 & 3 \\ \mu & 3 & 5 \end{vmatrix} = 0$$

$2(10 - 9) - 1(25 - 3\mu) + 1(15 - 2\mu) = 0$

$\mu = 8$

17. Official Ans. by NTA (1)

Sol.
$$\begin{vmatrix} \cos^2 x & 1+\sin^2 x & \sin 2x \\ 1+\cos^2 x & \sin^2 x & \sin 2x \\ \cos^2 x & \sin^2 x & 1+\sin 2x \end{vmatrix}$$

$R_1 \to R_1 - R_2,\ R_2 \to R_2 - R_3$

$$\begin{vmatrix} -1 & 1 & 0 \\ 1 & 0 & -1 \\ \cos^2 x & \sin^2 x & 1+\sin 2x \end{vmatrix}$$

$= -1\left(\sin^2 x\right) - 1\left(1 + \sin 2x + \cos^2 x\right)$

$= -\sin 2x - 2$

$m = -3,\ M = -1$

18. Official Ans. by NTA (3.00)

Sol. $(\lambda - 1)x + (3\lambda + 1)y + 2\lambda z = 0$

$(\lambda - 1)x + (4\lambda - 2)y + (\lambda + 3)z = 0$

$2x + (3\lambda + 1)y + (3\lambda - 3)z = 0$

$$\begin{vmatrix} \lambda-1 & 3\lambda+1 & 2\lambda \\ \lambda-1 & 4\lambda-2 & \lambda+3 \\ 2 & 3\lambda+1 & 3\lambda-3 \end{vmatrix} = 0$$

$R_1 \to R_1 - R_2$ & $R_2 \to R_2 - R_3$

$$\begin{vmatrix} 0 & 3-\lambda & \lambda-3 \\ \lambda-3 & \lambda-3 & -2(\lambda-3) \\ 2 & 3\lambda+1 & 3\lambda-3 \end{vmatrix} = 0$$

$$(\lambda-3)^2 \begin{vmatrix} 0 & -1 & 1 \\ 1 & 1 & -2 \\ 2 & 3\lambda+1 & 3\lambda-3 \end{vmatrix} = 0$$

$(\lambda - 3)^2 [(3\lambda + 1) + (3\lambda - 1)] = 0$

$6\lambda(\lambda - 3)^2 = 0 \Rightarrow \lambda = 0, 3$

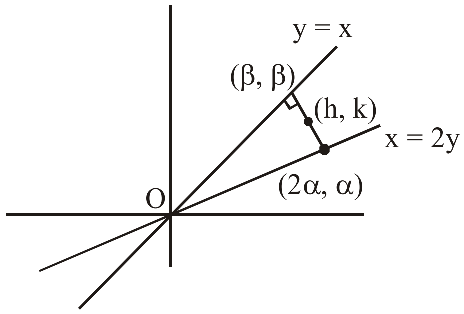
Sum = 3

STRAIGHT LINE

1. NTA Ans. (3)

Sol. $\dfrac{\alpha-\beta}{2\alpha-\beta}=-1$

$3\alpha = 2\beta$



$h = \dfrac{2\alpha+\beta}{2}$

$2h = \dfrac{7\alpha}{2}$

$k = \dfrac{\alpha+\beta}{2}$

$2k = \dfrac{5\alpha}{2}$

$\dfrac{h}{k}=\dfrac{7}{5}$

$5x = 7y$

2. NTA Ans. (5)

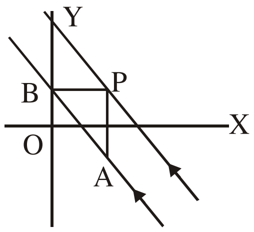
Sol. P is centroid of the triangle ABC

$\Rightarrow P \equiv \left(\dfrac{17}{6},\dfrac{8}{3}\right)$

$\Rightarrow PQ = 5$

3. NTA Ans. (3)

Sol. $\overleftrightarrow{AB}$: $3x + y - 2 = 0$



Also, $\dfrac{1}{2}\times\sqrt{10}\times h = 5$

$\Rightarrow h = \sqrt{10}$

$\Rightarrow \dfrac{|4\lambda-2|}{\sqrt{10}} = \sqrt{10} \Rightarrow \lambda = 3, -2$

4. NTA Ans. (2)

Sol. Centroid of $\Delta = (2, 2)$

line passing through intersection of

$x + 3y - 1 = 0$ and

$3x - y + 1 = 0$, be given by

$(x + 3y - 1) + \lambda(3x - y + 1) = 0$

$\because$ It passes through (2, 2)

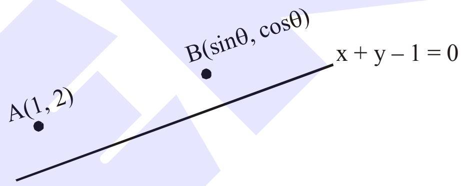
$\Rightarrow 7 + 5\lambda = 0 \Rightarrow \lambda = -\dfrac{7}{5}$

$\therefore$ Required line is $8x - 11y + 6 = 0$

$\because$ (–9, –6) satisfies this equation.

5. Official Ans. by NTA (4)

Sol. Given that both points (1, 2) & (sinθ, cosθ) lie on same side of the line x + y – 1 = 0



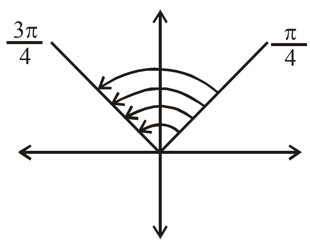
So, $\begin{pmatrix}\text{Put (1,2) in}\\ \text{given line}\end{pmatrix}\begin{pmatrix}\text{Put } (\sin\theta, \cos\theta \text{ in}\\ \text{given line}\end{pmatrix} > 0$

$\Rightarrow (1 + 2 - 1)(\sin\theta + \cos\theta - 1) > 0$

$\Rightarrow \sin\theta + \cos\theta > 1 \ \{\div \text{by}\sqrt{2}\}$

$\Rightarrow \dfrac{1}{\sqrt{2}}\sin\theta + \dfrac{1}{\sqrt{2}}\cos\theta > \dfrac{1}{\sqrt{2}}$

$\Rightarrow \sin\left(\theta+\dfrac{\pi}{4}\right) > \dfrac{1}{\sqrt{2}}$

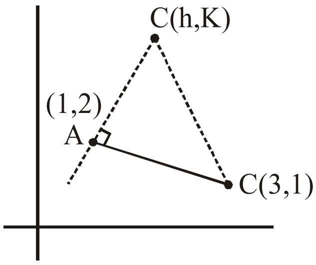


$\Rightarrow \dfrac{\pi}{4} < \theta + \dfrac{\pi}{4} < \dfrac{3\pi}{4}$

$\Rightarrow \boxed{0 < \theta < \dfrac{\pi}{2}}$

6. Official Ans. by NTA (3)

Sol.



$$\left(\frac{K-2}{h-1}\right)\left(\frac{1-2}{3-1}\right)=-1 \Rightarrow K = 2h \quad ...(1)$$

$$\sqrt{5}\,|h-1|=10$$

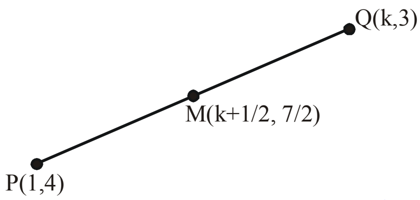
$$\because\ [\Delta ABC] = 5\sqrt{5}$$

$$\Rightarrow \frac{1}{2}\left(\sqrt{5}\right)\sqrt{(h-1)^2+(K-2)^2}=5\sqrt{5} \quad ....(2)$$

$$\Rightarrow\ h = 2\sqrt{5}+1\ \ (h>0)$$

7. Official Ans. by NTA (4)

Sol.



Slope = $m=\dfrac{1}{1-k}$

Equation of $\perp^r$ bisector is

$y + 4 = (k-1)(x-0)$

$\Rightarrow y + 4 = x(k-1)$

$$\Rightarrow \frac{7}{2}+4=\frac{k+1}{2}(k-1)$$

$$\Rightarrow \frac{15}{2}=\frac{k^2-1}{2}\Rightarrow k^2=16\Rightarrow k=4,-4$$

8. Official Ans. by NTA (30)

Sol. Apply distance between parallel line formula

$4x - 2y + \alpha = 0$

$4x - 2y + 6 = 0$

$$\left|\frac{\alpha-6}{255}\right|=\frac{1}{55}$$

$|\alpha - 6| = 2 \Rightarrow \alpha = 8, 4$

sum = 12

again

$6x - 3y + \beta = 0$

$6x - 3y + 9 = 0$

$$\left|\frac{\beta-9}{3\sqrt{5}}\right|=\frac{2}{\sqrt{5}}$$

$|\beta - 9| = 6 \Rightarrow \beta = 15, 3$

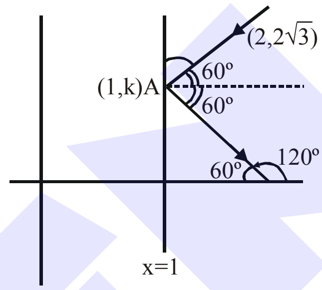
sum = 18

sum of all values of α and β is = 30

9. Official Ans. by NTA (2)

Sol. For point A

$$\tan 60^\circ=\frac{2\sqrt{3}-k}{2-1}$$



$$\sqrt{3}=2\sqrt{3}-k$$

$$\therefore\ k=\sqrt{3}$$

so point $A\left(1,\sqrt{3}\right)$

Now slope of line AB is $m_{AB} = \tan 120^\circ$

$m\,m_{AB}=-\sqrt{3}$

Now equation of line AB is

$$y-\sqrt{3}=-\sqrt{3}\left(x-1\right)$$

$$\sqrt{3}x+y=2\sqrt{3}$$

Now satisfy options

10. Official Ans. by NTA (3)

Sol. $L:\ \dfrac{x}{3}+\dfrac{y}{1}=1 \Rightarrow x + 3y - 3 = 0$

Image of point $(-1, -4)$

$$\frac{x+1}{1}=\frac{y+4}{3}=-2\left(\frac{-1-12-3}{10}\right)$$

$$\frac{x+1}{1}=\frac{y+4}{3}=\frac{16}{5}$$

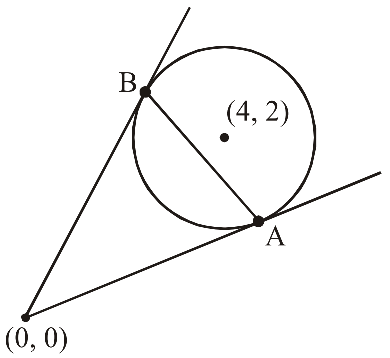
$$(x,y)\equiv\left(\frac{11}{5},\frac{28}{5}\right)$$

CIRCLE

1. NTA Ans. (4)

Sol. $R = \sqrt{16+4-16} = 2$

$L = \sqrt{S_1} = 4$



$AB(\text{Chord of contact}) = \dfrac{2LR}{\sqrt{L^2+R^2}} = \dfrac{8}{\sqrt{5}}$

$(AB)^2 = \dfrac{64}{5}$

2. NTA Ans. (2)

Sol. Slope of tangent to $x^2 + y^2 = 1$ at $P\left(\dfrac{1}{\sqrt{2}}, \dfrac{1}{\sqrt{2}}\right)$

$2x + 2yy' = 0 \Rightarrow m_T|_P = -1$

$y = mx + c$ is tangent to $(x - 3)^2 + y^2 = 1$

$y = x + c$ is tangent to $(x - 3)^2 + y^2 = 1$

$\left|\dfrac{c+3}{\sqrt{2}}\right| = 1 \Rightarrow c^2 + 6c + 7 = 0$

(2) Option

3. NTA Ans. (36)

Sol. Common tangent is $S_1 - S_2 = 0$

$\Rightarrow -6x + 8y - 8 + k = 0$

Use p = r for Ist circle

$\Rightarrow \dfrac{|-18-8+k|}{10} = 1$

$\Rightarrow k = 36$ or $16 \Rightarrow k_{max} = 36$

4. Official Ans. by NTA (9.00)

Sol. Circle $x^2 + y^2 - 2x - 4y + 4 = 0$

$\Rightarrow (x - 1)^2 + (y - 2)^2 = 1$

Centre : (1, 2) radius = 1

line $3x + 4y - k = 0$ intersects the circle at two distinct points.

$\Rightarrow$ distance of centre from the line < radius

$\Rightarrow \left|\dfrac{3\times1+4\times2-k}{\sqrt{3^2+4^2}}\right| < 1$

$\Rightarrow |11 - k| < 5$

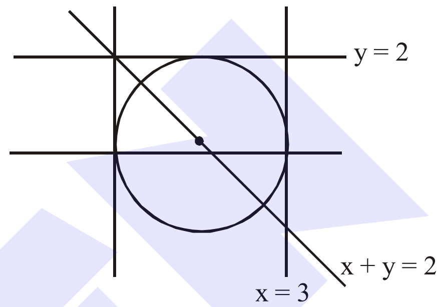
$\Rightarrow 6 < k < 16$

$\Rightarrow k \in \{7, 8, 9, \ldots\ldots 15\}$ since $k \in I$

Number of K is $\boxed{9}$

5. Official Ans. by NTA (3)

Sol.



$\because$ center lies on $x + y = 2$ and in 1st quadrant

center = $(\alpha, 2 - \alpha)$

where $\alpha > 0$ and $2 - \alpha > 0 \Rightarrow 0 < \alpha < 2$

$\because$ circle touches $x = 3$ and $y = 2$

$\Rightarrow |3 - \alpha| = |2 - (2 - \alpha)| =$ radius

$\Rightarrow |3 - \alpha| = |\alpha| \Rightarrow \alpha = \dfrac{3}{2}$

$\therefore$ radius = α

$\Rightarrow$ Diameter = $2\alpha = 3$.

6. Official Ans. by NTA (4)

Sol. Let S be the circle pasing through point of intersection of S_1 & S_2

$\therefore\ S = S_1 + \lambda S_2 = 0$

$\Rightarrow S : (x^2 + y^2 - 6x) + \lambda (x^2 + y^2 - 4y) = 0$

$\Rightarrow S : x^2 + y^2 - \left(\dfrac{6}{1+\lambda}\right)x - \left(\dfrac{4\lambda}{1+\lambda}\right)y = 0 \ldots(1)$

Centre $\left(\dfrac{3}{1+\lambda}, \dfrac{2\lambda}{1+\lambda}\right)$ lies on

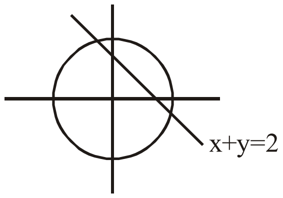
$2x - 3y + 12 = 0 \Rightarrow \lambda = -3$

put in (1) $\Rightarrow S : x^2 + y^2 + 3x - 6y = 0$

Now check options point (–3, 6) lies on S.

7. Official Ans. by NTA (7)

Sol. Let P $(3\cos\theta, 3\sin\theta)$



Q $(-3\cos\theta, -3\sin\theta)$

$$\Rightarrow \alpha\beta = \frac{|(3\cos\theta + 3\sin\theta)^2 - 4|}{2}$$

$$\Rightarrow \alpha\beta = \frac{5 + 9\sin 2\theta}{2} \le 7$$

8. Official Ans. by NTA (2)

Sol. Let chord

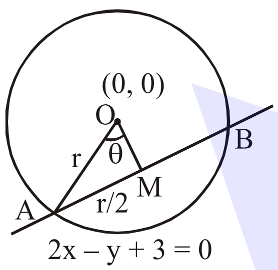
AB = r

$\because$ ΔAOM is right angled triangle

$\therefore$ $OM = \frac{r\sqrt{3}}{2}$ = perpendicular distance of line

AB from (0,0)

$$\frac{r\sqrt{3}}{2} = \left|\frac{3}{\sqrt{5}}\right|$$



$$r^2 = \frac{12}{5}$$

PERMUTATION & COMBINATION

1. NTA Ans. (1)

Sol. Total number of 6-digit numbers in which only and all the five digits 1, 3, 5, 7 and 9 is

$$^5C_1 \times \frac{6!}{2!}$$

2. NTA Ans. (2454)

Sol. N → 2, A → 2, I → 2, E, X, M, T, O → 1

Category	Selection	Arrangement
2 alike of one kind and 2 alike of other kind	$^3C_2 = 3$	$3 \times \frac{4!}{2!\,2!} = 18$
2 alike and 2 different	$^3C_1 \times {}^7C_2$	$^3C_1 \times {}^7C_2 \times \frac{4!}{2!} = 756$
All 4 different	$^8C_4$	$^8C_4 \times 4! = 1680$

Total = 2454

Ans. 2454.00

3. NTA Ans. (4)

Sol. $a = {}^{19}C_{10}$, $b = {}^{20}C_{10}$ and $c = {}^{21}C_{10}$

$$\Rightarrow a = {}^{19}C_9,\ b = 2({}^{19}C_9) \text{ and } c = \frac{21}{11}\left({}^{20}C_{10}\right)$$

$$\Rightarrow b = 2a \text{ and } c = \frac{21}{11}b = \frac{42a}{11}$$

$$\Rightarrow a : b : c = a : 2a : \frac{42a}{11} = 11 : 22 : 42$$

4. NTA Ans. (490.00)

ALLEN Ans. (490.00 OR 13.00)

Note: If same coloured marbles are identical then, answer is 13.00. However, NTA took them as distinct and kept only one answer as 490.00

Sol. The question does not mention that whether same coloured marbles are distinct or identical. So, assuming they are distinct our required answer = $^{12}C_4 - {}^5C_4 = 490$

And, if same coloured marbles are identical then required answer $= (2 + 3 + 4 + 4) = 13$

5. NTA Ans. (1)

Sol. _ _ _ 2 _

No. of five digits numbers =

No. of ways of filling remaining 4 places

= 8 × 8 × 7 × 6

$$\therefore\ k = \frac{8\times8\times7\times6}{336} = 8$$

6. Official Ans. by NTA (309.00)

Sol. MOTHER

$1 \rightarrow E$

$2 \rightarrow H$

$3 \rightarrow M$

$4 \rightarrow O$

$5 \rightarrow R$

$6 \rightarrow T$

So position of word MOTHER in dictionary

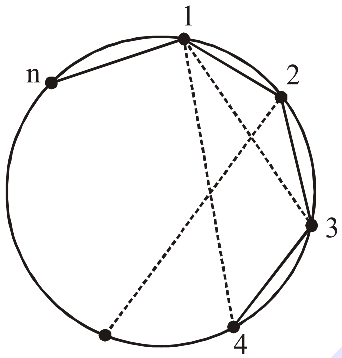
$2 \times 5! + 2 \times 4! + 3 \times 3! + 2! + 1$

$= 240 + 48 + 18 + 2 + 1$

$= \boxed{309}$

7. Official Ans. by NTA (3)

Sol.



Number of blue lines = Number of sides = n

Number of red lines = number of diagonals

$= {}^nC_2 - n$

${}^nC_2 - n = 99\,n \Rightarrow \dfrac{n(n-1)}{2} - n = 99\,n$

$\dfrac{n-1}{2} - 1 = 99 \Rightarrow n = 201$

8. Official Ans. by NTA (3)

Sol. $S = (2.{}^1p_0 - 3.{}^2p_1 + 4.{}^3p_2$ .... upto 51 terms) + (1! + 2! + 3! .... upto 51 terms) $[\because {}^np_{n-1} = n!]$

$\therefore S = (2 \times 1! - 3 \times 2! + 4 \times 3! \ldots + 52.51!) + (1! - 2! + 3! \ldots\ldots (51)!)$

$= (2! - 3! + 4! \ldots\ldots + 52!) + (1! - 2! + 3! - 4! + \ldots\ldots + (51)!)$

$= 1! + 52!.$

9. Official Ans. by NTA (54)

Sol. Let three digit number is xyz

$x + y + z = 10\ ;\ x \geq 1,\ y \geq 0\ z \geq 0$ ..... (1)

Let $T = x - 1 \Rightarrow x = T + 1$ where $T \geq 0$

Put in (1)

$T + y + z = 9\ ;\ 0 \leq T \leq 8,\ 0 \leq y, z \leq 9$

No. of non negative integral solution

$= {}^{9+3-1}C_{3-1} - 1$ (when $T = 9$)

$= 55 - 1 = 54$

10. Official Ans. by NTA (135)

Sol. Ways $= {}^6C_4 \cdot 1^4 \cdot 3^2$

$= 15 \times 9$

$= 135$

11. Official Ans. by NTA (240)

Sol. S_2YL_2ABU

ABCC type words

$= \underbrace{{}^2C_1}_{\text{selection of two alike letters}} \times \underbrace{{}^5C_2}_{\text{selection of of two distinct letters}} \times \underbrace{\dfrac{\lfloor 4}{\lfloor 2}}_{\text{arrangement of selected letters}}$

$= 240$

12. Official Ans. by NTA (4)

Sol.

A	B	C
$\boxed{5}$	$\boxed{5}$	$\boxed{5}$
1	2	2
2	1	2
2	2	1
1	1	3
1	3	1
3	1	1

Total number of selection

$= ({}^5C_1\ {}^5C_2\ {}^5C_2)\cdot 3 + ({}^5C_1\ {}^5C_1\ {}^5C_3)\cdot 3$

$= 5 \cdot 10 \cdot 10 \cdot 3 + 5 \cdot 5 \cdot 10 \cdot 3$

$= 2250$

13. Official Ans. by NTA (120.00)

Sol. LETTER

vowels = EE, consonant = LTTR

_ L _ T _ T _ R _

$\dfrac{4!}{2!} \times {}^5C_2 \times \dfrac{2!}{2!} = 12 \times 10 = 120$

BINOMIAL THEOREM

1. NTA Ans. (3)

Sol. $6\times{}^{35}C_r = (k^2-3)\,{}^{36}C_{r+1}$

$k^2 - 3 > 0 \Rightarrow k^2 > 3$

$k^2 - 3 = \dfrac{6\times{}^{35}C_r}{{}^{36}C_{r+1}} = \dfrac{r+1}{6}$

Possible values of r for integral values of k, are

r = 5, 35

number of ordered pairs are 4

(5, 2), (5, –2), (35, 3), (35, –3)

2. NTA Ans. (2)

Sol. Coefficient of x^7 is

$${}^{10}C_7 + {}^{9}C_6 + {}^{8}C_5 + \ldots + {}^{4}C_1 + {}^{3}C_0$$

$$\underbrace{{}^{4}C_0 + {}^{4}C_1}_{{}^{5}C_1} + {}^{5}C_2 + \ldots + {}^{10}C_7 = {}^{11}C_7 = 330$$

3. NTA Ans. (30)

Sol. Let $(1 + x + x^2 + \ldots + x^{2n})(1 - x + x^2 - x^3 + \ldots + x^{2n})$

$= a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + \ldots + a_{4n}x^{4n}$

So,

$a_0 + a_1 + a_2 + \ldots + a_{4n} = 2n + 1$ ...(1)

$a_0 - a_1 + a_2 - a_3 \ldots + a_{4n} = 2n + 1$ ...(2)

$\Rightarrow a_0 + a_2 + a_4 + \ldots + a_{4n} = 2n + 1$

$\Rightarrow 2n + 1 = 61 \quad \Rightarrow n = 30$

4. NTA Ans. (3)

Sol. $2[{}^6C_0x^6 + {}^6C_2x^4(x^2-1) + {}^6C_4x^2(x^2-1)^2 + {}^6C_6(x^2-1)^3]$

$\alpha = -96$ & $\beta = 36$

$\therefore \alpha - \beta = -132$

(3) Option

5. NTA Ans. (4)

Sol. $T_{r+1} = {}^{16}C_r\left(\dfrac{x}{\cos\theta}\right)^{16-r}\left(\dfrac{1}{x\sin\theta}\right)^{r}$

$= {}^{16}C_r\,(x)^{16-2r} \times \dfrac{1}{(\cos\theta)^{16-r}(\sin\theta)^r}$

For independent of x; $16 - 2r = 0 \Rightarrow r = 8$

$\Rightarrow T_9 = {}^{16}C_8\dfrac{1}{\cos^8\theta\sin^8\theta}$

$= {}^{16}C_8\dfrac{2^8}{(\sin2\theta)^8}$

for $\theta \in \left[\dfrac{\pi}{8}, \dfrac{\pi}{4}\right]$ ℓ_1 is least for $\theta_1 = \dfrac{\pi}{4}$

for $\theta \in \left[\dfrac{\pi}{16}, \dfrac{\pi}{8}\right]$ ℓ_2 is least for $\theta_2 = \dfrac{\pi}{8}$

$\dfrac{\ell_2}{\ell_1} = \dfrac{(\sin2\theta_1)^8}{(\sin2\theta_2)^8} = (\sqrt{2})^8 = \dfrac{16}{1}$

6. NTA Ans. (51)

Sol. $S = 1.{}^{25}C_0 + 5.{}^{25}C_1 + 9.{}^{25}C_2 + \ldots. + (101){}^{25}C_{25}$

$S = 101\,{}^{25}C_{25} + 97\,{}^{25}C_1 + \ldots\ldots\ldots + 1\,{}^{25}C_{25}$

$2S = (102)(2^{25})$

$S = 51(2^{25})$

7. NTA Ans. (615.00)

Sol. $(1 + x + x^2)^{10}$

$= {}^{10}C_0 + {}^{10}C_1x(1+x) + {}^{10}C_2x^2(1+x)^2 + {}^{10}C_3x^3(1+x)^3 + {}^{10}C_4x^4(1+x)^4 + \ldots\ldots.$

Coeff. of $x^4 = {}^{10}C_2 + {}^{10}C_3 \times {}^3C_1 + {}^{10}C_4 = 615.$

8. Official Ans. by NTA (2)

Sol. Let t_{r+1} denotes

r + 1th term of $\left(\alpha x^{\frac{1}{9}}+\beta x^{-\frac{1}{6}}\right)^{10}$

$$t_{r+1}={}^{10}C_r\alpha^{10-r}(x)^{\frac{10-r}{9}}.\beta^r x^{-\frac{r}{6}}$$

$$={}^{10}C_r\,\alpha^{10-r}\,\beta^r\,(x)^{\frac{10-r}{9}-\frac{r}{6}}$$

If t_{r+1} is independent of x

$$\frac{10-r}{9}-\frac{r}{6}=0 \quad\Rightarrow\quad r=4$$

maximum value of t_5 is 10 K (given)

$\Rightarrow\ {}^{10}C_4\,\alpha^6\,\beta^4$ is maximum

By AM $\geq$ GM (for positive numbers)

$$\frac{\frac{\alpha^3}{2}+\frac{\alpha^3}{2}+\frac{\beta^2}{2}+\frac{\beta^2}{2}}{4}\geq\left(\frac{\alpha^6\beta^4}{16}\right)^{\frac{1}{4}}$$

$\Rightarrow \boxed{\alpha^6\beta^4\leq 16}$

So, 10 K = ${}^{10}C_4 16$

$\Rightarrow$ K = 336

9. Official Ans. by NTA (118)

Sol. ${}^nC_{r-1} : {}^nC_r : {}^nC_{r+1}$ = 2:5:12

Now $\frac{{}^nC_{r-1}}{{}^nC_r}=\frac{2}{5}$

$\Rightarrow$ 7r = 2n + 2 …(1)

$$\frac{{}^nC_r}{{}^nC_{r+1}}=\frac{5}{12}$$

$\Rightarrow$ 17r = 5n – 12 …(2)

On solving (1) & (2)

$\Rightarrow$ n = 118

10. Official Ans. by NTA (2)

Sol. $T_{r+1}={}^nC_r\,(3)^{\frac{n-r}{2}}(5)^{\frac{r}{8}}$ $\quad (n\geq r)$

Clearly r should be a multiple of 8.

$\because$ there are exactly 33 integral terms

Possible values of r can be

0, 8, 16, ..., 32 × 8

$\therefore$ least value of n = 256.

11. Official Ans. by NTA (4)

Sol. $T_{r+1}={}^9C_r\left(\frac{3}{2}x^2\right)^{9-r}\left(-\frac{1}{3x}\right)^r$

$$T_{r+1}={}^9C_r\left(\frac{3}{2}\right)^{9-r}\left(-\frac{1}{3}\right)^r x^{18-3r}$$

For independent of x

18 – 3r = 0, r = 6

$$\therefore\ T_7={}^9C_6\left(\frac{3}{2}\right)^3\left(-\frac{1}{3}\right)^6=\frac{21}{54}=k$$

$$\therefore\ 18k=\frac{21}{54}\times 18=7$$

12. Official Ans. by NTA (2)

Sol. $\sum\limits_{r=0}^{20}{}^{50-r}C_6={}^{50}C_6+{}^{49}C_6+{}^{48}C_6+.....+{}^{30}C_6$

$$={}^{50}C_6+{}^{49}C_6+.....+{}^{31}C_6+\left({}^{30}C_6+{}^{30}C_7\right)-{}^{30}C_7$$

$$={}^{50}C_6+{}^{49}C_6+.....+\left({}^{31}C_6+{}^{31}C_7\right)-{}^{30}C_7$$

$$={}^{50}C_6+{}^{50}C_7-{}^{30}C_7$$

$$={}^{51}C_7-{}^{30}C_7$$

$$\boxed{{}^nC_r+{}^nC_{r-1}={}^{n+1}C_r}$$

13. Official Ans. by NTA (8)

Sol. Given $(2x^2+3x+4)^{10}=\sum\limits_{r=0}^{20}a_r\,x^r$ .... (1)

replace x by $\frac{2}{x}$ in above identity :-

$$\frac{2^{10}\left(2x^2+3x+4\right)^{10}}{x^{20}}=\sum_{r=0}^{20}\frac{a_r\,2^r}{x^r}$$

$$\Rightarrow 2^{10}\sum_{r=0}^{20}a_r\,x^r=\sum_{r=0}^{20}a_r\,2^r\,x^{(20-r)}\ \text{(from (i))}$$

now, comparing coefficient of x^7 from both sides

(take r = 7 in L.H.S. & r = 13 in R.H.S.)

$$2^{10}\,a_7=a_{13}\,2^{13}\Rightarrow\frac{a_7}{a_{13}}=2^3=8$$

14. Official Ans. by NTA (3)

Sol. Let n + 5 = N

$$^{N}C_{r-1} : {}^{N}C_{r} : {}^{N}C_{r+1} = 5 : 10 : 14$$

$$\Rightarrow \frac{^{N}C_{r}}{^{N}C_{r-1}} = \frac{N+1-r}{r} = 2$$

$$\frac{^{N}C_{r+1}}{^{N}C_{r}} = \frac{N-r}{r+1} = \frac{7}{5}$$

$\Rightarrow$ r = 4, N = 11

$\Rightarrow (1 + x)^{11}$

Largest coefficient = $^{11}C_6 = 462$

15. Official Ans. by NTA (13)

Sol. $T_{r+1} = {}^{22}C_r (x^m)^{22-r}\left(\frac{1}{x^2}\right)^r = {}^{22}C_r x^{22m-mr-2r}$

$= {}^{22}C_r\ x$

$\because\ {}^{22}C_3 = {}^{22}C_{19} = 1540$

$\therefore$ r = 3 or 19

22m – mr – 2r = 1

$$m = \frac{2r+1}{22-5}$$

$$r = 3,\ m = \frac{7}{19} \notin N$$

$$r = 19,\ m = \frac{38+1}{22-19} = \frac{39}{3} = 13$$

m = 13

16. Official Ans. by NTA (120.00)

Sol. $(1 + x + x^2 + x^3)^6 = \left((1+x)(1+x^2)\right)^6$

$= (1 + x)^6 (1 + x^2)^6$

$$= \sum_{r=0}^{6} {}^6C_r x^r \sum_{r=0}^{6} {}^6C_t x^{2t}$$

$$= \sum_{r=0}^{6}\sum_{t=0}^{6} {}^6C_r {}^6C_t\, x^{r+2t}$$

For coefficient of $x^4 \Rightarrow r + 2t = 4$

r	t
0	2
2	1
4	0

Coefficient of x^4

$= {}^6C_0\,{}^6C_2 + {}^6C_2\,{}^6C_1 + {}^6C_4\,{}^6C_0$

= 120

17. Official Ans. by NTA (1)

Sol. $$\left\{\frac{3^{200}}{8}\right\} = \left\{\frac{(3^2)^{100}}{8}\right\} = \left\{\frac{(1+8)^{100}}{8}\right\}$$

$$= \left\{\frac{1 + {}^{100}C_1.8 + {}^{100}C_2.8^2 + \ldots + {}^{100}C_{100}8^{100}}{8}\right\}$$

$$= \left\{\frac{1+8m}{8}\right\} = \frac{1}{8}$$

18. Official Ans. by NTA (3)

Sol. $$\left(\sqrt{x} - \frac{k}{x^2}\right)^{10}$$

$$T_{r+1} = {}^{10}C_r \left(\sqrt{x}\right)^{10-r}\left(\frac{-k}{x^2}\right)^r$$

$$T_{r+1} = {}^{10}C_r.x^{\frac{10-r}{2}}.(-k)^r.x^{-2r}$$

$$T_{r+1} = {}^{10}C_r x^{\frac{10-5r}{2}}(-k)^r$$

Constant term : $\frac{10-5r}{2} = 0 \Rightarrow r = 2$

$T_3 = {}^{10}C_2.(-k)^2 = 405$

$$k^2 = \frac{405}{45} = 9$$

$k = \pm 3 \Rightarrow |k| = 3$

SET

1. NTA Ans. (29.00)

Sol. n(A) = 25

n(B) = 7

$n(A \cap B) = 3$

$n(A \cup B) = 25 + 7 - 3 = 29$

2. Official Ans. by NTA (1)

Sol. $A : D \geq 0$

$\Rightarrow (m+1)^2 - 4(m+4) \geq 0$

$\Rightarrow m^2 + 2m + 1 - 4m - 16 \geq 0$

$\Rightarrow m^2 - 2m - 15 \geq 0$

$\Rightarrow (m-5)(m+3) \geq 0$

$\Rightarrow m \in (-\infty, -3] \cup [5, \infty)$

$\therefore A = (-\infty, -3] \cup [5, \infty)$

$B = [-3, 5)$

$A - B = (-\infty, -3) \cup [5, \infty)$

$A \cap B = \{-3\}$

$B - A = (-3, 5)$

$A \cup B = R$

3. Official Ans. by NTA (8)

Sol. $\Delta = \begin{vmatrix} 1 & -2 & 5 \\ -2 & 4 & 1 \\ -7 & 14 & 9 \end{vmatrix} = 0$

Let $x = k$

$\Rightarrow$ Put in (1) & (2)

$k - 2y + 5z = 0$

$-2k + 4y + z = 0$

$z = 0, \ y = \frac{k}{2}$

$\therefore$ x, y, z are integer

$\Rightarrow$ k is even integer

Now $x = k, y = \frac{k}{2}, z = 0$ put in condition

$15 \leq k^2 + \left(\frac{k}{2}\right)^2 + 0 \leq 150$

$12 \leq k^2 \leq 120$

$\Rightarrow k = \pm 4, \pm 6, \pm 8, \pm 10$

$\Rightarrow$ Number of element in S = 8.

4. Official Ans. by NTA (4)

Sol. $n(B) \leq n(A \cup B) \leq n(U)$

$\Rightarrow 76 \leq 76 + 63 - x \leq 100$

$\Rightarrow -63 \leq -x \leq -39$

$\Rightarrow 63 \geq x \geq 39$

5. Official Ans. by NTA (4)

Sol. $n(X_i) = 10. \ \bigcup_{i=1}^{50} X_i = T, \Rightarrow \ n(T) = 500$

each element of T belongs to exactly 20

elements of $X_i \Rightarrow \frac{500}{20} = 25$ distinct elements

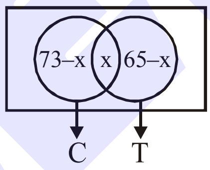
so $\frac{5n}{6} = 25 \Rightarrow n = 30$

6. Official Ans. by NTA (4)

Sol. $C \rightarrow$ person like coffee

$T \rightarrow$ person like Tea

$n(C) = 73$



$n(T) = 65$

$n(C \cup T) \leq 100$

$n(C) + n(T) - n(C \cap T) \leq 100$

$73 + 65 - x \leq 100$

$x \geq 38$

$73 - x \geq 0 \Rightarrow x \leq 73$

$65 - x \geq 0 \Rightarrow x \leq 65$

$\boxed{38 \leq x \leq 65}$

7. Official Ans. by NTA (28.00)

Sol. $2^m - 2^n = 112$

$m = 7, n = 4$

$(2^7 - 2^4 = 112)$

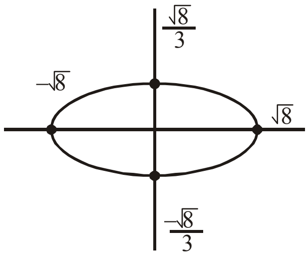
$m \times n = 7 \times 4 = 28$

RELATION

1. Official Ans. by NTA (2)

Sol. $R = \{(x, y) : x, y \in z, x^2 + 3y^2 \le 8\}$

For domain of R^{-1}



Collection of all integral of y's

For $x = 0$, $3y^2 \le 8$

$\Rightarrow y \in \{-1, 0, 1\}$

2. Official Ans. by NTA (4)

Sol. Let $a^2 + b^2 \in Q$ & $b^2 + c^2 \in Q$

eg. $a = 2 + \sqrt{3}$ & $b = 2 - \sqrt{3}$

$a^2 + b^2 = 14 \in Q$

Let $c = (1+2\sqrt{3})$

$b^2 + c^2 = 20 \in Q$

But $a^2 + c^2 = (2+\sqrt{3})^2 + (1+2\sqrt{3})^2 \notin Q$

for R_2 Let $a^2 = 1$, $b^2 = \sqrt{3}$ & $c^2 = 2$

$a^2 + b^2 \notin Q$ & $b^2 + c^2 \notin Q$

But $a^2 + c^2 \in Q$

FUNCTION

1. NTA Ans. (2)

Sol. $g(x) = x^2 + x - 1$

$g(f(x)) = 4x^2 - 10x + 5$

$= (2x - 2)^2 + (2 - 2x) - 1$

$= (2 - 2x)^2 + (2 - 2x) - 1$

$\Rightarrow f(x) = 2 - 2x$

$$f\left(\frac{5}{4}\right) = \frac{-1}{2}$$

2. NTA Ans. (4)

Sol. $$f(x) = \begin{cases} \dfrac{x}{x^2+1} & ; \quad x \in (1,2) \\ \dfrac{2x}{x^2+1} & ; \quad x \in [2,3) \end{cases}$$

$f(x)$ is decreasing function

$$\therefore\ f(x) \in \left(\frac{2}{5}, \frac{1}{2}\right) \cup \left(\frac{3}{5}, \frac{4}{5}\right]$$

(4) Option

3. NTA Ans. (2)

Sol. $$f(x) = \frac{2(2^x + 2^{-x}) + (3^x + 3^{-x})}{2} \ge 3$$

(A.M $\ge$ G.M)

4. NTA Ans. (3)

Sol. $$f(x) = y = \frac{8^{4x} - 1}{8^{4x} + 1} = 1 - \frac{2}{8^{4x} + 1}$$

so, $8^{4x} + 1 = \dfrac{2}{1-y} \Rightarrow 8^{4x} = \dfrac{1+y}{1-y}$

$$\Rightarrow x = \ell n\left(\frac{1+y}{1-y}\right) \times \frac{1}{4\ell n 8} = f^{-1}(y)$$

Hence, $f^{-1}(x) = \dfrac{1}{4}\log_8 e \ell n\left(\dfrac{1+x}{1-x}\right)$

5. Official Ans. by NTA (1)

Sol. $f(x + y) = f(x) + f(y)$

$\Rightarrow f(n) = nf(1)$

$f(n) = 2n$

$$g(n) = \sum_{k=1}^{n-1} 2n = 2\left(\frac{(n-1)n}{2}\right) = n(n-1)$$

$g(n) = 20 \Rightarrow n(n - 1) = 20$

$n = 5$

6. Official Ans. by NTA (4)

Sol. $[x]^2 + 2[x + 2] - 7 = 0$

$\Rightarrow [x]^2 + 2[x] + 4 - 7 = 0$

$\Rightarrow [x] = 1, -3$

$\Rightarrow x \in [1, 2) \cup [-3, -2)$

7. Official Ans. by NTA (19.00)

Sol. $C = \{f : A \to B | 2 \in f(A) \text{ and f is not one-one}\}$

Case-I : If f(x) = 2 $\forall$ x $\in$ A then number of function = 1

Case-II : If f(x) = 2 for exactly two elements then total number of many-one function = ${}^3C_2$ ${}^3C_1$ = 9

Case-III : If f(x) = 2 for exactly one element then total number of many-one functions = ${}^3C_1$ ${}^3C_1$ = 9

Total = 19

8. Official Ans. by NTA (2)

Sol. $f(x) = \dfrac{a-x}{a+x} \quad x \in R - \{-a\} \to R$

$$f(f(x)) = \frac{a - f(x)}{a + f(x)} = \frac{a - \left(\frac{a-x}{a+x}\right)}{a + \left(\frac{a-x}{a+x}\right)}$$

$$f(f(x)) = \frac{(a^2 - a) + x(a+1)}{(a^2 + a) + x(a-1)} = x$$

$\Rightarrow (a^2 - a) + x(a + 1) = (a^2 + a)x + x^2(a - 1)$

$\Rightarrow a(a - 1) + x(1 - a^2) - x^2(a - 1) = 0$

$\Rightarrow a = 1$

$f(x) = \dfrac{1-x}{1+x}$,

$$f\left(\frac{-1}{2}\right) = \frac{1+\frac{1}{2}}{1-\frac{1}{2}} = 3$$

9. Official Ans. by NTA (5.00)

Sol. $f(x + y) = f(x)\, f(y)$

put x = y = 1 $\quad f(2) = (f(1))^2 = 3^2$

put x = 2, y = 1 $f(3) = (f(1))^3 = 3^3$

$\vdots$

Similarly $f(x) = 3^x$

$$\sum_{i=1}^{n} f(i) = 363 \Rightarrow \sum_{i=1}^{n} 3^i = 363$$

$(3 + 3^2 + \dots + 3^n) = 363$

$$\frac{3(3^n - 1)}{2} = 363$$

$3^n - 1 = 242 \Rightarrow 3^n = 243$

$\Rightarrow n = 5$

INVERSE TRIGONOMETRY FUNCTION

1. Official Ans. by NTA (1)

Sol. $f(x) = \sin\left(\dfrac{|x|+5}{x^2+1}\right)$

For domain :

$$-1 \le \frac{|x|+5}{x^2+1} \le 1$$

Since |x| + 5 & x^2 + 1 is always positive

So $\dfrac{|x|+5}{x^2+1} \ge 0 \ \forall x \in R$

So for domain :

$$\frac{|x|+5}{x^2+1} \le 1$$

$\Rightarrow |x| + 5 \le x^2 + 1$

$\Rightarrow 0 \le x^2 - |x| - 4$

$$\Rightarrow 0 \le \left(|x| - \frac{1+\sqrt{17}}{2}\right)\left(|x| - \frac{1-\sqrt{17}}{2}\right)$$

$$\Rightarrow |x| \ge \frac{1+\sqrt{17}}{2} \text{ or } |x| \le \frac{1-\sqrt{17}}{2} \quad \text{(Rejected)}$$

$$\Rightarrow x \in \left(-\infty, -\frac{1+\sqrt{17}}{2}\right] \cup \left[\frac{1+\sqrt{17}}{2}, \infty\right)$$

So, $a = \dfrac{1+\sqrt{17}}{2}$

2. Official Ans. by NTA (3)

Sol. $$2\pi - \left(\sin^{-1}\left(\frac{4}{5}\right) + \sin^{-1}\left(\frac{5}{13}\right) + \sin^{-1}\left(\frac{16}{65}\right)\right)$$

$$= 2\pi - \left(\tan^{-1}\left(\frac{4}{3}\right) + \tan^{-1}\left(\frac{5}{12}\right) + \tan^{-1}\left(\frac{16}{63}\right)\right)$$

$$= 2\pi - \left(\tan^{-1}\left(\frac{63}{16}\right) + \tan^{-1}\left(\frac{16}{63}\right)\right)$$

$$= 2\pi - \frac{\pi}{2} = \frac{3\pi}{2}$$

3. Official Ans. by NTA (4)

Sol. $S = \tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{7}\right) + \tan^{-1}\left(\frac{1}{13}\right) + ...$

$S = \tan^{-1}\left(\frac{2-1}{1+1.2}\right) + \tan^{-1}\left(\frac{3-2}{1+2\times 3}\right) + \tan^{-1}\left(\frac{4-3}{1+3\times 4}\right) + ...+ \tan^{-1}\left(\frac{11-10}{1+10\times 11}\right)$

$S = (\tan^{-1}2 - \tan^{-1} 1) + (\tan^{-1} 3 - \tan^{-1} 2) + (\tan^{-1} 4 - \tan^{-1} 3) + ... + (\tan^{-1}(11) - \tan^{-1}(10))$

$S = \tan^{-1} 11 - \tan^{-1} 1 = \tan^{-1}\left(\frac{11-1}{1+11}\right)$

$\tan(S) = \frac{11-1}{1+11\times 1} = \frac{10}{12} = \frac{5}{6}$

LIMIT

1. NTA Ans. (36)

Sol. $\lim_{x\to 2}\frac{3^x + 3^{3-x} - 12}{3^{-x/2} - 3^{1-x}} \Rightarrow \lim_{x\to 2}\frac{3^{2x} - 12.3^x + 27}{3^{x/2} - 3}$

$= \lim_{x\to 2}\frac{(3^x - 9)(3^x - 3)}{(3^{x/2} - 3)}$

$= \lim_{x\to 2}\frac{(3^{x/2} + 3)(3^{x/2} - 3)(3^x - 3)}{(3^{x/2} - 3)}$

$= 36$

2. NTA Ans. (4)

Sol. Required limit $= e^{\lim_{x\to 0}\left(\frac{3x^2+2}{7x^2+2} - 1\right)\frac{1}{x^2}}$

$= e^{\lim_{x\to 0}\left(\frac{-4}{7x^2+2}\right)} = \frac{1}{e^2}$

3. Official Ans. by NTA (40.00)

Sol. $\lim_{x\to 1}\frac{x + x^2 + ... + x^2 - n}{x-1} = 820$

$\Rightarrow \lim_{x\to 1}\left(\frac{x-1}{x-1} + \frac{x^2-1}{x-1} + ... \frac{x^n - 1}{x-1}\right) = 820$

$\Rightarrow 1 + 2 + ... + n = 820$

$\Rightarrow n(n + 1) = 2 \times 820$

$\Rightarrow n(n + 1) = 40 \times 41$

Since $n \in N$, so $\boxed{n = 40}$

4. Official Ans. by NTA (4)

Sol. $\lim_{x\to 0}\left\{\tan\left(\frac{\pi}{4} + x\right)\right\}^{1/x}$

$= e^{\lim_{x\to 0}\frac{1}{x}\left\{\tan\left(\frac{\pi}{4}+x\right) - 1\right\}}$

$= e^{\lim_{x\to 0}\left(\frac{1+\tan x - 1 + \tan x}{x(1-\tan x)}\right)}$

$= e^{\lim_{x\to 0}\frac{2\tan x}{x(1-\tan x)}}$

$= e^2$

5. Official Ans. by NTA (2)

Sol. LHL : $\lim_{x\to 0^-}\left|\frac{1-x-x}{\lambda - x - 1}\right| = \left|\frac{1}{\lambda - 1}\right|$

RHL : $\lim_{x\to 0^+}\left|\frac{1-x+x}{\lambda - x + 1}\right| = \left|\frac{1}{\lambda}\right|$

For existence of limit

LHL = RHL

$\Rightarrow \frac{1}{|\lambda - 1|} = \frac{1}{|\lambda|} \Rightarrow \lambda = \frac{1}{2}$

$\therefore\ L = \frac{1}{|\lambda|} = 2$

6. Official Ans. by NTA (8)

Sol. $\lim_{x\to 0}\left\{\frac{1}{x^8}\left(1 - \cos\frac{x^2}{2} - \cos\frac{x^2}{4} + \cos\frac{x^2}{2}\cos\frac{x^2}{4}\right)\right\} = 2^{-k}$

$\Rightarrow \lim_{x\to 0}\frac{\left(1-\cos\frac{x^2}{2}\right)}{4\left(\frac{x^2}{2}\right)^2}\frac{\left(1-\cos\frac{x^2}{4}\right)}{16\left(\frac{x^2}{4}\right)^2} = \frac{1}{8}\times\frac{1}{32} = 2^{-k}$

$\Rightarrow 2^{-8} = 2^{-k} \Rightarrow k = 8.$

7. Official Ans. by NTA (1)

Sol. Required limit

$$L = \lim_{h\to 0}\frac{(a+2(a+h))^{1/3}-(3(a+h))^{1/3}}{(3a+a+h)^{1/3}-(4(a+h))^{1/3}}$$

$$= \lim_{h\to 0}\frac{(3a)^{1/3}\left(1+\frac{2h}{3a}\right)^{1/3}-(3a)^{1/3}\left(1+\frac{h}{a}\right)^{1/3}}{(4a)^{1/3}\left(1+\frac{h}{4a}\right)^{1/3}-(4a)^{1/3}\left(1+\frac{h}{a}\right)^{1/3}}$$

$$= \lim_{h\to 0}\left(\frac{3^{1/3}}{4^{1/3}}\right)\left[\frac{\left(1+\frac{2h}{9a}\right)-\left(1+\frac{h}{3a}\right)}{\left(1+\frac{h}{12a}\right)-\left(1+\frac{h}{3a}\right)}\right]$$

$$= \left(\frac{3}{4}\right)^{1/3}\frac{\left(\frac{2}{9}-\frac{1}{3}\right)}{\left(\frac{1}{12}-\frac{1}{3}\right)} = \left(\frac{3}{4}\right)^{1/3}\left(\frac{8-12}{3-12}\right)$$

$$= \left(\frac{3}{4}\right)^{1/3}\left(\frac{-4}{-9}\right) = \frac{4^{1-\frac{1}{3}}}{3^{2-\frac{1}{3}}} = \frac{4^{2/3}}{3^{5/3}}$$

$$= \frac{(8\times 2)^{1/3}}{(27\times 9)^{1/3}} = \frac{2}{3}\left(\frac{2}{9}\right)^{1/3}$$

8. Official Ans. by NTA (4)

Sol. $L = \lim\limits_{t\to x}\frac{t^2f^2(x)-x^2f^2(t)}{t-x}$

using L.H. rule

$L = \lim\limits_{t\to x}\frac{2tf^2(x)-x^2\cdot 2f'(t).f(t)}{1}$

$\Rightarrow$ L = 2xf(x) (f(x) – x f'(x)) = 0 (given)

$\Rightarrow f(x) = xf'(x) \Rightarrow \int\frac{f'(x)dx}{f(x)} = \int\frac{dx}{x}$

$\Rightarrow \ell n\,|f(x)| = \ell n\,|x| + C$

$\because f(1) = e,\ x > 0,\ f(x) > 0$

$\Rightarrow f(x) = ex$, if $f(x) = 1 \Rightarrow x = \frac{1}{e}$

9. Official Ans. by NTA (1)

Sol. $x^2 - x - 2 = 0$

roots are 2 & –1

$$\Rightarrow \lim_{x\to 2^+}\frac{\sqrt{1-\cos(x^2-x-2)}}{(x-2)}$$

$$= \lim_{x\to 2^+}\frac{\sqrt{2\sin^2\frac{(x^2-x-2)}{2}}}{(x-2)}$$

$$= \lim_{x\to 2^+}\frac{\sqrt{2}\sin\left(\frac{(x-2)(x+1)}{2}\right)}{(x-2)} = \frac{3}{\sqrt{2}}$$

10. Official Ans. by NTA (4)

Sol. $\lim\limits_{x\to 0}\frac{x\left(e^{\left(\sqrt{1+x^2+x^4}-1\right)/x}-1\right)}{\sqrt{1+x^2+x^4}-1}$

$\because \lim\limits_{x\to 0}\frac{\sqrt{1+x^2+x^4}-1}{x}$ $\left(\frac{0}{0}\text{ from}\right)$

$\lim\limits_{x\to 0}\frac{(1+x^2+x^4)-1}{x(\sqrt{1+x^2+x^4}+1}$

$\lim\limits_{x\to 0}\frac{x(1+x^2)}{\left(\sqrt{1+x^2+x^4}+1\right)} = 0$

So $\lim\limits_{x\to 0}\frac{x\left(e^{\left(\frac{\sqrt{1+x^2+x^4}-1}{x}\right)}-1\right)}{\sqrt{1+x^2+x^4}-1}$ $\left(\frac{0}{0}\text{ from}\right)$

$\lim\limits_{x\to 0}\frac{e^{\frac{\sqrt{1+x^2+x^4}-1}{x}}-1}{\left(\frac{\sqrt{1+x^2+x^4}-1}{x}\right)} = 1$

11. Official Ans. by NTA (1)
Official Ans. by ALLEN
(Bonus-Answers musbe zero)

Sol. $\lim\limits_{x\to 1}\dfrac{\int_0^{(x-1)^2} t\cos(t^2)dt}{(x-1)\sin(x-1)}\left(\dfrac{0}{0}\right)$

Apply L Hopital Rule

$$=\lim_{x\to 1}\frac{2(x-1).(x-1)^2\cos(x-1)^4-0}{(x-1).\cos(x-1)+\sin(x-1)}\left(\frac{0}{0}\right)$$

$$=\lim_{x\to 1}\frac{2(x-1)^3.\cos(x-1)^4}{(x-1)\left[\cos(x-1)+\dfrac{\sin(x-1)}{(x-1)}\right]}$$

$$=\lim_{x\to 1}\frac{2(x-1)^2\cos(x-1)^4}{(x-1)\left[\cos(x-1)+\dfrac{\sin(x-1)}{(x-1)}\right]}$$

$$=\lim_{x\to 1}\frac{2(x-1)^2\cos(x-1)^4}{\cos(x-1)+\dfrac{\sin(x-1)}{(x-1)}}$$

on taking limit

$$=\frac{0}{1+1}=0$$

CONTINUITY

1. NTA Ans. (5.00)

Sol. $k = \lim\limits_{x\to 0}\left(\dfrac{\ell n(1+3x)}{x}-\dfrac{\ell n(1-2x)}{x}\right)$

$k = 3 + 2 = 5$

2. NTA Ans. (2)

Sol. $A = \lim\limits_{x\to 0} x\left[\dfrac{4}{x}\right]=\lim\limits_{x\to 0} x\left(\dfrac{4}{x}\right)-x\left\{\dfrac{4}{x}\right\}=4$

$f(x) = [x^2]\sin(\pi x)$ will be discontinuous at nonintegers

$\therefore\ x=\sqrt{A+1}$ i.e. $\sqrt{5}$

3. NTA Ans. (4)

Sol. $\lim\limits_{x\to 0^-} f(x)=\lim\limits_{x\to 0}\left(\dfrac{\sin(a+2)x}{x}+\dfrac{\sin x}{x}\right)=a+3$

$\lim\limits_{x\to 0^-} f(x)=\lim\limits_{x\to 0}\dfrac{(x+3x^2)^{1/3}-x^{1/3}}{x^{4/3}}$

$=\lim\limits_{x\to 0}\dfrac{(1+3x)^{1/3}-1}{x}=1$

$f(0) = b$

for continuity at $x = 0$

$\lim\limits_{x\to 0^-} f(x)=f(0)=\lim\limits_{x\to 0^+} f(x)$

$\Rightarrow a + 3 = b = 1$

$\therefore a = -2,\ b = 1$

$\therefore a + 2b = 0$

4. Official Ans. by NTA (8)

Sol. $x \in (-10, 10)$

$\dfrac{x}{2}\in(-5, 5) \rightarrow 9$ integers

check continuity at $x = 0$

$\left.\begin{array}{l} f(0) = 0 \\ f(0^+) = 0 \\ f(0^-) = 0 \end{array}\right\}$ continuous at $x = 0$

function will be distcontinuous when

$\dfrac{x}{2}=\pm 4,\ \pm 3,\ \pm 2,\ \pm 1$

8 points of discontinuity

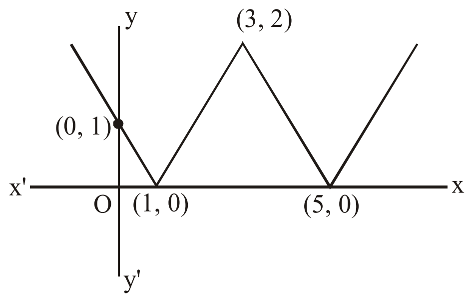
DIFFERENTIABILITY

1. NTA Ans. (3)

Sol. $f(x) = |2 - |x - 3||$

f is not differentiable at

$x = 1, 3, 5$



$\Rightarrow \sum\limits_{x\in s} f(f(x))=f(f(1))+f(f(3))+f(f(5))$

$= f(0) + f(2) + f(0)$

$= 1 + 1 + 1 = 3$

2. Official Ans. by NTA (4)

Sol. $f(x)=\begin{cases} ae^{x}+be^{-x}, & -1\le x<1 \\ cx^{2}, & 1\le x\le 3 \\ ax^{2}+2cx, & 3<x\le 4\end{cases}$

For continuity at x = 1

$\lim\limits_{x\to 1^-} f(x)=\lim\limits_{x\to 1^+} f(x)$

$\Rightarrow \boxed{ae+be^{-1}=c} \Rightarrow \boxed{b=ce-ae^{2}}$

...(1)

For continuity at x = 3

$\lim\limits_{x\to 3^-} f(x)=\lim\limits_{x\to 3^+} f(x)$

$\Rightarrow$ 9c = 9a + 6c

$\Rightarrow$ c = 3a ...(2)

$f'(0) + f'(2) = e$

$(ae^{x} - be^{x})_{x=0} + (2cx)_{x=2} = e$

$\Rightarrow \boxed{a-b+4c=e}$ ...(3)

From (1), (2) & (3)

$a - 3ae + ae^{2} + 12a = e$

$\Rightarrow a(e^{2} + 13 - 3e) = e$

$\Rightarrow a=\dfrac{e}{e^{2}-3e+13}$

3. Official Ans. by NTA (10)

Sol. Since, $\lim\limits_{x\to 0}\dfrac{f(x)}{x}$ exist $\Rightarrow f(0) = 0$

Now, $f'(x)=\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$

$=\lim\limits_{h\to 0}\dfrac{f(h)+xh^{2}+x^{2}h}{h}$ (take y = h)

$=\lim\limits_{h\to 0}\dfrac{f(h)}{h}+\lim\limits_{h\to 0}(xh)+x^{2}$

$\Rightarrow f'(x) = 1 + 0 + x^{2}$

$\Rightarrow f'(3) = 10$

4. Official Ans. by NTA (1)

Sol. $f(x)=\begin{cases} \dfrac{\pi}{4}+\tan^{-1}x & , \quad x\in(-\infty,-1]\cup[1,\infty) \\ -\dfrac{(x+1)}{2} & , \quad x\in(-1,0] \\ \dfrac{x-1}{2} & , \quad x\in(0,1)\end{cases}$

for continuity at x = –1

L.H.L. $=\dfrac{\pi}{4}-\dfrac{\pi}{4}=0$

R.H.L. = 0

so, continuous at x = –1

for continuity at x = 1

L.H.L. = 0

R.H.L. $=\dfrac{\pi}{4}+\dfrac{\pi}{4}=\dfrac{\pi}{2}$

so, not continuous at x = 1

For differentiability at x = –1

L.H.D. $=\dfrac{1}{1+1}=\dfrac{1}{2}$

R.H.D. $=-\dfrac{1}{2}$

so, non differentiable at x = –1

5. Official Ans. by NTA (1)

Sol. f(x) is continuous and differentiable

$f(\pi^{-}) = f(\pi) = f(\pi^{+})$

$-1 = -k_2$

$\boxed{k_2=1}$

$f'(x)=\begin{cases} 2k_1(x-\pi)\,; & x\le\pi \\ -k_2\sin x\,; & x>\pi\end{cases}$

$f'(\pi^{-}) = f'(\pi^{+})$

0 = 0

so, differentiable at x = 0

$f''(x)=\begin{cases} 2k_1\,; & x\le\pi \\ -k_2\cos x\,; & x>\pi\end{cases}$

$f''(\pi^{-}) = f''(\pi^{+})$

$2k_1 = k_2$

$\boxed{k_1=\dfrac{1}{2}}$

$(k_1, k_2)=\left(\dfrac{1}{2}, 1\right)$

6. Official Ans. by NTA (5.00)

Sol. $f(x)=x^5.\sin\frac{1}{x}+5x^2 \quad$ if $x<0$

$f(x)=0 \quad$ if $x=0$

$f(x)=x^5.\cos\frac{1}{x}+\lambda x^2 \quad$ if $x>0$

LHD of $f'(x)$ at $x=0$ is 10

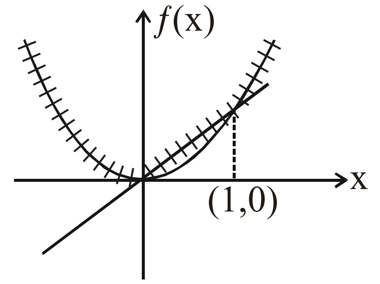
RHD of $f'(x)$ at $x=0$ is 2λ

if $f''(0)$ exists then

$2\lambda = 10 \Rightarrow \lambda = 5$

7. Official Ans. by NTA (1)

Sol. $f(x) = \max(x, x^2)$



Non-differentiable at $x = 0, 1$

$S = \{0, 1\}$

METHOD OF DIFFERENTIATION

1. NTA Ans. (2)

Sol. Put $x = \sin\theta$, $y = \sin\alpha$

$y\sqrt{1-x^2}=k-x\sqrt{1-y^2}$

$\Rightarrow \sin\alpha \cdot \cos\theta + \cos\alpha \cdot \sin\theta = k$

$\Rightarrow \sin(\alpha + \theta) = k$

$\Rightarrow \alpha + \theta = \sin^{-1}k$

$\Rightarrow \sin^{-1}x + \sin^{-1}y = \sin^{-1}k$

$\Rightarrow \frac{1}{\sqrt{1-x^2}}+\frac{1}{\sqrt{1-y^2}}\times\frac{dy}{dx}=0$

at $x = \frac{1}{2}$, $y = \frac{-1}{4}$

$\frac{dy}{dx}=\frac{-\sqrt{5}}{2}$

2. NTA Ans. (1)

Sol. $y(\alpha)=\sqrt{2\frac{(\tan\alpha+\cot\alpha)}{1+\tan^2\alpha}+\frac{1}{\sin^2\alpha}}$, $\alpha\in\left(\frac{3\pi}{4},\pi\right)$

$=\frac{|\sin\alpha+\cos\alpha|}{|\sin\alpha|}=\frac{-(\sin\alpha+\cos\alpha)}{\sin\alpha}$

$= -1 - \cot\alpha$

$y'(\alpha) = \mathrm{cosec}^2\alpha$

$y'\left(\frac{5\pi}{6}\right)=4$

3. NTA Ans. (3)

Sol. $x^k + y^k = a^k$ $(a, k > 0)$

$kx^{k-1}+ky^{k-1}\frac{dy}{dx}=0$

$\frac{dy}{dx}+\left(\frac{x}{y}\right)^{k-1}=0 \Rightarrow k-1=-\frac{1}{3} \Rightarrow k = 2/3$

4. NTA Ans. (1)

ALLEN Ans. (BONUS)

Note: The given information is insufficient to find y(x) for $x<-1$. So, it should be bonus, but NTA retained its answer as options.

Sol. Let $\tan^{-1}x=\theta$, $\theta\in\left(-\frac{\pi}{2},-\frac{\pi}{4}\right)\cup\left(\frac{\pi}{4},\frac{\pi}{2}\right)$

$f(x)=(\sin\theta+\cos\theta)^2-1=\sin2\theta=\frac{2x}{1+x^2}$

Now, $\frac{dy}{dx}=\frac{1}{2}\frac{d}{dx}\sin^{-1}\left(\frac{2x}{1+x^2}\right)$

$=-\frac{1}{1+x^2}, |x|>1$

Since, we can integrate only in the continuous interval. So we have to take integral in two cases separtely namely for $x<-1$ and for $x>1$.

$\Rightarrow y=\begin{cases}-\tan^{-1}x+c_1 & ; \quad x>1\\ -\tan^{-1}x+c_2 & ; \quad x<-1\end{cases}$

so, $c_1=\frac{\pi}{2}$ as $y(\sqrt{3})=\frac{\pi}{6}$

But we cannot find c_2 as we do not have any other additional information for $x<-1$. So, all of the given options may be correct as c_2 is unknown so, it should be bonus.

5. NTA Ans. (BONUS)

Note: This question has been cancelled by NTA as none option matches.

Sol. $x = 2\sin\theta - \sin 2\theta$

$$\Rightarrow \frac{dx}{d\theta} = 2\cos\theta - 2\cos 2\theta = 4\sin\left(\frac{\theta}{2}\right)\sin\left(\frac{3\theta}{2}\right)$$

$y = 2\cos\theta - \cos 2\theta$

$$\Rightarrow \frac{dy}{d\theta} = -2\sin\theta + 2\sin 2\theta = 4\sin\frac{\theta}{2}\cos\frac{3\theta}{2}$$

$$\Rightarrow \frac{dy}{dx} = \cot\left(\frac{3\theta}{2}\right) \Rightarrow \frac{d^2y}{dx^2} = \frac{-\frac{3}{2}\operatorname{cosec}^2\left(\frac{3\theta}{2}\right)}{4\sin\left(\frac{\theta}{2}\right)\sin\frac{3\theta}{2}}$$

$$\Rightarrow \left(\frac{d^2y}{dx^2}\right)_{\theta=\pi} = \frac{3}{8}$$

Alternate :-

$$\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{-2\sin\theta + 2\sin 2\theta}{2\cos\theta - 2\cos 2\theta} = \frac{\sin\theta - \sin 2\theta}{-\cos\theta + \cos 2\theta}$$

$$\frac{d^2y}{dx^2}.\frac{dx}{d\theta} = \frac{(-\cos\theta + \cos 2\theta)(\cos\theta - 2\cos 2\theta) - (\sin\theta - \sin 2\theta)(\sin\theta - 2\sin 2\theta)}{(-\cos\theta + \cos 2\theta)^2}$$

$$\frac{d^2y}{dx^2}.(-2-2) = \frac{(+1+1)(-1-2)-(0)}{(1+1)^2}$$

$$\frac{d^2y}{dx^2}(-4) = \frac{2\times -3}{4} = -\frac{3}{2}$$

$$\frac{d^2y}{dx^2} = \frac{3}{8}$$

Answer should be $\frac{3}{8}$. No options is correct.

6. NTA Ans. (3)

Sol. $f(g(x)) = x$

$f'(g(x))\, g'(x) = 1$

put $x = a$

$\Rightarrow f'(b)\, g'(a) = 1$

$f'(b) = \frac{1}{5}$

7. Official Ans. by NTA (91)

Sol. Put $\cos\alpha = \frac{3}{5}, \sin\alpha = \frac{4}{5} \quad 0 < \alpha < \frac{\pi}{2}$

Now $\frac{3}{5}\cos kx - \frac{4}{5}\sin kx$

$= \cos\alpha \,.\, \cos kx - \sin\alpha \,.\, \sin kx$

$= \cos(\alpha + kx)$

As we have to find derivate at $x = 0$

We have $\cos^{-1}(\cos(\alpha + kx))$

$= (\alpha + kx)$

$$\Rightarrow y = \sum_{k=1}^{6}(\alpha + kx)$$

$$\Rightarrow \left.\frac{dy}{dx}\right|_{at\ x=0} = \sum_{k=x}^{6} k = \frac{6\times 7\times 13}{6} = 91$$

8. Official Ans. by NTA (1)

Sol. $y^2 + \ln(\cos^2 x) = y \quad x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

for $x = 0$ $y = 0$ or 1

Differentiating wrt x

$\Rightarrow 2yy' - 2\tan x = y'$

At (0, 0) $y' = 0$

At (0, 1) $y' = 0$

Differentiating wrt x

$2yy'' + 2(y')^2 - 2\sec^2 x = y''$

At (0, 0) $y'' = -2$

At (0, 1) $y'' = 2$

$\therefore |y''(0)| = 2$

9. Official Ans. by NTA (2)

Sol. $\left(a + \sqrt{2}b\cos x\right)\left(a - \sqrt{2}b\cos y\right) = a^2 - b^2$

$$\Rightarrow a^2 - \sqrt{2}ab\cos y + \sqrt{2}ab\cos x - 2b^2\cos x\cos y = a^2 - b^2$$

Differentiating both sides :

$$0 - \sqrt{2}ab\left(-\sin y\frac{dy}{dx}\right) + \sqrt{2}ab(-\sin x)$$

$$-2b^2\left[\cos x\left(-\sin y\frac{dy}{dx}\right) + \cos y(-\sin x)\right] = 0$$

At $\left(\frac{\pi}{4}, \frac{\pi}{4}\right)$:

$$ab\frac{dy}{dx} - ab - 2b^2\left(-\frac{1}{2}\frac{dy}{dx} - \frac{1}{2}\right) = 0$$

$$\Rightarrow \frac{dx}{dy} = \frac{ab + b^2}{ab - b^2} = \frac{a+b}{a-b} \ ; a, b > 0$$

10. Official Ans. by NTA (2)

Sol. Let f = $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$

Put x = tan θ ⇒ θ = $\tan^{-1}$ x

$f = \tan^{-1}\left(\frac{\sec\theta-1}{\tan\theta}\right)$

$f = \tan^{-1}\left(\frac{1-\cos\theta}{\sin\theta}\right) = \frac{\theta}{2}$

$f = \frac{\tan^{-1}x}{2} \Rightarrow \frac{df}{dx} = \frac{1}{2(1+x^2)}$ ....(i)

Let $g = \tan^{-1}\left(\frac{2x\sqrt{1-x^2}}{1-2x^2}\right)$

Put x = sin θ ⇒ θ = $\sin^{-1}$ x

$g = \tan^{-1}\left(\frac{2\sin\theta\cos\theta}{1-2\sin^2\theta}\right)$

$g = \tan^{-1}(\tan 2\theta) = 2\theta$

$g = 2\sin^{-1}x$

$\frac{dg}{dx} = \frac{2}{\sqrt{1-x^2}}$ ....(ii)

$\frac{df}{dg} = \frac{1}{2(1+x^2)}\,\frac{\sqrt{1-x^2}}{2}$

at $x = \frac{1}{2}$ $\left(\frac{df}{dg}\right)_{x=\frac{1}{2}} = \frac{\sqrt{3}}{10}$

INDEFINITE INTEGRATION

1. NTA Ans. (1)

Sol. $\int\frac{\cos x\,dx}{\sin^3 x\left(1+\sin^6 x\right)^{2/3}} = \frac{-6}{-6}\int\frac{\cos x\,dx}{\sin^7 x\left(\frac{1}{\sin^6 x}+1\right)^{2/3}}$

$= -\frac{1}{6}\times 3\left(\frac{1}{\sin^6 x}+1\right)^{\frac{1}{3}}+c$

$= -\frac{1}{2}\frac{\left(1+\sin^6 x\right)^{\frac{1}{3}}}{\sin^2 x}+c$

Hence, λ = 3 and $f(x) = -\frac{1}{2\sin^2 x}$

so, $\lambda f\left(\frac{\pi}{3}\right) = -2$

REMARK : Technically, this question should be marked as bonus. Because $f(x)$ and λ cannot be found uniquely.
For example, another such $f(x)$ and λ can be

$-\frac{\left(1+\sin^6 x\right)^{\frac{1}{6}}}{2\sin^2 x}$ and 6 respectively.

2. NTA Ans. (1)

Sol. $I = \int\frac{d\theta}{\cos^2\theta(\tan 2\theta+\sec 2\theta)}$

$= \int\frac{\sec^2\theta\,d\theta}{\frac{2\tan\theta}{1-\tan^2\theta}+\frac{1+\tan^2\theta}{1-\tan^2\theta}} = \int\frac{(1-\tan^2\theta)\sec^2\theta\,d\theta}{(1+\tan\theta)^2}$

tanθ = t ⇒ sec²θ dθ = dt

$I = \int\frac{1-t^2}{(1+t)^2}dt = \int\frac{(1-t)(1+t)}{(1+t)^2}dt$

$= \int\frac{1}{1+t}-\frac{t}{1+t}\,dt$

$= \ell n|1+t| - \int\left(\frac{1+t}{1+t}-\frac{1}{1+t}\right)dt$

$= \ell n|1+t| - t + \ell n|1+t| = 2\ell n|1+t| - t + C$

$= 2\ell n|1+\tan\theta| - \tan\theta + C$

λ = –1, f(θ) = 1 + tanθ

3. NTA Ans. (1)

Sol. $I = \int\frac{dx}{(x+4)^{\frac{8}{7}}(x-3)^{\frac{6}{7}}} = \int\frac{dx}{\left(\frac{x+4}{x-3}\right)^{\frac{8}{7}}(x-3)^2}$

Let $\frac{x+4}{x-3} = t \Rightarrow \frac{dx}{(x-3)^2} = -\frac{1}{7}dt$

$\Rightarrow I = -\frac{1}{7}\int\frac{dt}{t^{8/7}} = -\frac{1}{7}\int t^{-8/7}dt$

$= t^{-1/7}+C = +\left(\frac{x+4}{x-3}\right)^{-1/7}+C = \left(\frac{x-3}{x+4}\right)^{1/7}+C$

4. Official Ans. by NTA (3)

Sol. Put $x = \tan^2\theta \Rightarrow dx = 2\tan\theta\sec^2\theta\, d\theta$

$$\int \theta.(2\tan\theta\cdot\sec^2\theta)d\theta$$

$\downarrow$ $\downarrow$

I II (By parts)

$$=\theta.\tan^2\theta - \int \tan^2\theta\, d\theta$$

$$=\theta.\tan^2\theta - \int (\sec^2\theta - 1)\, d\theta$$

$= \theta(1 + \tan^2\theta) - \tan\theta + C$

$$=\tan^{-1}\left(\sqrt{x}\right)(1+x) - \sqrt{x} + C$$

5. Official Ans. by NTA (4)

Sol. $\int\left(\frac{x}{x\sin x+\cos x}\right)^2 dx = \int\left(\frac{x}{\cos x}\right).\frac{x\cos x\, dx}{(x\sin x+\cos x)^2}$

$$= \frac{x}{\cos x}\left(-\frac{1}{x\sin x+\cos x}\right)$$

$$+\int\left(\frac{\cos x + x\sin x}{\cos^2 x}\right)\left(\frac{1}{x\sin x+\cos x}\right)dx =$$

$$-\frac{x\sec x}{x\sin x+\cos x} + \int \sec^2 x\, dx$$

$$= -\frac{x\sec x}{x\sin x+\cos x} + \tan x + C$$

6. Official Ans. by NTA (1)

Sol. $e^{2x} + 2e^x - e^{-x} - 1$

$= e^x(e^x + 1) - e^{-x}(e^x + 1) + e^x = [(e^x + 1)(e^x - e^{-x}) + e^x]$

so $I = \int (e^x+1)(e^x-e^{-x})e^{e^x+e^{-x}} + \int e^x\cdot e^{e^x+e^{-x}}dx =$

$$(e^x+1)e^{e^x+e^{-x}} - \int e^x.e^{e^x+e^{-x}}dx + \int e^x.e^{e^x+e^{-x}}dx$$

$= (e^x+1)e^{e^x+e^{-x}} + C$ $\therefore$ $g(x) = e^x + 1 \Rightarrow g(0) = 2$

7. Official Ans. by NTA (4)

Sol. $\int \frac{\cos\theta\, d\theta}{5+7\sin\theta-2\cos^2\theta}$

$$\int \frac{\cos\theta\, d\theta}{3+7\sin\theta+2\sin^2\theta} \qquad \boxed{\begin{array}{l}\sin\theta = t\\ \cos\theta d\theta = dt\end{array}}$$

$$\int \frac{dt}{2t^2+7t+3} = \int \frac{dt}{(2t+1)(t+3)} =$$

$$\frac{1}{5}\int\left(\frac{2}{2t+1} - \frac{1}{t+3}\right)dt$$

$$= \frac{1}{5}\ell n\left|\frac{2t+1}{t+3}\right| + C = \frac{1}{5}\ell n\left|\frac{2\sin\theta+1}{\sin\theta+3}\right| + C$$

$A = \frac{1}{5}$ and $B(\theta) = \frac{2\sin\theta+1}{\sin\theta+3}$

DEFINITE INTEGRATION

1. NTA Ans. (4)

Sol. $2\cos^2\theta - 5\sin\theta + 4\sin^2\theta = 0$

$3\sin^2\theta - 5\sin\theta + 2 = 0$

$\sin\theta = \frac{1}{2}, 2$ (Rejected)

$$\int_{\theta_1}^{\theta_2} \cos^2 3\theta d\theta = \int_{\pi/6}^{5\pi/6} \frac{1+\cos 6\theta}{2} d\theta$$

$$= \frac{1}{2}\left(\frac{5\pi}{6} - \frac{\pi}{6}\right) = \frac{2\pi}{6} = \frac{\pi}{3}$$

2. NTA Ans. (3)

Sol. $4\alpha\left[\int_{-1}^{0} e^{\alpha x}dx + \int_{0}^{2} e^{-\alpha x}dx\right] = 5$

$$\Rightarrow 4\alpha\left(\left[\frac{e^{\alpha x}}{\alpha}\right]_{-1}^{0} + \left[\frac{e^{-\alpha x}}{-\alpha}\right]_{0}^{2}\right) = 5$$

$\Rightarrow 4e^{-2\alpha} + 4e^{-\alpha} - 3 = 0$

Let $e^{-\alpha} = t$, $4t^2 + 4t - 3 = 0$, $t = \frac{1}{2}, \frac{-3}{2}$ (Rejected)

$e^{-\alpha} = \frac{1}{2}$; $\alpha = \ell n2$

3. NTA Ans. (1)

ALLEN Ans. (1 OR 3)

Note: In this Question, both options (1) as well as (3) are correct, but NTA accepte only option (1).

Sol. f(x + 1) = f (a + b – x)

$$I=\frac{1}{(a+b)}\int_a^b x\left(f(x)+f(x+1)\right)dx \quad .....(1)$$

$$I=\frac{1}{(a+b)}\int_a^b (a+b-x)\left(f(x+1)+f(x)\right)dx \quad ...(2)$$

from (1) and (2)

$$2I=\int_a^b\left(f(x)+f(x+1)\right)dx$$

$$2I=\int_a^b f\left(a+b-x\right)dx+\int_a^b f(x+1)dx$$

$$2I=2\int_a^b f(x+1)dx\Rightarrow I=\int_a^b f(x+1)dx$$

$$=\int_{a+1}^{b+1} f(x)dx$$

OR

$$I=\frac{1}{(a+b)}\int_a^b x\left(f(x)+f(x+1)\right)dx \quad ...(1)$$

$$=\frac{1}{(a+b)}\int_a^b (a+b-x)\left(f(a+b-x)+f(a+b+1-x)\right)dx$$

$$I=\frac{1}{(a+b)}\int_a^b (a+b-x)\left(f(x+1)+f(x)\right)dx \quad ...(2)$$

equation (1) + (2)

$$2I=\frac{1}{(a+b)}\int_a^b (a+b)\left(f(x+1)+f(x)\right)dx$$

$$I=\frac{1}{2}\left[\int_a^b f(x+1)dx+\int_a^b f(x)dx\right]$$

$$=\frac{1}{2}\left[\int_a^b f(a+b+1-x)dx+\int_a^b f(x)dx\right]$$

$$=\frac{1}{2}\left[\int_a^b f(x)dx+\int_a^b f(x)dx\right]$$

$$I=\int_a^b f(x)dx$$

Let x = T + 1

$$=\int_{a-1}^{b-1} f(T+1)dT$$

$$I=\int_{a-1}^{b-1} f(x+1)dx$$

4. NTA Ans. (1)

Sol. $$f(x)=\frac{1}{\sqrt{2x^3-9x^2+12x+4}}$$

$$f'(x)=\frac{-6(x-1)(x-2)}{2\left(2x^3-9x^2+12x+4\right)^{3/2}}$$

$\therefore$ $f(x)$ is decreasing in (1,2)

$$f(1)=\frac{1}{3};\ f(2)=\frac{1}{\sqrt{8}}$$

$$\frac{1}{3}<I<\frac{1}{\sqrt{8}}\Rightarrow I^2\in\left(\frac{1}{9},\frac{1}{8}\right)$$

(1) Option

5. NTA Ans. (1)

Sol. Using L.H. Rule

$$\lim_{x\to 0}\frac{x\sin(10x)}{1}=0$$

(1) Option

6. NTA Ans. (4)

Sol. $$I=\int_0^{2\pi}\frac{x\sin^8 x}{\sin^8 x+\cos^8 x}dx \quad ....(1)$$

$$=\left[\int_0^{\pi}\frac{x\sin^8 x}{\sin^8 x\cos^8 x}dx+\int_0^{\pi}\frac{(2\pi-x)\sin^8 x}{\sin^8 x+\cos^8 x}dx\right]$$

$$=2\pi\int_0^{\pi}\frac{\sin^8 x}{\sin^8 x+\cos^8 x}dx$$

$$I=2\pi\left[\int_0^{\pi/2}\frac{\sin^8 x}{\sin^8 x+\cos^8 x}dx+\int_0^{\pi/2}\frac{\cos^8 xdx}{\sin^8 x+\cos^8 x}dx\right]$$

$$=2\pi\int_0^{\pi/2}1dx=2\pi.\frac{\pi}{2}=\pi^2$$

7. NTA Ans. (3)

Sol. $f(x) = a + bx + cx^2$

$$\int_0^1 f(x)dx = \left[ax + \frac{bx^2}{2} + \frac{cx^3}{3}\right]_0^1$$

$$= a + \frac{b}{2} + \frac{c}{3} = \frac{1}{6}[6a + 3b + c]$$

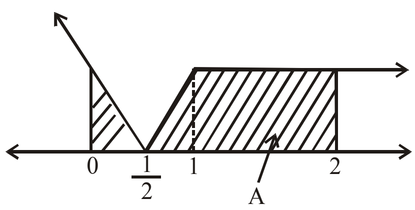
$$= \frac{1}{6}\left[f(0) + f(1) + 4f\left(\frac{1}{2}\right)\right]$$

8. Official Ans. by NTA (1.50)

Sol. $\int_0^2 |\,|x-1| - x\,|\,dx$

Let f(x) $||x - 1| - x|$

$$= \begin{cases} 1, & x \geq 1 \\ |1 - 2x|, & x \leq 1 \end{cases}$$



$$A = \frac{1}{2} + 1 = \frac{3}{2}$$

OR

$$\int_0^{1/2} (1-2x)dx + \int_{1/2}^{1} (2x-1) + \int_0^2 1dx$$

$$= \left[x - x^2\right]_0^{\frac{1}{2}} + \left[x^2 - x\right]_{1/2}^1 + \left[x\right]_1^2$$

$$= \boxed{3/2}$$

9. Official Ans. by NTA (1.0)

Sol. $3 < 3x < 6$

Take cases when $3 < 3x < 4$, $4 < 3x < 5$, $5 < 3x < 6$;

Now $\int_1^2 |2x - [3x]|\,dx$

$$= \int_1^{4/3} (3-2x)dx + \int_{4/3}^{5/3} (4-2x)\,dx + \int_{5/3}^{2} (5-2x)dx$$

$$= \frac{2}{9} + \frac{3}{9} + \frac{4}{9} = 1$$

10. Official Ans. by NTA (1)

Sol. $\int_{-\pi}^{\pi} |\pi - |x||dx = 2\int_0^{\pi} |\pi - x|dx$

$$= 2\int_0^{\pi} (\pi - x)\,dx$$

$$= 2\left[\pi x - \frac{x^2}{2}\right]_0^{\pi} = \pi^2$$

11. Official Ans. by NTA (1)

Sol. $\int_0^{1/2} \frac{((x^2-1)+1)}{(1-x^2)^{3/2}}dx$

$$\int_0^{1/2} \frac{dx}{(1-x^2)^{3/2}} - \int_0^{1/2} \frac{dx}{\sqrt{1-x^2}}$$

$$\int_0^{1/2} \frac{x^{-3}}{(x^{-2}-1)^{3/2}}dx - (\sin^{-1}x)_0^{1/2}$$

Let $x^{-2} - 1 = t^2 \Rightarrow x^{-3}\,dx = -\,t\,dt$

$$\int_{\infty}^{\sqrt{3}} \frac{-t\,dt}{t^3} - \frac{\pi}{6} = \int_{\sqrt{3}}^{\infty} \frac{dt}{t^2} - \frac{\pi}{6} = \frac{1}{\sqrt{3}} - \frac{\pi}{6} = \frac{k}{6}$$

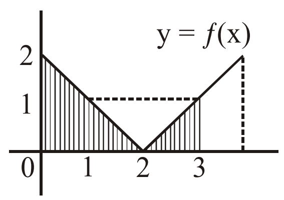
$$k = 2\sqrt{3} - \pi$$

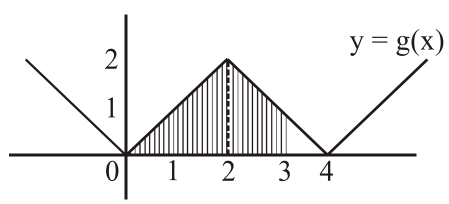
12. Official Ans. by NTA (4)

Sol. $\int_0^3 g(x) - f(x) = \int_0^3 ||x-2|-2|dx - \int_0^3 |x-2|dx$

$$= \left(\frac{1}{2}\times 2\times 2 + 1 + \frac{1}{2}\times 1\times 1\right) - \left(\frac{1}{2}\times 2\times 2 + \frac{1}{2}\times 1\times 1\right)$$

$$= \left(2 + 1 + \frac{1}{2}\right) - \left(2 + \frac{1}{2}\right) = 1$$





13. Official Ans. by NTA (4)

Sol. $f(x) = \int_1^3 \frac{\sqrt{x}\,dx}{(1+x)^2} = \int_1^{\sqrt{3}} \frac{t.2t\,dt}{(1+t^2)^2}$ $\left(\text{put } \sqrt{x} = t\right)$

$= \left(-\frac{t}{1+t^2}\right)_1^{\sqrt{3}} + (\tan^{-1} t)_1^{\sqrt{3}}$ [Appling by parts]

$= -\left(\frac{\sqrt{3}}{4} - \frac{1}{2}\right) + \frac{\pi}{3} - \frac{\pi}{4}$

$= \frac{1}{2} - \frac{\sqrt{3}}{4} + \frac{\pi}{12}$

14. Official Ans. by NTA (3)

Sol. $I = \int_{\pi/6}^{\pi/3} ((2\tan^3 x \cdot \sec^2 x \cdot \sin^4 3x) + (3\tan^4 x \ \cdot \sin^3 3x$

$\cdot \cos 3x))dx$

$\Rightarrow I = \frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} d((\sin 3x)^4 (\tan x)^4)$

$\Rightarrow I = ((\sin 3x)^4 (\tan x)^4)_{\pi/6}^{\pi/3}$

$\Rightarrow I = -\frac{1}{18}$

15. Official Ans. by NTA (21)

Sol. $\int_0^n \{x\}dx = n\int_0^1 \{x\}dx = n\int_0^1 x\ dx = \frac{n}{2}$

$\int_0^n [x]dx = \int_0^n (x - \{x\})dx = \frac{n^2}{2} - \frac{n}{2}$

$\Rightarrow \left(\frac{n^2 - n}{2}\right)^2 = \frac{n}{2} \cdot 10 \cdot n(n-1)$ (where $n > 1$)

$\Rightarrow \frac{n-1}{4} = 5 \Rightarrow n = 21$

16. Official Ans. by NTA (4)

Sol. $I = \int_{-\pi/2}^{\pi/2} \frac{1}{1+e^{\sin x}}dx$ ....(1)

Apply King property

$I = \int_{-\pi/2}^{\pi/2} \frac{1}{1+e^{-\sin x}}dx = \int_{-\pi/2}^{\pi/2} \frac{e^{\sin x}}{1+e^{\sin x}}dx$ ...(2)

Add (1) & (2)

$2I = \int_{-\pi/2}^{\pi/2} dx = \pi$

$I = \frac{\pi}{2}$

17. Official Ans. by NTA (1)

Sol. $I_1 = \int_0^1 \left(1 - x^{50}\right)^{100} dx$ and $I_2 = \int_0^1 \left(1 - x^{50}\right)^{101} dx$

and $I_1 = \lambda I_2$

$I_2 = \int_0^1 \left(1 - x^{50}\right)^{101} dx$

$I_2 = \int_0^1 \left(1 - x^{50}\right)\left(1 - x^{50}\right)^{100} dx$

$I_2 = \int_0^1 \left(1 - x^{50}\right)dx - \int_0^1 x^{50}.\left(1 - x^{50}\right)^{100} dx$

$I_2 = I_1 - \int_0^1 \underbrace{x}_{\mathrm{I}}.\underbrace{x^{49}.\left(1 - x^{50}\right)^{100}}_{\mathrm{II}} dx$

Now apply IBP

$I_2 = I_1 - \left[x\int x^{49}.\left(1-x^{50}\right)^{100}dx - \int \frac{d(x)}{dx}.\int \frac{d(x)}{dx}.\int x^{49}.\left(1-x^{50}\right)^{100}dx\right]$

Let $(1 - x^{50}) = t$

$-50x^{49}dx = dt$

$I_2 = I_1 - \left[x.\left(-\frac{1}{50}\right)\frac{\left(1-x^{50}\right)^{101}}{101}\Bigg|_{x=0}^{x=1} - \int_0^1 \left(-\frac{1}{50}\right)\frac{\left(1-x^{50}\right)^{101}}{101}dx\right]$

$I_2 = I_1 - 0 - \frac{1}{50}.\frac{1}{101}.I_2 = I_1 - \frac{1}{5050}I_2$

$I_2 + \frac{1}{5050}I_2 = I_1 \Rightarrow \frac{5051}{5050}I_2 = I_1$

$\therefore\ \alpha = \frac{5050}{5051}$

$I_2 = \frac{5050}{5051}I_1$

$\because I_2 = \alpha.I_1$

18. Official Ans. by NTA (4)

Sol. $\int_1^2 e^x.x^x\left(2+\log_e x\right)dx$

$\int_1^2 e^x\left(2x^x+x^x\log_e x\right)dx$

$\int_1^2 e^x\left(\underbrace{x^x}_{f(x)}+\underbrace{x^x\left(1+\log_e x\right)}_{f'(x)}\right)dx$

$\left(e^x.x^x\right)_1^2=4e^2-e$

TANGENT & NORMAL

1. NTA Ans. (2)

Sol. $x^2 + 2xy - 3y^2 = 0$

m_N = slope of normal drawn to curve at (2,2) is –1

L : x + y = 4.

perpendicular distance of L from (0,0)

$=\frac{|0+0-4|}{\sqrt{2}}=2\sqrt{2}$

(2) Option

2. NTA Ans. (4.00)

Sol. Let $P(\alpha,\beta)$

so, $\beta^2-3\alpha^2+\beta+10=0$ ...(i)

Now, $2yy'-6x+y'=0$

$\Rightarrow m=\frac{6\alpha}{2\beta+1}$ ....(ii)

Also, $\frac{\beta-\frac{3}{2}}{\alpha}=-\frac{1}{m}$

$\Rightarrow \frac{2\beta-3}{2\alpha}=-\frac{(2\beta+1)}{6\alpha}$ (from (ii))

$\Rightarrow \beta=1\Rightarrow \alpha^2=4$ (from (1))

Hence, $|m|=\frac{12}{3}=4.00$

3. Official Ans. by NTA (3)

Sol. Slope of tangent to the curve $y=x+\sin y$

at (a, b) is $\frac{2-\frac{3}{2}}{\frac{1}{2}-0}=1$

$\Rightarrow \left.\frac{dy}{dx}\right]_{x=a}=1$

$\frac{dy}{dx}=1+\cos y.\frac{dy}{dx}$ (from equation of curve)

$\left.\frac{dy}{dx}\right]_{x=a}=1+\cos b.\left.\frac{dy}{dx}\right]_{x=a}$

$\Rightarrow$ cos b = 0

$\Rightarrow$ sin b = ±1

Now, from curve y = x + sin y

b = a + sin b

$\Rightarrow$ |b – a| = |sin b| = 1

4. Official Ans. by NTA (2)

Sol. Given equation of curve

$y = (1 + x)^{2y} + \cos^2(\sin^{-1}x)$

at $\boxed{x=0}$

$y = (1 + 0)^{2y} + \cos^2(\sin^{-1}0)$

$y = 1 + 1$

$\boxed{y=2}$

So we have to find the normal at (0, 2)

Now $y = e^{2y\ln(1+x)} + \cos^2\left(\cos^{-1}\sqrt{1-x^2}\right)$

$y = e^{2y\ln(1+x)} + \left(\sqrt{1-x^2}\right)^2$

$y = e^{2y\ln(1+x)} + (1-x^2)$ ...(1)

Now differentiate w.r.t. x

$y' = e^{2y\ln(1+x)}\left[2y\cdot\left(\frac{1}{1+x}\right) + \ln(1+x).2y'\right] - 2x$

Put x = 0 & y = 2

$y' = e^{2\times 2\ln 1}\left[2\times 2\left(\frac{1}{1+0}\right) + \ln(1+0).2y'\right] - 2\times 0$

$y' = e^0[4 + 0] - 0$

$y' = 4$ = slope of tangent to the curve

so slope of normal to the curve $= -\frac{1}{4}$ $\{m_1m_2=-1\}$

Hence equation of normal at (0, 2) is

$y - 2 = -\frac{1}{4}(x - 0)$

$\Rightarrow 4y - 8 = -x$

$\Rightarrow \boxed{x + 4y = 8}$

5. Official Ans. by NTA (1)

Sol. $\frac{d}{dt}(6a^2) = 3.6 \Rightarrow 12a\frac{da}{dt} = 3.6$

$a\frac{da}{dt} = 0.3$

$\frac{dv}{dt} = \frac{d}{dt}(a^3) = 3a\left(a\frac{da}{dt}\right)$

$= 3 \times 10 \times 0.3 = 9$

6. Official Ans. by NTA (4)

Sol. $y = e^x \Rightarrow \frac{dy}{dx} = e^x$

$m = \left(\frac{dy}{dx}\right)_{(c,e^c)} = e^c$

$\Rightarrow$ Tangent at (c, e^c)

$y - e^c = e^c(x - c)$

it intersect x-axis

Put $y = 0 \Rightarrow x = c - 1$

......(1)

Now $y^2 = 4x \Rightarrow \frac{dy}{dx} = \frac{2}{y} \Rightarrow \left(\frac{dy}{dx}\right)_{(1,\,2)} = 1$

$\Rightarrow$ Slope of normal = –1

Equation of normal $y - 2 = -1(x - 1)$

$x + y = 3$ it intersect x-axis

Put $y = 0 \Rightarrow x = 3$

......(2)

Points are same

$\Rightarrow x = c - 1 = 3$

$\Rightarrow c = 4$

7. Official Ans. by NTA (0.50)

Sol. $y = x^2 - 3x + 2$

At x-axis $y = 0 = x^2 - 3x + 2$

$x = 1, 2$

$\frac{dy}{dx} = 2x - 3$

A(1, 0) B(2, 0)

$\left(\frac{dy}{dx}\right)_{x=1} = -1$ and $\left(\frac{dy}{dx}\right)_{x=2} = 1$

$x + y = a \Rightarrow \frac{dy}{dx} = -1$ So A(1, 0) lies on it

$\Rightarrow 1 + 0 = a \Rightarrow \boxed{a=1}$

$x - y = b \Rightarrow \frac{dy}{dx} = 1$ So B(2, 0) lies on it

$2 - 0 = b \Rightarrow \boxed{b=2}$

$\frac{a}{b} = 0.50$

8. Official Ans. by NTA (4)

Sol. $\dfrac{f(t_2)-f(t_1)}{t_2-t_1}=2at+b$

$\dfrac{a(t_2^2-t_1^2)+b(t_2-t_1)}{t_2-t_1}=2at+b$

$\Rightarrow a(t_2+t_1)+b=2at+b$

$\Rightarrow t=\dfrac{t_1+t_2}{2}$

MONOTONICITY

1. NTA Ans. (4)

Sol. $f(0)=11$

$f(1)=16$

$\dfrac{f(1)-f(0)}{1-0}=3c^2-8c+8$

$\Rightarrow 3c^2-8c+8=5$

$\Rightarrow 3c^2-8c+3=0$

$c\in[0,1] \Rightarrow c=\dfrac{4-\sqrt{7}}{3}$

2. NTA Ans. (2)

Sol. Using LMVT in $[-7,-1]$

$\dfrac{f(-1)-f(-7)}{-1-(-7)}\le 2$

$f(-1)-f(-7)\le 12$

$\Rightarrow f(-1)\le 9$ ....(1)

Using LMVT in $[-7,0]$

$\dfrac{f(0)-f(-7)}{0-(-7)}\le 2$

$f(0)-f(-7)\le 14$

$f(0)\le 11$ ....(2)

from (1) and (2)

$f(0)+f(-1)\le 20$

3. NTA Ans. (2)

ALLEN Ans. (BONUS)

Note: None of the options is correct for all *f* in S. Thus, it should be bonus, but NTA did not accept it.

Sol. Option (1), (2), (3) are incorrect for $f(x)$ = constant and option (4) is incorrect

$\dfrac{f(1)-f(c)}{1-c}=f'(a)$ where $c<a<1$ (use LMVT)

Also for $f(x)=x^2$ option (4) is incorrect.

4. NTA Ans. (2)

Sol. $\dfrac{9+\alpha}{21}=\dfrac{16+\alpha}{28}\Rightarrow\alpha=12$

Also, $f'(x)=\dfrac{7x}{x^2+12}\times\dfrac{x^2-12}{7x^2}=\dfrac{x^2-12}{x(x^2+12)}$

Hence, $c=2\sqrt{3}$

Now, $f''(c)=\dfrac{1}{12}$

5. NTA Ans. (1)

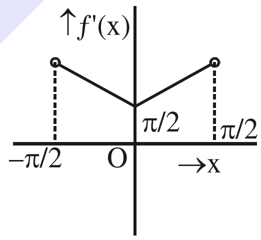
Sol. $f(x)$ is an odd function.

Now, if $x\ge 0$, then $f(x)=x\cos^{-1}(-\sin x)$

$=x\left(\dfrac{\pi}{2}-\sin^{-1}(-\sin x)\right)=x\left(\dfrac{\pi}{2}+x\right)$

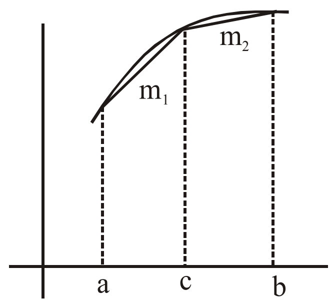
Hence, $f(x)=\begin{cases}x\left(\dfrac{\pi}{2}+x\right) & ; \quad x\in\left[0,\dfrac{\pi}{2}\right]\\ x\left(\dfrac{\pi}{2}-x\right) & ; \quad x\in\left[-\dfrac{\pi}{2},0\right)\end{cases}$

so, $f'(x)=\begin{cases}\dfrac{\pi}{2}+2x & ; \quad x\in\left[0,\dfrac{\pi}{2}\right)\\ \dfrac{\pi}{2}-2x & ; \quad x\in\left(-\dfrac{\pi}{2},0\right)\end{cases}$



6. NTA Ans. (3)

Sol.



it is clear from graph that $m_1>m_2$

$\Rightarrow \dfrac{f(c)-f(a)}{c-a}>\dfrac{f(b)-f(c)}{b-c}$

$\Rightarrow \dfrac{f(c)-f(a)}{f(b)-f(c)}>\dfrac{c-a}{b-c}$

7. Official Ans. by NTA (1)

Sol. $f'(x)=\dfrac{\dfrac{x}{1+x}-\ell n(1+x)}{x^2}$

$=\dfrac{x-(1+x)\,\ell n(1+x)}{x^2(1+x)}$

Suppose $h(x) = x - (1 + x)\ \ell n(1+x)$

$\Rightarrow\ h'(x) = 1 - \ell n(1+x) - 1 = -\ell n(1+x)$

$h'(x) > 0,\ \forall\ x \in (-1, 0)$

$h'(x) < 0,\ \forall\ x \in (0, \infty)$

$h(0) = 0 \Rightarrow h'(x) < 0\ \forall\ x \in (-1, \infty)$

$\Rightarrow f'(x) < 0\ \forall\ x\in(-1, \infty)$

$\Rightarrow$ f(x) is a decreasing function for all $x\in(-1, \infty)$

8. Official Ans. by NTA (2)

Sol. $f(x) = (3x - 7)x^{2/3}$

$\Rightarrow f(x) = 3x^{5/3} - 7x^{2/3}$

$\Rightarrow f'(x) = 5x^{2/3} - \dfrac{14}{3x^{1/3}} = \dfrac{15x-14}{3x^{1/3}} > 0$

+ – +
0 14/15

$\therefore\ f'(x) > 0\ \forall\ x\in(-\infty, 0)\cup\left(\dfrac{14}{15}, \infty\right)$

9. Official Ans. by NTA (3)

Sol. $f(2) = 8, f'(2) = 5, f'(x)\geq 1, f''(x) \geq 4, \forall x\in(1,6)$

$f''(x) = \dfrac{f'(5)-f'(2)}{5-2} \geq 4 \Rightarrow f'(5) \geq 17$ ...(1)

$f'(x) = \dfrac{f(5)-f(2)}{5-2} \geq 1 \Rightarrow f(5) \geq 11$ ...(2)

$\overline{f'(5)+f(5)\geq 28}$

10. Official Ans. by NTA (1)

Sol. $f(0) = f(1) = f'(0) = 0$

Apply Rolles theorem on $y = f(x)$ in $x \in [0, 1]$

$f(0) = f(1) = 0$

$\Rightarrow f'(\alpha) = 0$ where $\alpha \in (0, 1)$

Now apply Rolles theorem on $y = f'(x)$ in $x \in [0, \alpha]$

$f'(0) = f'(\alpha) = 0$ and $f'(x)$ is continuous and differentiable

$\Rightarrow f''(\beta) = 0$ for some , $\beta \in (0, \alpha) \in (0, 1)$

$\Rightarrow f''(x) = 0$ for some $x \in (0, 1)$

11. Official Ans. by NTA (3)

Sol. $f(x) = x\log_e x$

$f'(x)\big|_{(c,f(c))} = \dfrac{e-0}{e-1}$

$f'(x) = 1 + \log_e x$

$f'(x)\big|_{(c,f(c))} = 1+\log_e c = \dfrac{e}{e-1}$

$\log_e c = \dfrac{e-(e-1)}{e-1} = \dfrac{1}{e-1} \Rightarrow c = e^{\frac{1}{e-1}}$

MAXIMA & MINIMA

1. NTA Ans. (2)

Sol. $\lim\limits_{x\to 0}\left(2+\dfrac{f(x)}{x^3}\right) = 4$

$\Rightarrow f(x) = 2x^3 + ax^4 + bx^5$

$f'(x) = 6x^2 + 4ax^3 + 5bx^4$

$f'(1) = 0, f'(-1) = 0$

$a = 0, b = \dfrac{-6}{5} \Rightarrow f(x) = 2x^3 - \dfrac{6}{5}x^5$

$f'(x) = 6x^2 - 6x^4$

$= 6x^2(1 - x)(1 + x)$

Sign scheme for f'(x)

–ve +ve +ve –ve
–1 0 1

Minima at x = –1

Maxima at x = 1

2. NTA Ans. (3)

Sol. $f''(x) = \lambda(x - 1)$

$f'(x) = \dfrac{\lambda x^2}{2} - \lambda x + C \Rightarrow f'(-1) = 0 \Rightarrow c = \dfrac{-3\lambda}{2}$

$f(x) = \dfrac{\lambda x^3}{6} - \dfrac{\lambda x^2}{2} - \dfrac{3\lambda}{2}x + d$

$f(1) = -6 \Rightarrow -11\lambda + 6d = -36$ ...(i)

$f(-1) = 10 \Rightarrow 5\lambda + 6d = 60$ ...(ii)

from (i) & (ii) $\lambda = 6$ & $d = 5$

$f(x) = x^3 - 3x^2 - 9x + 5$

Which has minima at x = 3

Ans. 3.00

3. NTA Ans. (1)

Sol. $F'(x) = x^2 g(x) = x^2\int_1^x f(u)\,du \Rightarrow F'(1) = 0$

$F''(x) = x^2 f(x) - 2x\int_1^x f(u)\,du$

$F''(1) = 1.f(1) - 2 \times 0$

$F''(1) = 3$

$F'(1) = 0$ and $F''(1) = 3 > 0$ So, Minima

4. NTA Ans. (3)

Sol. Let thickness of ice be 'h'.

Vol. of ice = $v = \frac{4\pi}{3}\left((10+h)^3 - 10^3\right)$

$\frac{dv}{dt} = \frac{4\pi}{3}\left(3(10+h)^2\right).\frac{dh}{dt}$

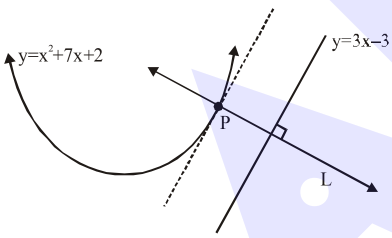
Given $\frac{dv}{dt} = 50cm^3/min$ and h = 5cm

$\Rightarrow 50 = \frac{4\pi}{3}\left(3(10+5)^2\right)\frac{dh}{dt}$

$\Rightarrow \frac{dh}{dt} = \frac{50}{4\pi \times 15^2} = \frac{1}{18\pi}$ cm/min

5. Official Ans. by NTA (4)

Sol.



Let L be the common normal to parabola $y = x^2 + 7x + 2$ and line $y = 3x - 3$

$\Rightarrow$ slope of tangent of $y = x^2 + 7x + 2$ at P = 3

$\Rightarrow \left.\frac{dy}{dx}\right]_{\text{For P}} = 3$

$\Rightarrow 2x + 7 = 3 \Rightarrow x = -2 \Rightarrow y = -8$

So P(–2, –8)

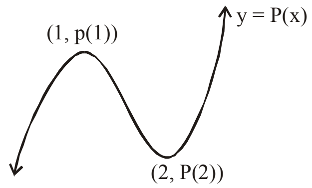
Normal at P : $x + 3y + C = 0$

$\Rightarrow C = 26$ (P satisfies the line)

$\boxed{\text{Normal}: x + 3y + 26 = 0}$

6. Official Ans. by NTA (4)

Sol. Since p(x) has realtive extreme at



x = 1 & 2

so $p'(x) = 0$ at x = 1 & 2

$\Rightarrow p'(x) = A(x-1)(x-2)$

$\Rightarrow p(x) = \int A(x^2 - 3x + 2)dx$

$p(x) = A\left(\frac{x^3}{3} - \frac{3x^2}{2} + 2x\right) + C$ ...(1)

P(1) = 8

From (1)

$8 = A\left(\frac{1}{3} - \frac{3}{2} + 2\right) + C$

$\Rightarrow 8 = \frac{5A}{6} + C \Rightarrow \boxed{48 = 5A + 6C}$ ...(3)

P(2) = 4

$\Rightarrow 4 = A\left(\frac{8}{3} - 6 + 4\right) + C$

$\Rightarrow 4 = \frac{2A}{3} + C \Rightarrow \boxed{12 = 2A + 3C}$ ...(4)

From 3 & 4, C = –12

So P(0) = C = $\boxed{-12}$

7. Official Ans. by NTA (3)

Sol. $f'(x) = x(x+1)(x-1) = x^3 - x$

$\int df(x) = \int x^3 - x\,dx$

$f(x) = \frac{x^4}{4} - \frac{x^2}{2} + C$

$f(x) = f(0)$

$\frac{x^4}{4} - \frac{x^2}{2} = 0$

$x^2(x^2 - 2) = 0$

$x = 0, 0, \sqrt{2}, -\sqrt{2}$

$x_1^2 + x_2^2 + x_3^2 = 0 + 2 + 2 = 4$

8. Official Ans. by NTA (1)

Sol. $f(x) = (3x^2 + ax - 2 - a)e^x$

$f'(x) = (3x^2 + ax - 2 - a)e^x + e^x(6x + a)$

$= e^x(3x^2 + x(6 + a) - 2)$

$f'(x) = 0$ at $x = 1$

$\Rightarrow 3 + (6 + a) - 2 = 0$

$a = -7$

$f'(x) = e^x(3x^2 - x - 2)$

$= e^x(x - 1)(3x + 2)$

+ − +

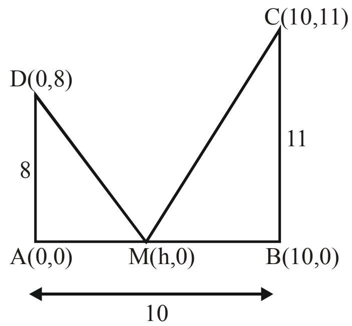
−2/3 1

$x = 1$ is point of local minima

$x = \frac{-2}{3}$ is point of local maxima

9. Official Ans. by NTA (5.00)

Sol.



$(MD)^2 + (MC)^2 = h^2 + 64 + (h - 10)^2 + 121$

$= 2h^2 - 20h + 64 + 100 + 121$

$= 2(h^2 - 10h) + 285$

$= 2(h - 5)^2 + 235$

it is minimum if $h = 5$

10. Official Ans. by NTA (4)

Sol. $f(x) = (1 - \cos^2 x)(\lambda + \sin x)$ $\quad x \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

$f(x) = \lambda\sin^2 x + \sin^3 x$

$f'(x) = 2\lambda\sin x\cos x + 3\sin^2 x\cos x$

$f'(x) = \sin x\cos x(2\lambda + 3\sin x)$

$\sin x = 0, \frac{-2\lambda}{3}, (\lambda \neq 0)$

for exactly one maxima & minima

$\frac{-2\lambda}{3} \in (-1, 1) \Rightarrow \lambda \in \left(\frac{-3}{2}, \frac{3}{2}\right)$

$\lambda \in \left(-\frac{3}{2}, \frac{3}{2}\right) - \{0\}$

DIFFERENTIAL EQUATION

1. NTA Ans. (3)

Sol. $(y^2 - x)\frac{dy}{dx} = 1$

$\Rightarrow \frac{dx}{dy} + x = y^2$

I.F. $= e^{\int dy} = e^y$

Solution is given by

$x e^y = \int y^2 e^y dy + C$

$\Rightarrow x e^y = (y^2 - 2y + 2)e^y + C$

$x = 0, y = 1,$ gives $C = -e$

If $y = 0$, then $x = 2 - e$

2. NTA Ans. (4)

Sol. $e^y\frac{dy}{dx} - e^y = e^x$, Let $e^y = t$

$\Rightarrow e^y\frac{dy}{dx} = \frac{dt}{dx}$

$\frac{dt}{dx} - t = e^x$

I.F. $= e^{\int -dx} = e^{-x}$

$t e^{-x} = x + c \Rightarrow e^{y-x} = x + c$

$y(0) = 0 \Rightarrow c = 1$

$e^{y-x} = x + 1 \Rightarrow y(1) = 1 + \log_e 2$

3. NTA Ans. (1)

Sol. $2x = 4by' \Rightarrow y' = \frac{2x}{4b}$

Required D.E. is $x^2 = \frac{2x}{y'}y + \left(\frac{x}{y'}\right)^2$

$x(y')^2 = 2yy' + x$

(1) Option

4. NTA Ans. (2)

ALLEN Ans. (BONUS)

Note: As per the given informaton, x cannot be negative. So, it is invalid to ask y(x) for x < 0. Hence, it should be bonus but, NTA retained its answer as option (2).

Sol. $\frac{dy}{dx}=-\frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$ so, $\frac{dy}{\sqrt{1-y^2}}+\frac{dx}{\sqrt{1-x^2}}=0$

Integrating, $\sin^{-1}x+\sin^{-1}y=c$

so, $\frac{\pi}{6}+\frac{\pi}{3}=c$

Hence, $\sin^{-1}x+\sin^{-1}y=\frac{\pi}{2}$

Put $x=-\frac{1}{\sqrt{2}},\sin^{-1}y=\frac{3\pi}{4}$ (Not possible)

5. NTA Ans. (4)

Sol. $\frac{dy}{dx}=\frac{xy}{x^2+y^2}$

Let y = vx

$\frac{dy}{dx}=v+x.\frac{dv}{dx}$

$v+x\frac{dv}{dx}=\frac{xvx}{x^2+v^2x^2}=\frac{v}{1+v^2}$

$x\frac{dv}{dx}=\frac{v}{1+v^2}-v=\frac{v-v-v^3}{1+v^2}=-\frac{v^3}{1+v^2}$

$\int\frac{1+v^2}{v^3}.dv=\int-\frac{dx}{x}$

$\Rightarrow \int v^{-3}.dv+\int\frac{1}{v}dv=-\int\frac{dx}{x}$

$\Rightarrow \frac{v^{-2}}{-2}+\ell n\,v=-\ell n\,x+\lambda$

$\Rightarrow -\frac{1}{2v^2}+\ell n\left(\frac{y}{x}\right)=-\ell n\,x+\lambda$

$\Rightarrow -\frac{1}{2}\frac{x^2}{y^2}+\ell n\,y-\ell n\,x=-\ell n\,x+\lambda$

$\Rightarrow -\frac{1}{2}+0=\lambda\Rightarrow\lambda=-\frac{1}{2}$

$\Rightarrow -\frac{1}{2}\frac{x^2}{y^2}+\ell n\,y+\frac{1}{2}=0$ at y = e

$\Rightarrow -\frac{1}{2}\frac{x^2}{e^2}+1+\frac{1}{2}=0\Rightarrow\frac{x^2}{2e^2}=\frac{3}{2}\Rightarrow x^2=3e^2$

$\therefore x=\sqrt{3}e$

6. NTA Ans. (3)

Sol. $f'(x)=\tan^{-1}(\sec x+\tan x)$

$f'(x)=\tan^{-1}\left(\frac{1+\sin x}{\cos x}\right)=\tan^{-1}\left(\frac{1+\tan\frac{x}{2}}{1-\tan\frac{x}{2}}\right)$

$=\tan^{-1}\left(\tan\left(\frac{\pi}{4}+\frac{\pi}{2}\right)\right)$

$\because -\frac{\pi}{2}<x<\frac{\pi}{2}\Rightarrow 0<\frac{\pi}{4}+\frac{x}{2}<\frac{\pi}{2}$

$\Rightarrow f'(x)=\frac{\pi}{4}+\frac{x}{2}$

$\therefore f(x)=\frac{\pi}{4}.x+\frac{x^2}{4}+c$

$\because f(0)=0\Rightarrow c=0$

$\Rightarrow f(x)=\frac{\pi}{4}x+\frac{x^2}{4}$

$\therefore f(1)=\frac{\pi+1}{4}$

7. NTA Ans. (3.00)

Sol. $(x+1)dy-ydx=((x+1)^2-3)dx$

$\Rightarrow \frac{(x+1)dy-ydx}{(x+1)^2}=\left(1-\frac{3}{(x+1)^2}\right)dx$

$\Rightarrow d\left(\frac{y}{(x+1)}\right)=\left(1-\frac{3}{(x+1)^2}\right)dx$

integrating both sides

$\frac{y}{x+1}=x+\frac{3}{(x+1)}+C$

Given $y(2)=0\Rightarrow c=-3$

$\therefore y=(x+1)\left(x+\frac{3}{(x+1)}-3\right)$

$\therefore y(3)=3.00$

8. Official Ans. by NTA (4)

Sol. $\frac{2+\sin x}{y+1}\frac{dy}{dx} = -\cos x,\ y > 0$

$\Rightarrow \frac{dy}{y+1} = \frac{-\cos x}{2+\sin x}dx$

By integrating both sides :

$\ell n|y+1| = -\ell n|2+\sin x| + \ell nK$

$\Rightarrow y + 1 = \frac{K}{2+\sin x} \quad (y + 1 > 0)$

$\Rightarrow y(x) = \frac{K}{2+\sin x} - 1$

Given $y(0) = 1 \Rightarrow K = 4$

So, $y(x) = \frac{4}{2+\sin x} - 1$

$a = y(\pi) = 1$

$b = \left.\frac{dy}{dx}\right]_{x=\pi} = \left.\frac{-\cos x}{2+\sin x}(y(x)+1)\right]_{x=\pi} = 1$

So, $(a, b) = (1, 1)$

9. Official Ans. by NTA (2)

Sol. $2x^2dy = (2xy + y^2)\,dx$

$\Rightarrow \frac{dy}{dx} = \frac{2xy+y^2}{2x^2}$ {Homogeneous D.E.}

$\left\{\begin{array}{l}\text{let } y = xt \\ \Rightarrow \frac{dy}{dx} = t + x\frac{dt}{dx}\end{array}\right\}$

$\Rightarrow t + x\frac{dt}{dx} = \frac{2x^2t + x^2t^2}{2x^2}$

$\Rightarrow t + x\frac{dt}{dx} = t + \frac{t^2}{2}$

$\Rightarrow x\frac{dt}{dx} = \frac{t^2}{2}$

$\Rightarrow 2\int\frac{dt}{t^2} = \int\frac{dx}{x}$

$\Rightarrow 2\left(-\frac{1}{t}\right) = \ell n(x) + C \quad \left\{\text{Put } t = \frac{y}{x}\right\}$

$\Rightarrow -\frac{2x}{y} = \ell n x + C \quad \left\{\begin{array}{l}\text{Put } x = 1 \ \&\ y = 2 \\ \text{then we get } C = -1\end{array}\right\}$

$\Rightarrow \frac{-2x}{y} = \ell n(x) - 1 \Rightarrow y = \frac{2x}{1-\ell n x}$

$\Rightarrow f(x) = \frac{2x}{1-\log_e x}$

so, $\boxed{f\left(\frac{1}{2}\right) = \frac{1}{1+\log_e 2}}$

10. Official Ans. by NTA (1)

Sol. $(1 + e^{-x})(1 + y^2)\frac{dy}{dx} = y^2$

$\Rightarrow (1 + y^{-2})\,dy = \left(\frac{e^x}{1+e^x}\right)dx$

$\Rightarrow \left(y - \frac{1}{y}\right) = \ell n(1+e^x) + c$

$\therefore$ It passes through (0, 1) $\Rightarrow c = -\ell n 2$

$\Rightarrow y^2 = 1 + y\,\ell n\left(\frac{1+e^x}{2}\right)$

11. Official Ans. by NTA (2)

Sol. $x^3dy + xy\,dx = x^2\,dy + 2y\,dx$

$\Rightarrow dy(x^3 - x^2) = dx\,(2y - xy)$

$$\Rightarrow -\int \frac{1}{y}dy = \int \frac{x-2}{x^2(x-1)}dx$$

$$\Rightarrow -\ell ny = \int\left(\frac{A}{x}+\frac{B}{x^2}+\frac{C}{(x-1)}\right)dx$$

Where A = 1, B = +2, C = –1

$$\Rightarrow -\ell ny = \ell n\,x - \frac{2}{x} - \ell n\,(x-1) + \lambda$$

$\Rightarrow y(2) = e$

$\Rightarrow -1 = \ell n\ 2 - 1 - 0 + \lambda$

$\therefore \lambda = -\ \ell n\,2$

$$\Rightarrow \ell n\,y = -\ \ell nx + \frac{2}{x} + \ell n(x-1) + \ell n\,2$$

Now put x = 4 in equation

$$\Rightarrow \ell n\,y = -\ell n4 + \frac{1}{2} + \ell n3 + \ell n2$$

$$\Rightarrow \ell n\,y = \ell n\left(\frac{3}{2}\right) + \frac{1}{2}\ell n\,e$$

$$\Rightarrow y = \frac{3}{2}\sqrt{e}$$

12. Official Ans. by NTA (1)

Sol. $x\dfrac{dy}{dx} - y = x^2(x\cos x + \sin x),\ \ x > 0$

$$\frac{dy}{dx} - \frac{y}{x} = x(x\cos x + \sin x)\ \Rightarrow \frac{dy}{dx} + Py = Q$$

$$\text{so, I.F.} = e^{\int -\frac{1}{x}dx} = \frac{1}{|x|} = \frac{1}{x}\ \ (x > 0)$$

$$\text{Thus, } \frac{y}{x} = \int \frac{1}{x}\big(x(x\cos x + \sin x)\big)dx$$

$$\Rightarrow \frac{y}{x} = x\sin x + C$$

$\because y(\pi) = \pi\ \Rightarrow\ C = 1$

so, $y = x^2\sin x + x \Rightarrow (y)_{\pi/2} = \dfrac{\pi^2}{4} + \dfrac{\pi}{2}$

Also, $\dfrac{dy}{dx} = x^2\cos x + 2x\sin x + 1$

$$\Rightarrow \frac{d^2y}{dx^2} = -x^2\sin x + 4x\cos x + 2\sin x$$

$$\Rightarrow \left.\frac{d^2y}{dx^2}\right|_{\frac{\pi}{2}} = -\frac{\pi^2}{4} + 2$$

$$\text{Thus, } y_{\left(\frac{\pi}{2}\right)} + \frac{d^2y}{dx^2}_{\left(\frac{\pi}{2}\right)} = \frac{\pi}{2} + 2$$

13. Official Ans. by NTA (3)

Sol. $\ell n(y + 3x) = z$ (let)

$$\frac{1}{y+3x}\cdot\left(\frac{dy}{dx}+3\right) = \frac{dz}{dx}$$

..(1)

$$\frac{dy}{dx} + 3 = \frac{y+3x}{\ell n(y+3x)} \qquad \text{(given)}$$

$$\frac{dz}{dx} = \frac{1}{z}$$

$$\Rightarrow z\,dz = dx \Rightarrow \frac{z^2}{2} = x + C$$

$$\Rightarrow \frac{1}{2}\ell n^2(y+3x) = x + C$$

$$\Rightarrow x - \frac{1}{2}(\ell n(y + 3x))^2 = C$$

14. Official Ans. by NTA (2)

Sol. $\dfrac{(5+e^x)}{2+y}\dfrac{dy}{dx} = -e^x$

$$\int \frac{dy}{2+y} = \int \frac{-e^x}{e^x+5}dx$$

$\ln(y + 2) = -\ln(e^x + 5) + k$

$(y + 2)(e^x + 5) = C$

$\because y(0) = 1$

$\Rightarrow C = 18$

$$y + 2 = \frac{18}{e^x+5}$$

at $x = \ln 13$

$$y + 2 = \frac{18}{13+5} = 1$$

$\boxed{y = -1}$

15. Official Ans. by NTA (1)

Sol. $\cos x \dfrac{dy}{dx} + 2y \sin x = \sin 2x$

$\dfrac{dy}{dx} + \dfrac{2\sin x}{\cos x} y = 2 \sin x$

I.F. $= e^{\int 2\frac{\sin x}{\cos x}dx}$

$= e^{2 \ln \sec x} = \sec^2 x$

$y \cdot \sec^2 x = \int 2\sin x \cdot \sec^2 x dx$

$y \sec^2 x = 2 \int \tan x \sec x \, dx$

$y \sec^2 x = 2 \sec x + c$

At $x = \dfrac{\pi}{3}$, $y = 0$

$\Rightarrow 0 = 2 \sec \dfrac{\pi}{3} + C \Rightarrow C = -4$

$\boxed{y \sec^2 x = 2\sec x - 4}$

Put $x = \dfrac{\pi}{4}$

$y \cdot 2 = 2\sqrt{2} - 4$

$y = \sqrt{2} - 2$

16. Official Ans. by NTA (2)

Sol. $x^4 e^y + 2\sqrt{y+1} = 3$

d.w.r. to x

$x^4 e^y y' + e^y 4x^3 + \dfrac{2y'}{2\sqrt{y+1}} = 0$

at P(1, 0)

$y'_P + 4 + y'_P = 0$

$\Rightarrow y'_P = -2$

Tangent at P(1, 0) is

$y - 0 = -2 (x - 1)$

$2x + y = 2$

(–2, 6) lies on it

17. Official Ans. by NTA (1)

Sol. $\sqrt{1+x^2+y^2+x^2y^2} + xy\dfrac{dy}{dx} = 0$

$\Rightarrow \sqrt{(1+x)^2(1+y^2)} + xy\dfrac{dy}{dx} = 0$

$\Rightarrow \sqrt{1+x^2}\sqrt{1+y^2} = -xy\dfrac{dy}{dx}$

$\Rightarrow \int \dfrac{ydy}{\sqrt{1+y^2}} = -\int \dfrac{\sqrt{1+x^2}}{x}dx$ ...(1)

Now put $1 + x^2 = u^2$ and $1 + y^2 = v^2$

$2xdx = 2udu$ and $2ydy = 2vdv$

$\Rightarrow xdx = udu$ and $ydy = vdv$

substitude these values in equation (1)

$\int \dfrac{vdv}{v} = -\int \dfrac{u^2.du}{u^2-1}$

$\Rightarrow \int dv = -\int \dfrac{u^2-1+1}{u^2-1}du$

$\Rightarrow v = -\int \left(1+\dfrac{1}{u^2-1}\right)du$

$\Rightarrow v = -u - \dfrac{1}{2}\log_e\left|\dfrac{u-1}{u+1}\right| + c$

$\Rightarrow \sqrt{1+y^2} = -\sqrt{1+x^2} + \dfrac{1}{2}\log_e\left|\dfrac{\sqrt{1+x^2}+1}{\sqrt{1+x^2}-1}\right| + c$

$\Rightarrow \sqrt{1+y^2} + \sqrt{1+x^2} = \dfrac{1}{2}\log_e\left|\dfrac{\sqrt{1+x^2}+1}{\sqrt{1+x^2}-1}\right| + c$

18. Official Ans. by NTA (1)

Sol. $y = \left(\dfrac{2x}{\pi} - 1\right)\operatorname{cosec} x$ ...(1)

$\dfrac{dy}{dx} = \dfrac{2}{\pi}\operatorname{cosec} x - \left(\dfrac{2x}{\pi}-1\right)\operatorname{cosec} x \cot x$

$\dfrac{dy}{dx} = \dfrac{2\operatorname{cosec} x}{\pi} - y\cot x$

using equation (1)

$\dfrac{dy}{dx} + y\cot x = \dfrac{2\operatorname{cosec} x}{\pi}$

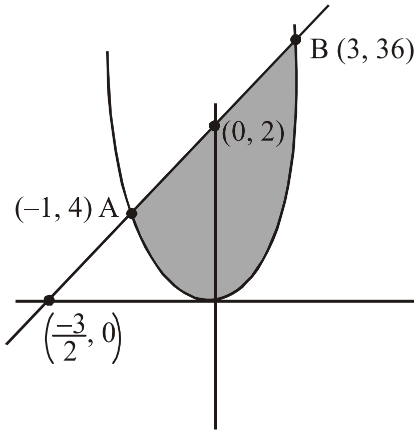
$\dfrac{dy}{dx} + p(x).y = \dfrac{2\operatorname{cosec} x}{\pi}$ $x \in \left(0, \dfrac{\pi}{2}\right)$

Compare : $p(x) = \cot x$

AREA UNDER THE CURVE

1. NTA Ans. (4)

Sol. $4x^2 - y \le 0$ and $8x - y + 12 \ge 0$



On solving $y = 4x^2$

and $y = 8x + 12$

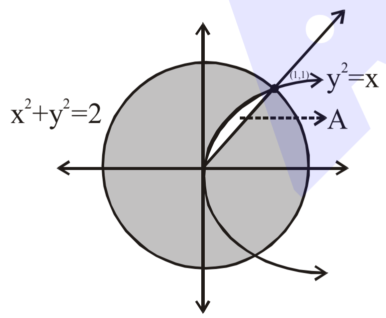
We get A (–1, 4) & B(3, 36)

Required area = area of the shaded region

$$= \int_{-1}^{3} (8x + 12 - 4x^2)dx = \frac{128}{3}$$

2. NTA Ans. (2)

Sol. $A = \int_0^1 \left(\sqrt{x} - x\right)dx$

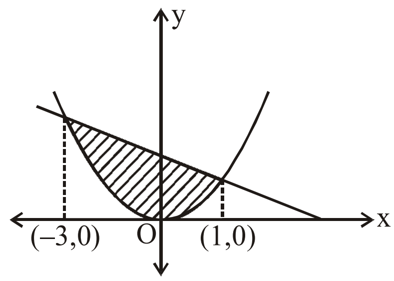


$$= \left[\frac{2}{3}x^{3/2} - \frac{x^2}{2}\right]_0^1 = \frac{1}{6}$$

Required Area : $\pi r^2 - \frac{1}{6} = \frac{1}{6}(12\pi - 1)$

3. NTA Ans. (4)

Sol.

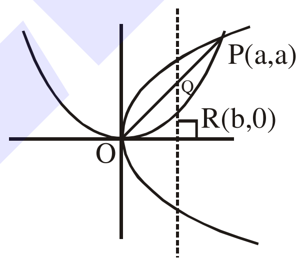


$$\text{Area} = \int_{-3}^{1} \left(3 - 2x - x^2\right)dx = \frac{32}{3}$$

(4) option

4. NTA Ans. (1)

Sol. $\int_0^b \left(\sqrt{ax} - \frac{x^2}{a}\right)dx = \frac{1}{2} \times \frac{16\left(\frac{a}{4}\right)\left(\frac{a}{4}\right)}{3}$



$$\Rightarrow \left[\frac{2\sqrt{a}}{3}x^{3/2} - \frac{x^3}{3a}\right]_0^b = \frac{a^2}{6}$$

$$\Rightarrow \frac{2\sqrt{a}}{3}b^{3/2} - \frac{b^3}{3a} = \frac{a^2}{6}$$

...(i)

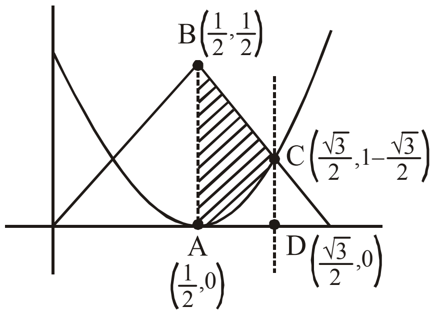
Also, $\frac{1}{2} \times b^2 = \frac{1}{2} \Rightarrow b = 1$

so, $\frac{2\sqrt{a}}{3} - \frac{1}{3a} = \frac{a^2}{6} \Rightarrow a^3 - 4a^{3/2} + 2 = 0$

$\Rightarrow a^6 + 4a^3 + 4 = 16a^3 \Rightarrow a^6 - 12a^3 + 4 = 0$

5. NTA Ans. (2)

Sol.



Required area = Area of trepezium ABCD –

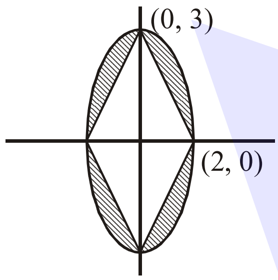
Area of parabola between $x = \frac{1}{2}$ & $x = \frac{\sqrt{3}}{2}$

$$A = \frac{1}{2}\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\right)\left(\frac{1}{2}+1-\frac{\sqrt{3}}{2}\right) - \int_{1/2}^{\sqrt{3}/2}\left(x-\frac{1}{2}\right)^2 dx = \frac{\sqrt{3}}{4}-\frac{1}{3}$$

6. Official Ans. by NTA (2)

Sol. $\frac{|x|}{2}+\frac{|y|}{3}=1$

$$\frac{x^2}{4}+\frac{y^2}{9}=1$$



Area of Ellipse = $\pi ab = 6\pi$

Required area,

= $\pi \times 2 \times 3$ – (Area of quadrilateral)

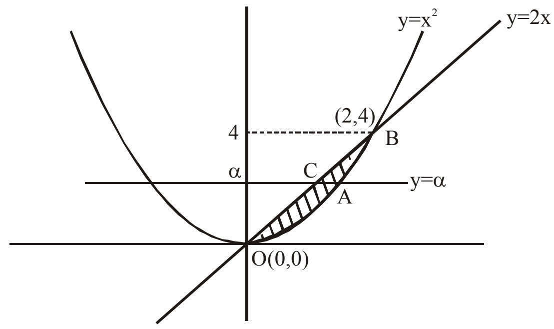
$= 6\pi - \frac{1}{2}6\times 4$

$= 6\pi - 12$

$= 6(\pi - 2)$

7. Official Ans. by NTA (4)

Sol.



* $y \geq x^2 \Rightarrow$ upper region of $y = x^2$

$y \leq 2x \Rightarrow$ lower region of $y = 2x$

According to ques, area of OABC = 2 area of OAC

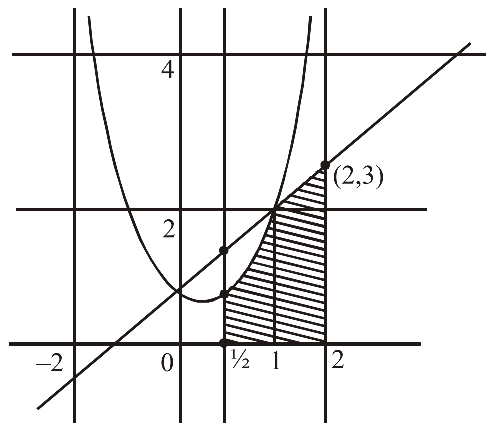
$$\Rightarrow \int_0^4\left(\sqrt{y}-\frac{y}{2}\right)dy = 2\int_0^{\alpha}\left(\sqrt{y}-\frac{y}{2}\right).dy$$

$$\Rightarrow \frac{4}{3} = 2\left[\frac{2}{3}\alpha^{3/2}-\frac{1}{4}.\alpha^2\right]$$

$$\Rightarrow \boxed{3\alpha^2-8\alpha^{3/2}+8=0}$$

8. Official Ans. by NTA (3)

Sol. $0 \leq y \leq x^2 + 1,\ 0 \leq y \leq x + 1,\ \frac{1}{2} \leq x \leq 2$



Required area = $\int_{1/2}^{1}(x^2+1)dx + \frac{1}{2}(2+3)\times 1$

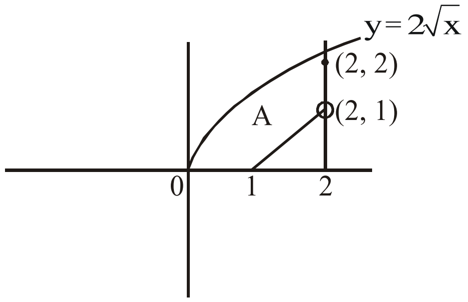
$$= \frac{19}{24}+\frac{5}{2} = \frac{79}{24}$$

9. Official Ans. by NTA (1)

Sol. $(x-1)[x] \le y \le 2\sqrt{x}$, $\boxed{0 \le x \le 2}$

Draw $y = 2\sqrt{x} \Rightarrow y^2 = 4x$ $\boxed{x \ge 0}$

$$y = (x-1)[x] = \begin{cases} 0 & , 0 \le x < 1 \\ x-1, & 1 \le x < 2 \\ 2 & , \quad x = 2 \end{cases}$$



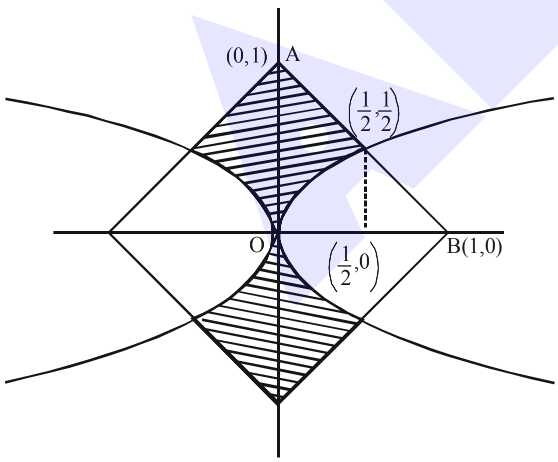
$$A = \int_0^2 2\sqrt{x}\,dx - \frac{1}{2}1\cdot1$$

$$A = 2\cdot\left[\frac{x^{3/2}}{(3/2)}\right]_0^2 - \frac{1}{2} = \frac{8\sqrt{2}}{3} - \frac{1}{2}$$

10. Official Ans. by NTA (4)

Sol. $|x| + |y| \le 1$

$2y^2 \ge |x|$



For point of intersection

$x + y = 1 \Rightarrow x = 1 - y$

$$y^2 = \frac{x}{2} \Rightarrow 2y^2 = x$$

$2y^2 = 1 - y \Rightarrow 2y^2 + y - 1 = 0$

$(2y - 1)(y + 1) = 0$

$$y = \frac{1}{2} \text{ or } -1$$

Now Area of $\Delta OAB = \frac{1}{2}\times1\times1 = \frac{1}{2}$

Area of Region $R_1 = \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2} = \frac{1}{8}$

Area of Region $R_2 = \frac{1}{\sqrt{2}}\int_0^{\frac{1}{2}}\sqrt{x}\,dx = \frac{1}{6}$

Now area of shaded region in first quadrant

= Area of $\Delta OAB - R_1 - R_2$

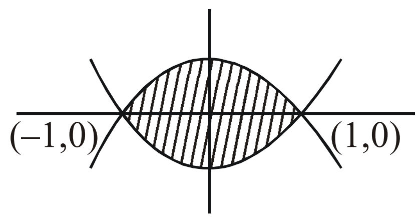
$$= \frac{1}{2} - \left(\frac{1}{6}\right) - \left(\frac{1}{8}\right) = \frac{5}{24}$$

So required area $= 4\left(\frac{5}{24}\right) = \frac{5}{6}$

so option (4) is correct.

11. Official Ans. by NTA (2)

Sol. $y = x^2 - 1$ and $y = 1 - x^2$



$$A = \int_{-1}^{1}\left((1-x^2)-(x^2-1)\right)dx$$

$$A = \int_{-1}^{1}(2-2x^2)\,dx = 4\int_0^1(1-x^2)\,dx$$

$$A = 4\left(x - \frac{x^3}{3}\right)_0^1 = 4\left(\frac{2}{3}\right) = \frac{8}{3}$$

MATRICES

1. NTA Ans. (4)

Sol. $b_{ij} = (3)^{(i+j-2)} a_{ij}$

$$B = \begin{bmatrix} a_{11} & 3a_{12} & 3^2 a_{13} \\ 3a_{21} & 3a_{22} & 3a_{23} \\ 3^2 a_{31} & 3^2 a_{32} & 3^2 a_{33} \end{bmatrix}$$

$$\Rightarrow |B| = 3 \times 3^2 \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ 3a_{21} & 3a_{22} & 3a_{23} \\ 3^2 a_{31} & 3^2 a_{32} & 3^2 a_{33} \end{vmatrix}$$

$= 3^6|A|$

$$\Rightarrow |A| = \frac{81}{27 \times 27} = \frac{1}{9}$$

2. NTA Ans. (1)

Sol. $x^2 + x + 1 = 0$

$\alpha = \omega$

$\alpha^2 = \omega^2$

$$A = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$\Rightarrow A^4 = A^2 \,.\, A^2 = I_3$

$A^{31} = A^{28}.\ A^3 = A^3.$

3. NTA Ans. (2)

Sol. $A = \begin{pmatrix} 2 & 2 \\ 9 & 4 \end{pmatrix}$; $|A| = 8 - 18 = -10$

$$A^{-1} = \frac{\text{adj}A}{|A|} = \frac{\begin{pmatrix} 4 & -2 \\ -9 & 2 \end{pmatrix}}{-10}$$

$$10A^{-1} = \begin{pmatrix} -4 & 2 \\ 9 & -2 \end{pmatrix} = A - 6I$$

(2) Option

4. NTA Ans. (672.00)

Sol. trace $(AA^T) = \Sigma a_{ij}^2 = 3$

Hence, number of such matrices

$= {}^9C_3 \times 2^3 = 672.00$

5. NTA Ans. (3)

Sol. $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 3 & 4 \\ 1 & -1 & 3 \end{bmatrix}$

$\Rightarrow \quad |A| = 6$

$$\frac{|\text{adj}B|}{|c|} = \frac{|\text{adj}(\text{adj}A)|}{|9A|} = \frac{|A|^4}{3^3|A|} = \frac{|A|^3}{3^3}$$

$$= \frac{(6)^3}{(3)^3} = 8$$

6. Official Ans. by NTA (4)

Sol. $|A| \neq 0$

For (P) : $A \neq I_2$

So, $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ or $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ or $\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$ or $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

or $\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$

$|A|$ can be -1 or 1

So (P) is false.

For (Q); $|A| = 1$

$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ or $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ or $\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$

$\Rightarrow$ tr(A) = 2

$\Rightarrow$ Q is true

7. Official Ans. by NTA (4)

Sol. $A^TA = I$

$\Rightarrow a^2 + b^2 + c^2 = 1$

and $ab + bc + ca = 0$

Now, $(a + b + c)^2 = 1$

$\Rightarrow a + b + c = \pm 1$

So, $a^3 + b^3 + c^3 - 3abc$

$= (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

$= \pm 1 \ (1 - 0) = \pm 1$

$\Rightarrow 3\ abc = 2 \pm 1 = 3, 1$

$\Rightarrow abc = 1, \frac{1}{3}$

8. Official Ans. by NTA (2)

Sol. Given $P = \begin{bmatrix} 1 & 2 & 1 \\ -2 & 3 & -4 \\ 1 & 9 & 1 \end{bmatrix}$, Here |P| = 0 & also

given PX = 0

$$\Rightarrow \begin{bmatrix} 1 & 2 & 1 \\ -2 & 3 & -4 \\ 1 & 9 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$\Rightarrow \left.\begin{array}{l} x+2y+z=0 \\ -2x+3y-4z=0 \\ x+9y-z=0 \end{array}\right\} \because D=0, \text{so system have}$$

infinite many solutions,

By solving these equation

we get $x = \frac{-11\lambda}{2}; y = \lambda; z = \frac{7\lambda}{2}$

Also given, $x^2 + y^2 + z^2 = 1$

$$\Rightarrow \left(\frac{-11\lambda}{2}\right)^2 + (\lambda)^2 + \left(\frac{7\lambda}{2}\right)^2 = 1$$

$$\Rightarrow \lambda = \pm\frac{1}{\sqrt{\frac{121}{4}+1+\frac{49}{4}}}$$

so, there are 2 values of λ.

$\therefore$ so, there are 2 solution set of (x,y,z).

9. Official Ans. by NTA (10)

Sol. $A = \begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}$

$$A^2 = \begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} x^2+1 & x \\ x & 1 \end{bmatrix}$$

$$A^4 = \begin{bmatrix} x^2+1 & x \\ x & 1 \end{bmatrix}\begin{bmatrix} x^2+1 & x \\ x & 1 \end{bmatrix}$$

$$= \begin{bmatrix} (x^2+1)^2+x^2 & x(x^2+1)+x \\ x(x^2+1)+x & x^2+1 \end{bmatrix}$$

$a_{11} = (x^2 + 1)^2 + x^2 = 109$

$\Rightarrow x = \pm 3$

$a_{22} = x^2 + 1 = 10$

10. Official Ans. by NTA (3)

Sol. $C = \text{adj } A = \begin{vmatrix} +2 & -1 & 1 \\ -1 & 0 & 2 \\ 1 & -2 & -1 \end{vmatrix}$

$|C| = |\text{adj } A| = +2(0 + 4) + 1.(1 - 2) + 1.(2-0) = +8 - 1 + 2$

$|\text{adj } A| = |A|^2 = 9 = 9$

$\lambda = |A| = \pm 3$

$|\lambda| = 3$

$B = \text{adj } C$

$|B| = |\text{adj } C| = |C|^2 = 81$

$|(B^{-1})^T| = |B|^{-1} = \frac{1}{81}$

$(|\lambda|, \mu) = \left(3, \frac{1}{81}\right)$

11. Official Ans. by NTA (4)

Sol. $Ax_1 = b_1$

$Ax_2 = b_2$

$Ax_3 = b_3$

$$\Rightarrow |A|\begin{vmatrix} 1 & 0 & 0 \\ 1 & 2 & 0 \\ 1 & 1 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{vmatrix}$$

$$\Rightarrow |A| = \frac{4}{2} = 2$$

12. Official Ans. by NTA (2)

Sol. $A=\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$

$A^2=\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$

$A^2=\begin{bmatrix}\cos 2\theta & \sin 2\theta\\ -\sin 2\theta & \cos 2\theta\end{bmatrix}$

$B = A + A^4$

$=\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}+\begin{bmatrix}\cos 4\theta & \sin 4\theta\\ -\sin 4\theta & \cos 4\theta\end{bmatrix}$

$B=\begin{bmatrix}(\cos\theta+\cos 4\theta) & (\sin\theta+\sin 4\theta)\\ -(\sin\theta+\sin 4\theta) & (\cos\theta+\cos 4\theta)\end{bmatrix}$

$|B| = (\cos\theta + \cos 4\theta)^2 + (\sin\theta + \sin 4\theta)^2$

$|B| = 2 + 2\cos 3\theta$, when $\theta=\frac{\pi}{5}$

$|B|=2+2\cos\frac{3\pi}{5}=2(1-\sin 18)$

$|B|=2\left(1-\frac{\sqrt{5}-1}{4}\right)=2\left(\frac{5-\sqrt{5}}{4}\right)=\frac{5-\sqrt{5}}{2}$

VECTORS

1. NTA Ans. (1)

Sol. $\vec{a}+\vec{b}+\vec{c}=\vec{0}$

$\Rightarrow |\vec{a}|^2+|\vec{b}|^2+|\vec{c}|^2+2(\vec{a}\cdot\vec{b})+2(\vec{b}\cdot\vec{c})+2(\vec{c}\cdot\vec{a})=0$

$\lambda = \vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}=\frac{-3}{2}$

$\vec{d} = \vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}$

$\vec{a}+\vec{b}+\vec{c}=\vec{0}$

$\Rightarrow \vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}$

$\Rightarrow \vec{d} = 3(\vec{a}\times\vec{b})$

2. NTA Ans. (4)

ALLEN Ans. (BONUS)

Note: None of the given options matches. So, it should be bonus but NTA did not accept our claim

Sol. $\vec{a}=\lambda\left(\hat{b}+\hat{c}\right)=\lambda\left(\frac{\hat{i}+\hat{j}}{\sqrt{2}}+\frac{\hat{i}-\hat{j}+4\hat{k}}{3\sqrt{2}}\right)$

$\vec{a}=\frac{\lambda}{3\sqrt{2}}\left(4\hat{i}+2\hat{j}+4\hat{k}\right)\Rightarrow\frac{\lambda}{3\sqrt{2}}(4\hat{i}+2\hat{j}+4\hat{k})$

$=\alpha\hat{i}+2\hat{j}+\beta\hat{k}$

$\Rightarrow \alpha = 4$ and $\beta = 4$

So, $\vec{a}=4\hat{i}+2\hat{j}+4\hat{k}$

None of the given options is correct

3. NTA Ans. (3)

Sol. $\vec{b}\times\vec{c}-\vec{b}\times\vec{a}=\vec{0}$

$\vec{b}\times(\vec{c}-\vec{a})=\vec{0}$

$\vec{b}=\lambda(\vec{c}-\vec{a})$ ...(i)

$\vec{a}\cdot\vec{b}=\lambda\left(\vec{a}\cdot\vec{c}-\vec{a}^2\right)$

$4 = \lambda(0 - 6) \Rightarrow \lambda=\frac{-4}{6}=\frac{-2}{3}$

from (i) $\vec{b}=\frac{-2}{3}(\vec{c}-\vec{a})$

$\vec{c}=\frac{-3}{2}\vec{b}+\vec{a}=\frac{-1}{2}\left(\hat{i}+\hat{j}+\hat{k}\right)$

$\boxed{\vec{b}\cdot\vec{c}=\frac{1}{2}}$ (3) Option

4. NTA Ans. (1)

Sol. $\left\|\begin{matrix}1 & 1 & \lambda\\ 1 & 1 & 3\\ 2 & 1 & 1\end{matrix}\right\|=1\Rightarrow\lambda=2,4$

Now, $\cos\theta=\frac{\vec{u}.\vec{w}}{|\vec{u}||\vec{w}|}$

$=\frac{5}{\sqrt{6}\sqrt{6}}$ or $\frac{7}{\sqrt{6}\sqrt{18}}=\frac{5}{6}$ or $\frac{7}{6\sqrt{3}}$

5. NTA Ans. (30)

Sol. $\vec{b}\cdot\vec{c}=10 \Rightarrow 5|\vec{c}|\cos\frac{\pi}{3}=10 \Rightarrow |\vec{c}|=4$

$\left|\vec{a}\times\left(\vec{b}\times\vec{c}\right)\right|=|\vec{a}|\,|\vec{b}\times\vec{c}|$

$=\sqrt{3}\,.\,5\,.\,4\,.\sin\frac{\pi}{4}=30$

6. NTA Ans. (1.00)

Sol. $\vec{p} = (a+1)\hat{i} + a\hat{j} + a\hat{k}$,

$\vec{q} = a\hat{i} + (a+1)\hat{j} + a\hat{k}$ and

$\vec{r} = a\hat{i} + a\hat{j} + (a+1)\hat{k}$

$\because$ $\vec{p}, \vec{q}, \vec{r}$ are coplanar

$\Rightarrow [\vec{p}\ \ \vec{q}\ \ \vec{r}] = 0$

$$\Rightarrow \begin{vmatrix} a+1 & a & a \\ a & a+1 & a \\ a & a & a+1 \end{vmatrix} = 0$$

$\Rightarrow 3a + 1 = 0 \Rightarrow a = -\dfrac{1}{3}$

$\vec{p}.\vec{q} = -\dfrac{1}{3}$, $\vec{r}.\vec{q} = -\dfrac{1}{3}$

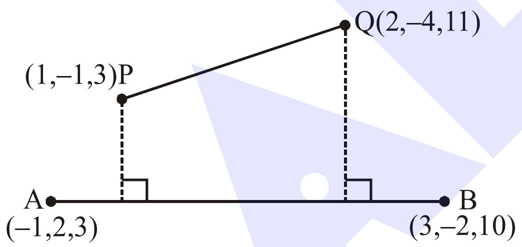
$|\vec{r}|^2 = |\vec{q}|^2 = \dfrac{2}{3}$

$\therefore 3(\vec{p}.\vec{q})^2 - \lambda|\vec{r}\times\vec{q}|^2 = 0$

$$\Rightarrow \lambda = \frac{3(\vec{p}.\vec{q})^2}{|\vec{r}\times\vec{q}|^2} = \frac{3(\vec{p}.\vec{q})^2}{|\vec{r}|^2|\vec{q}|^2 - (\vec{r}.\vec{q})^2} = 1.00$$

7. NTA Ans. (8.00)

Sol.



Projection of $\overline{PQ}$ on $\overline{AB} = \left|\dfrac{\overline{PQ}\cdot\overline{AB}}{|\overline{AB}|}\right|$

$$= \left|\frac{(\hat{i} - 3\hat{j} + 8\hat{k})\cdot(4\hat{i} - 4\hat{j} + 7\hat{k})}{9}\right| = 8$$

8. Official Ans. by NTA (2.00)

Sol. $|\vec{a}| = |\vec{b}| = |\vec{c}| = 1$

$\left|\vec{a} - \vec{b}\right|^2 + \left|\vec{a} - \vec{b}\right|^2 = 8$

$\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a}.\vec{b} + |\vec{a}|^2 + |\vec{c}|^2 - 2\vec{a}.\vec{c} = 8$

$\Rightarrow 4 - 2(\vec{a}.\vec{b} + \vec{a}.\vec{c}) = 8$

$\Rightarrow \boxed{\vec{a}.\vec{b} + \vec{a}.\vec{c} = -2}$

$|\vec{a} + 2\vec{b}|^2 + |\vec{a} + 2\vec{c}|^2$

$= |a^2| + 4|\vec{b}|^2 + 4\vec{a}.\vec{b} + |\vec{a}|^2 + 4|\vec{c}|^2 + 4\vec{a}.\vec{c}$

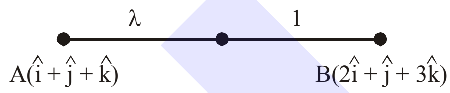
$= 10 + 4(\vec{a}.\vec{b} + \vec{a}.\vec{c})$

$= 10 - 8$

$= \boxed{2}$

9. Official Ans. by NTA (0.8)

Sol.



Using section formula we get

$$\overline{OP} = \frac{2\lambda+1}{\lambda+1}\hat{i} + \frac{\lambda+1}{\lambda+1}\hat{j} + \frac{3\lambda+1}{\lambda+1}\hat{k}$$

Now $\overline{OB}\cdot\overline{OP} = \dfrac{4\lambda + 2 + \lambda + 1 + 9\lambda + 3}{\lambda+1}$

$$= \frac{14\lambda+6}{\lambda+1}$$

$$\overline{OA}\times\overline{OP} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ \dfrac{2\lambda+1}{\lambda+1} & 1 & \dfrac{3\lambda+1}{\lambda+1} \end{vmatrix}$$

$$= \frac{2\lambda+1}{\lambda+1}\hat{i} + \frac{-\lambda}{\lambda+1}\hat{j} + \frac{-\lambda}{\lambda+1}\hat{k}$$

$$|\overline{OA}\times\overline{OP}|^2 = \frac{(2\lambda+1)^2 + \lambda^2 + \lambda^2}{(\lambda+1)^2}$$

$$= \frac{6\lambda^2+1}{(\lambda+1)^2}$$

$$\Rightarrow \frac{14\lambda+6}{\lambda+1} - 3\times\frac{(6\lambda^2+1)}{(\lambda+1)^2} = 6$$

$\Rightarrow 10\lambda^2 - 8\lambda = 0$

$\Rightarrow \lambda = 0,\ \dfrac{8}{10} = 0.8$

$\Rightarrow \lambda = 0.8$

10. Official Ans. by NTA (3)

Sol. $\vec{r} = \hat{i}(1+2\ell) + \hat{j}(-1) + \hat{k}(\ell)$

$\vec{r} = \hat{i}(2+m) + \hat{j}(m-1) + \hat{k}(-m)$

For intersection

$1 + 2\ell = 2 + m$ ...... (i)

$-1 = m - 1$ ...... (ii)

$\ell = -m$ ...... (iii)

from (ii) $m = 0$

from (iii) $\ell = 0$

These values of m and ℓ do not satisfy equation (1).

Hence the two lines do not intersect for any values of ℓ and m.

11. Official Ans. by NTA (5)

Sol. Dr's normal to plane

$$= \begin{vmatrix} i & j & k \\ 1 & 1 & 0 \\ 0 & 1 & -1 \end{vmatrix} = -\hat{i} + \hat{j} + \hat{k}$$

Equation of plane

$-1(x - 1) + 1(y - 0) + 1(z - 0) = 0$

$x - y - z - 1 = 0$

......(1)

Now $\dfrac{\alpha-1}{1} = \dfrac{\beta-0}{-1} = \dfrac{\gamma-1}{-1} = -\dfrac{(1-0-1-1)}{3}$

$$\frac{\alpha-1}{1} = \frac{\beta}{-1} = \frac{\gamma-1}{-1} = \frac{1}{3}$$

$$\alpha = \frac{4}{3}, \beta = -\frac{1}{3}, \gamma = \frac{2}{3}$$

$$3(\alpha+\beta+\gamma) = 3\left(\frac{4}{3} - \frac{1}{3} + \frac{2}{3}\right) = 5$$

12. Official Ans. by NTA (4)

Sol. $f(x) = \vec{a}\,.\left(\vec{b}\times\vec{c}\right) = \begin{vmatrix} x & -2 & 3 \\ -2 & x & -1 \\ 7 & -2 & x \end{vmatrix} = x^3 - 27x + 26$

$f'(x) = 3x^2 - 27 = 0 \Rightarrow x = \pm 3$

and $f''(-3) < 0$

$\Rightarrow$ local maxima at $x = x_0 = -3$

Thus, $\vec{a} = -3\hat{i} - 2\hat{j} + 3\hat{k}$,

$\vec{b} = -2\hat{i} - 3\hat{j} - \hat{k}$,

and $\vec{c} = 7\hat{i} - 2\hat{j} - 3\hat{k}$

$\Rightarrow \vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a} = 9 - 5 - 26 = -22$

13. Official Ans. by NTA (18)

Sol. $\Sigma|\vec{a} - (\vec{a}\cdot i)i|^2$

$\Rightarrow \Sigma\left(|a|^2 + (\vec{a}\cdot i)^2 - 2(\vec{a}\cdot i)^2\right)$

$\Rightarrow 3|\vec{a}|^2 - \Sigma(\vec{a}\cdot i)^2$

$\Rightarrow 2|\vec{a}|^2$

$\Rightarrow 18$

14. Official Ans. by NTA (2)

Sol. $v = \left[\vec{a}\;\vec{b}\;\vec{c}\right]$

$$158 = \begin{vmatrix} 1 & 1 & n \\ 2 & 4 & -n \\ 1 & n & 3 \end{vmatrix}, n \geq 0$$

$158 = 1\,(12 + n^2) - (6 + n) + n(2n - 4)$

$158 = n^2 + 12 - 6 - n + 2n^2 - 4n$

$3n^2 - 5n - 152 = 0$

$n = 8, -\dfrac{38}{6}$ (rejected)

$\vec{a}\cdot\vec{c} = 1 + n + 3n = 1 + 4n = 33$

$\vec{b}\cdot\vec{c} = 2 + 4n - 3n = 2 + n = 10$

15. Official Ans. by NTA (6.00)

Sol. Projection of $\vec{b}$ on $\vec{a}$ = projection of $\vec{c}$ on $\vec{a}$

$$\Rightarrow \frac{\vec{b}\cdot\vec{a}}{|\vec{a}|} = \frac{\vec{c}\cdot\vec{a}}{|\vec{a}|} \Rightarrow \vec{b}\cdot\vec{a} = \vec{c}\cdot\vec{a}$$

$\because$ $\vec{b}$ is perpendicular to $\vec{c}$ $\Rightarrow \vec{b}\cdot\vec{c} = 0$

Let $|\vec{a} + \vec{b} - \vec{c}| = k$

Square both sides

$k^2 = \vec{a}^2 + \vec{b}^2 + \vec{c}^2 + 2\vec{a}\cdot\vec{b} - 2\vec{a}\cdot\vec{c} - 2\vec{b}\cdot\vec{c}$

$k^2 = \vec{a}^2 + \vec{b}^2 + \vec{c}^2 = 36$

$k = 6 = |\vec{a} + \vec{b} - \vec{c}|$

16. Official Ans. by NTA (4.00)

Sol. $\sqrt{3}|\vec{a}+\vec{b}|+|\vec{a}-\vec{b}|$

$=\sqrt{3}\left(\sqrt{2+2\cos\theta}\right)+\sqrt{2-2\cos\theta}$

$=\sqrt{6}\left(\sqrt{1+\cos\theta}\right)+\sqrt{2}\left(\sqrt{1-\cos\theta}\right)$

$=2\sqrt{3}\left|\cos\frac{\theta}{2}\right|+2\left|\sin\frac{\theta}{2}\right|$

$\le\sqrt{\left(2\sqrt{3}\right)^2+(2)^2}=4$

17. Official Ans. by NTA (1.00)

Sol. $|\vec{x}+\vec{y}|=|\vec{x}|$

$\sqrt{|\vec{x}|^2+|\vec{y}|^2+2\vec{x}.\vec{y}}=|\vec{x}|$

$|\vec{y}|^2+2\vec{x}.\vec{y}=0$ .... (1)

Now $(2\vec{x}+\lambda\vec{y}).\vec{y}=0$

$2\vec{x}.\vec{y}+\lambda|\vec{y}|^2=0$

from (1)

$-|\vec{y}|^2+\lambda|\vec{y}|^2=0$

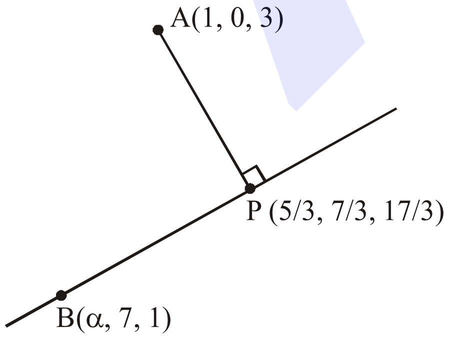
$(\lambda-1)|\vec{y}|^2=0$

given $|\vec{y}|\neq 0 \Rightarrow \lambda = 1$

3D

1. NTA Ans. (4.00)

Sol. D.R. of BP = $<\frac{5}{3}-\alpha,\frac{7}{3}-7,\frac{17}{3}-1>$



D.R. of AP = $<\frac{5}{3}-1,\frac{7}{3}-0,\frac{17}{3}-3>$

BP $\perp^r$ AP

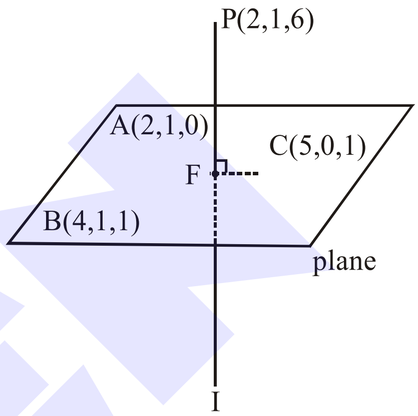
$\Rightarrow \alpha = 4$

2. NTA Ans. (1)

Sol. Plane passing through : (2, 1, 0), (4, 1, 1) and (5, 0, 1)

$$\begin{vmatrix} x-2 & y-1 & z \\ 2 & 0 & 1 \\ 3 & -1 & 1 \end{vmatrix}=0$$

$\Rightarrow x + y - 2z = 3$



Let I and F are respectively image and foot of perpendicular of point P in the plane.

eq^n of line PI $\frac{x-2}{1}=\frac{y-1}{1}=\frac{z-6}{-2}=\lambda(\text{say})$

Let I $(\lambda + 2 , \lambda + 1, -2\lambda + 6)$

$\Rightarrow F\left(2+\frac{\lambda}{2},1+\frac{\lambda}{2},-\lambda+6\right)$

F lies in the plane

$\Rightarrow 2+\frac{\lambda}{2}+1+\frac{\lambda}{2}+2\lambda-12-3=0$

$\Rightarrow \lambda = 4$

$\Rightarrow$ I (6, 5, –2)

3. NTA Ans. (4)

Sol. Point on plane $R\left(\frac{-2}{3},\frac{1}{3},\frac{4}{3}\right)$

Normal vector of plane is $\frac{10}{3}\hat{i}+\frac{10}{3}\hat{j}+\frac{10}{3}\hat{k}$

Equation of require plane is x + y + z = 1

Hence (1, –1, 1) lies on plane

(4) Option

4. NTA Ans. (2)

Sol. Shortest distance $= \dfrac{\left\|\begin{matrix} 6 & 15 & -3 \\ 3 & -1 & 1 \\ -3 & 2 & 4 \end{matrix}\right\|}{\sqrt{11\times 29-49}} = \dfrac{270}{\sqrt{270}}$

$= \sqrt{270} = 3\sqrt{30}$

5. NTA Ans. (3)

Sol. If $\lambda = -7$, then planes will be parallel & distance between them will be $\dfrac{3}{\sqrt{633}} \Rightarrow k = 3$

But if $\lambda \neq -7$, then planes will be intersecting & distance between them will be 0

6. NTA Ans. (1)

Sol. For planes to intersect on a line

$\Rightarrow$ there should be infinite solution of the given system of equations

for infinite solutions

$$\Delta = \begin{vmatrix} 1 & 4 & -2 \\ 1 & 7 & -5 \\ 1 & 5 & \alpha \end{vmatrix} = 0 \Rightarrow 3\alpha + 9 = 0 \Rightarrow \alpha = -3$$

$$\Delta_z = \begin{vmatrix} 1 & 4 & 1 \\ 1 & 7 & \beta \\ 1 & 5 & 5 \end{vmatrix} = 0 \Rightarrow 13 - \beta = 0 \Rightarrow \beta = 13$$

Also for $\alpha = -3$ and $b = 13$ $\Delta_x = \Delta_y = 0$

$\therefore\ \alpha + \beta = -3 + 13 = 10$

7. Official Ans. by NTA (2)

Sol. Two points on the line (L say) $\dfrac{x}{3} = \dfrac{y}{2}, z = 1$ are (0, 0, 1) & (3, 2, 1)

So dr's of the line is < 3, 2, 0 >

Line passing through (1, 2, 1), parallel to L and coplanar with given plane is

$\vec{r} = \hat{i} + 2\hat{j} + \hat{k} + t(3\hat{i} + 2\hat{j}), t \in R$ (–2, 0, 1) satisfies the line (for t = –1)

$\Rightarrow$ (–2, 0, 1) lies on given plane.

Answer of the question is (2)

We can check other options by finding eqution of plane

Equation plane : $\begin{vmatrix} x-1 & y-2 & z-1 \\ 1+2 & 2-0 & 1-1 \\ 2+2 & 1-0 & 2-1 \end{vmatrix} = 0$

$\Rightarrow 2(x - 1) - 3(y - 2) - 5(z - 1) = 0$

$\Rightarrow 2x - 3y - 5z + 9 = 0$

8. Official Ans. by NTA (2)

Sol. Hence normal is $\perp^r$ to both the lines so normal vector to the plane is

$\vec{n} = (\hat{i} - 2\hat{j} + 2\hat{k}) \times (2\hat{i} + 3\hat{j} - \hat{k})$

$$\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 2 \\ 2 & 3 & -1 \end{vmatrix} = \hat{i}(2-6) - \hat{j}(-1-4) + \hat{k}(3+4)$$

$\vec{n} = -4\hat{i} + 5\hat{j} + 7\hat{k}$

Now equation of plane passing through (3,1,1) is

$\Rightarrow -4(x-3) + 5(y - 1) + 7(z - 1) = 0$

$\Rightarrow -4x + 12 + 5y - 5 + 7z - 7 = 0$

$\Rightarrow -4x + 5y + 7z = 0 \quad \ldots(1)$

Plane is also passing through $(\alpha, -3, 5)$ so this point satisfies the equation of plane so put in equation (1)

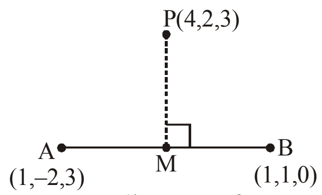
$-4\alpha + 5 \times (-3) + 7 \times (5) = 0$

$\Rightarrow -4\alpha - 15 + 35 = 0$

$\Rightarrow \boxed{\alpha = 5}$

9. Official Ans. by NTA (4)

Sol. Equation of AB $= \vec{r} = (\hat{i} + \hat{j}) + \lambda(3\hat{j} - 3\hat{k})$



Let coordinates of M = $(1, (1 + 3\lambda), -3\lambda)$.

$\overrightarrow{PM} = -3\hat{i} + (3\lambda - 1)\hat{j} - 3(\lambda + 1)\hat{k}$

$\overrightarrow{AB} = 3\hat{j} - 3\hat{k}$

$\because\ \overrightarrow{PM} \perp \overrightarrow{AB} \Rightarrow \overrightarrow{PM} . \overrightarrow{AB} = 0$

$\Rightarrow 3(3\lambda - 1) + 9(\lambda + 1) = 0$

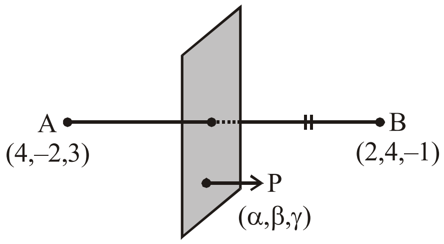
$\Rightarrow \lambda = -\dfrac{1}{3}$

$\therefore$ M = (1, 0, 1)

Clearly M lies on 2x + y – z = 1.

10. Official Ans. by NTA (1)

Sol. PA = PB



$\Rightarrow PA^2 = PB^2$

$\Rightarrow (\alpha - 4)^2 + (\beta + 2)^2 + (\gamma - 3)^2$

$= (\alpha - 2)^2 + (\beta - 4)^2 + (\gamma + 1)^2$

$\Rightarrow -4\alpha + 12\beta - 8\gamma = -8$

$\Rightarrow 2x - 6y + 4z = 4$

11. Official Ans. by NTA (3)

Sol. $D_1 = \begin{vmatrix} -7 & 4 & -1 \\ 8 & 1 & 5 \\ 15 & b & 6 \end{vmatrix} = 0 \Rightarrow b = -3$

$D = \begin{vmatrix} 1 & 4 & -1 \\ 3 & 1 & 5 \\ a & b & 6 \end{vmatrix} = 0 \Rightarrow 21a - 8b - 66 = 0$ .... (1)

P : $2x - 3y + 6z = 15$

so required distance $= \frac{21}{7} = 3$

12. Official Ans. by NTA (2)

Sol. equation of line parallel to $\frac{x}{2} = \frac{y}{3} = \frac{z}{-6}$ passes through (1, –2, 3) is

$\frac{x-1}{2} = \frac{y+2}{3} = \frac{z-3}{-6} = r$

$x = 2r + 1$

$y = 3r - 2,$

$z = -6r + 3$

So $2r + 1 - 3r + 2 - 6r + 3 = 5$

$\Rightarrow -7r + 1 = 0$

$r = \frac{1}{7}$

$x = \frac{9}{7},\ y = \frac{-11}{7},,\ z = \frac{15}{7}$

Distance is $= \sqrt{\left(\frac{9}{7}-1\right)^2 + \left(2-\frac{11}{7}\right)^2 + \left(3-\frac{15}{7}\right)^2}$

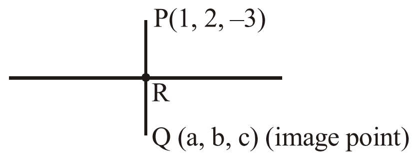
$= \sqrt{\left(\frac{2}{7}\right)^2 + \left(\frac{3}{7}\right)^2 + \left(\frac{6}{7}\right)^2}$

$= \frac{1}{7}\sqrt{4+9+36}$

$= \frac{1}{7}\sqrt{49} = 1$

13. Official Ans. by NTA (2)

Sol. Line is $\frac{x+1}{2} = \frac{y-3}{-2} = \frac{z}{-1} = \lambda$: Let point R is $(2\lambda - 1, -2\lambda + 3, -\lambda)$



Direction ratio of PQ $\equiv (2\lambda - 2, -2\lambda + 1, 3 - \lambda)$

PQ is $\perp^r$ to line

$\Rightarrow 2(2\lambda - 2) - 2(-2\lambda + 1) - 1(3 - \lambda) = 0$

$4\lambda - 4 + 4\lambda - 2 - 3 + \lambda = 0$

$9\lambda = 9 \Rightarrow \lambda = 1$

$\Rightarrow$ Point R is (1, 1, –1)

$\frac{a+1}{2} = 1$ | $\frac{b+2}{2} = 1$ | $\frac{c-3}{2} = -1$

a = 1 b = 0 c = 1

$\Rightarrow a + b + c = 2$

14. Official Ans. by NTA (4)

Sol. $L_1 \equiv \frac{x+1}{2} = \frac{y-2}{-1} = \frac{z-1}{1}$

$L_2 \equiv \frac{x+2}{\alpha} = \frac{y+1}{5-\alpha} = \frac{z+1}{1}$

Point A(–1, 2, 1) B(–2, –1, –1)

$\because$ L_1 and L_2 are coplanar

$$\Rightarrow \begin{vmatrix} 2 & -1 & 1 \\ \alpha & 5-\alpha & 1 \\ 1 & 3 & 2 \end{vmatrix} = 0$$

$\alpha = -4$

$L_2 \equiv \frac{x+2}{-4} = \frac{y+1}{9} = \frac{z+1}{1}$

Check options (2, –10, –2) lies on L_2

15. Official Ans. by NTA (4)

Sol. Line of intersection of planes

$x + y + z + 1 = 0$ ...(1)

$2x - y + z + 3 = 0$ ...(2)

eliminate y

$3x + 2z + 4 = 0$

$x = \frac{-2z-4}{3}$ ...(3)

put in equaiton (1)

$z = -3y + 1$ ...(4)

from (3) and (4)

$$\frac{3x+4}{-2} = -3y+1 = z$$

$$\frac{x - \left(-\frac{4}{3}\right)}{-\frac{2}{3}} = \frac{y - \frac{1}{3}}{-\frac{1}{3}} = \frac{z-0}{1}$$

now shortest distance between skew lines

$$\frac{x-1}{0} = \frac{y+1}{-1} = \frac{z}{1}$$

$$\frac{x - \left(-\frac{4}{3}\right)}{-\frac{2}{3}} = \frac{y - \left(\frac{1}{3}\right)}{-\frac{1}{3}} = \frac{z-0}{1}$$

$$\text{S.D.} = \left| \frac{(\vec{b}-\vec{a}).(\vec{c}\times\vec{d})}{|\vec{c}\times\vec{d}|} \right|$$

where $\vec{a} = (1,-1,0)$

$\vec{b} = \left(-\frac{4}{3}, \frac{1}{3}, 0\right)$

$\vec{c} = (0,-1,1)$

$\vec{d} = \left(-\frac{2}{3}, -\frac{1}{3}, 1\right)$

$\Rightarrow \text{S.D} = \frac{1}{\sqrt{3}}$

16. Official Ans. by NTA (2)

Sol. $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$

$A \equiv (a, 0, 0)$, $B \equiv (0, b, 0)$, $C \equiv (0, 0, c)$

Centroid $\equiv \left(\frac{a}{3}, \frac{b}{3}, \frac{c}{3}\right) = (1, 1, 2)$

$a = 3$, $b = 3$, $c = 6$

Plane : $\frac{x}{3} + \frac{y}{3} + \frac{z}{6} = 1$

$2x + 2y + z = 6$

line $\perp$ to the plane (DR of line = $2\hat{i} + 2\hat{j} + \hat{k}$)

$$\frac{x-1}{2} = \frac{y-1}{2} = \frac{z-2}{1}$$

PARABOLA

1. NTA Ans. (3)

Sol. $y = mx + 4$ is tangent to $y^2 = 4x$

$\Rightarrow m = \frac{1}{4}$

$y = \frac{1}{4}x + 4$ is tangent to $x^2 = 2by$

$\Rightarrow x^2 - \frac{b}{2}x - 8b = 0$

$\Rightarrow D = 0$

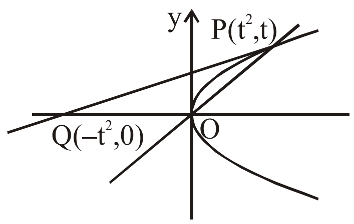
$b^2 + 128b = 0$

$\Rightarrow b = -128, 0$

$b \neq 0 \Rightarrow b = -128$

2. NTA Ans. (0.50)

Sol. $\Delta OPQ = 4$



$$\frac{1}{2}\left\|\begin{matrix} 0 & 0 & 1 \\ t^2 & t & 1 \\ -t^2 & 0 & 1 \end{matrix}\right\| = 4$$

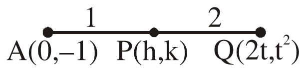
$t = 2$ ($\because$ $t > 0$)

$\therefore\ m = \dfrac{1}{2}$

Ans. 0.50

3. NTA Ans. (2)

Sol.

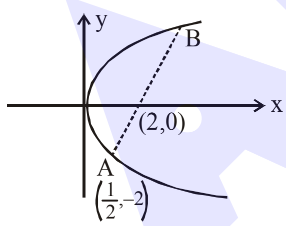


$\Rightarrow 3h = 2t$ and $3k = t^2 - 2$

$\Rightarrow\ 3y = \left(\dfrac{3x}{2}\right)^2 - 2 \Rightarrow 12y = 9x^2 - 8$

4. NTA Ans. (2)

Sol. $y^2 = 8x$



$4t_1 = -2 \Rightarrow t_1 = -\dfrac{1}{2}$,

$t_1.t_2 = -1$

$t_2 = -\dfrac{1}{t_1}$

$\Rightarrow t_2 = 2$

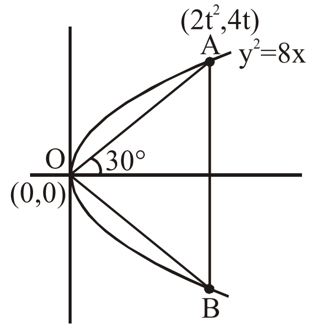
So coordinate of B is (8, 8)

$\therefore$ Equation of tangent at B is

$8y = 4(x + 8) \Rightarrow 2y = x + 8$

5. Official Ans. by NTA (3)

Sol.



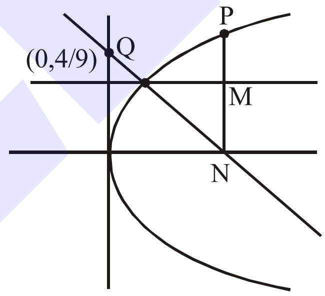
$\tan 30° = \dfrac{4t}{2t^2} = \dfrac{2}{t} \Rightarrow\ t = 2\sqrt{3}$

$AB = 8t = 16\sqrt{3}$

$\text{Area} = 256.3\cdot\dfrac{\sqrt{3}}{4} = 192\sqrt{3}$

6. Official Ans. by NTA (3)

Sol. Let $P = (3t^2, 6t)$; $N = (3t^2, 0)$



$M = (3t^2, 3t)$

Equation of MQ : $y = 3t$

$\therefore\ Q = \left(\dfrac{3}{4}t^2, 3t\right)$

Equation of NQ

$$y = \frac{3t}{\left(\frac{3}{4}t^2 - 3t^2\right)}(x - 3t^2)$$

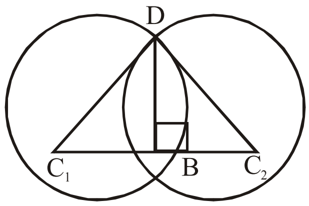
y-intercept of NQ $= 4t = \dfrac{4}{3} \Rightarrow\ t = \dfrac{1}{3}$

$\therefore\ MQ = \dfrac{9}{4}t^2 = \dfrac{1}{4}$

$PN = 6t = 2$

7. Official Ans. by NTA (1)

Sol. Length of latus rectum = 4



DB = 2

$C_1B = \sqrt{(C_1D)^2 - (DB)^2} = 4$

$C_1C_2 = 8$

8. Official Ans. by NTA (1)

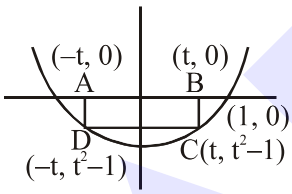
Sol. Area (A) = $2t \cdot (1 - t^2)$

$(0 < t < 1)$

$A = 2t - 2t^3$

$\frac{dA}{dt} = 2 - 6t^2$

$t = \frac{1}{\sqrt{3}}$



$\Rightarrow A_{max} = \frac{2}{\sqrt{3}}\left(1 - \frac{1}{3}\right) = \frac{4}{3\sqrt{3}}$

9. Official Ans. by NTA (3)

Sol. $y = mx + \frac{1}{m}$ (tangent at $y^2 = 4x$)

$y = mx - m^2$ (tangent at $x^2 = 4y$)

$\frac{1}{m} = -m^2$ (for common tangent)

$m^3 = -1$

$\boxed{m = -1}$

$y = -x - 1$

$x + y + 1 = 0$

This line touches circle

$\therefore$ apply p = r

$c = \left|\frac{0+0+1}{\sqrt{2}}\right| = \frac{1}{\sqrt{2}}$

10. Official Ans. by NTA (1)

Sol. $y^2 = 4(x + 1)$

equation of tangent $y = m(x + 1) + \frac{1}{m}$

$y = mx + m + \frac{1}{m}$

$y^2 = 8(x + 2)$

equation of tangent $y = m'(x+2) + \frac{2}{m'}$

$y = m'x + 2\left(m' + \frac{1}{m'}\right)$

since lines intersect at right angles

$\therefore mm' = -1$

Now $y = mx + m + \frac{1}{m}$ ...(1)

$y = m'x + 2\left(m' + \frac{1}{m'}\right)$

$y = -\frac{1}{m}x + 2\left(-\frac{1}{m} - m\right)$

$y = -\frac{1}{m}x - 2\left(m + \frac{1}{m}\right)$ ...(2)

From equation (1) and (2)

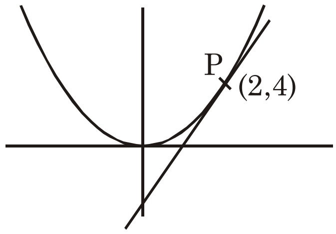
$mx + m + \frac{1}{m} = -\frac{1}{m}x - 2\left(m + \frac{1}{m}\right)$

$\left(m + \frac{1}{m}\right)x + 3\left(m + \frac{1}{m}\right) = 0$

$\therefore x + 3 = 0$

11. Official Ans. by NTA (2)

Sol. $y = x^2$



$$\left.\frac{dy}{dx}\right|_P = 4$$

$(y - 4) = 4(x - 2)$

$4x - y - 4 = 0$

Circle : $(x - 2)^2 + (y - 4)^2 + \lambda(4x - y - 4) = 0$

passes through (0, 1)

$4 + 9 + \lambda(-5) = 0 \qquad \Rightarrow \lambda = \dfrac{13}{5}$

Circle : $x^2 + y^2 + x(4\lambda - 4) + y(-\lambda - 8) + (20 - 4\lambda) = 0$

Centre : $\left(2 - 2\lambda, \dfrac{\lambda + 8}{2}\right) \equiv \left(\dfrac{-16}{5}, \dfrac{53}{10}\right)$

ELLIPSE

1. NTA Ans. (2)

Sol. $3x + 4y = 12\sqrt{12}$ is tangent to $\dfrac{x^2}{a^2} + \dfrac{y^2}{9} = 1$

$c^2 = m^2a^2 + b^2$

$\Rightarrow a^2 = 16$

$e = \sqrt{1 - \dfrac{9}{16}} = \dfrac{\sqrt{7}}{4}$

Distance between focii = $2ae = 2\sqrt{7}$

2. NTA Ans. (3)

Sol. Given $2ae = 6 \Rightarrow \boxed{ae = 3}$ ......(1)

and $\dfrac{2a}{e} = 12 \Rightarrow \boxed{a = 6e}$ ....(2)

from (1) and (2)

$6e^2 = 3 \Rightarrow \boxed{e = \dfrac{1}{\sqrt{2}}}$

$\Rightarrow \boxed{a = 3\sqrt{2}}$

Now, $b^2 = a^2(1 - e^2)$

$\Rightarrow b^2 = 18\left(1 - \dfrac{1}{2}\right) = 9$

Length of L.R = $\dfrac{2(9)}{3\sqrt{2}} = 3\sqrt{2}$

3. NTA Ans. (4)

Sol. Any normal to the ellipse is

$$\frac{x\sec\theta}{\sqrt{2}} - y\,\mathrm{cosec}\,\theta = -\frac{1}{2}$$

$$\Rightarrow \frac{x}{\left(\dfrac{-\cos\theta}{\sqrt{2}}\right)} + \frac{y}{\left(\dfrac{\sin\theta}{2}\right)} = 1$$

$$\Rightarrow \frac{\cos\theta}{\sqrt{2}} = \frac{1}{3\sqrt{2}} \text{ and } \frac{\sin\theta}{2} = \beta$$

$$\Rightarrow \beta = \frac{\sqrt{2}}{3}$$

4. NTA Ans. (2)

Sol. Let $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$; $a > b$;

$2b = \dfrac{4}{\sqrt{3}} \Rightarrow b = \dfrac{2}{\sqrt{3}} \Rightarrow b^2 = \dfrac{4}{3}$

tangent $y = \dfrac{-x}{6} + \dfrac{4}{3}$ compare with

$y = mx \pm \sqrt{a^2m^2 + b^2}$

$\Rightarrow m = \dfrac{-1}{6} \Rightarrow \sqrt{\dfrac{a^2}{36} + \dfrac{4}{3}} = \dfrac{4}{3} \Rightarrow a = 4;$

$e = \sqrt{1 - \dfrac{b^2}{a^2}} = \dfrac{1}{2}\sqrt{\dfrac{11}{3}}$

5. Official Ans. by NTA (1)

Sol. For ellipse $\frac{x^2}{25}+\frac{y^2}{b^2}=1$ (b < 5)

Let e_1 is eccentricity of ellipse

$\therefore\ b^2 = 25\ (1 - e_1^2)$

.... (1)

Again for hyperbola

$\frac{x^2}{16}-\frac{y^2}{b^2}=1$

Let e_2 is eccentricity of hyperbola.

$\therefore\ b^2 = 16(e_2^2 - 1)$

.... (2)

by (1) & (2)

$25(1 - e_1^2) = 16(e_2^2 - 1)$

Now $e_1.e_2 = 1$ (given)

$\therefore\ 25(1 - e_1^2) = 16\left(\frac{1-e_1^2}{e_1^2}\right)$

or $e_1=\frac{4}{5}$ $\therefore\ e_2 = \frac{5}{4}$

Now distance between foci is 2ae

$\therefore$ distance for ellipse $= 2 \times 5 \times \frac{4}{5} = 8 = \alpha$

distance for hyperbola $= 2 \times 4 \times \frac{5}{4} = 10 = \beta$

$\therefore\ (\alpha, \beta) \equiv (8, 10)$

6. Official Ans. by NTA (1)

Sol. $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1\ (a>b)$; $\frac{2b^2}{a}=10 \Rightarrow b^2 = 5a$ ...(i)

Now, $\phi(t)=\frac{5}{12}+t-t^2=\frac{8}{12}-\left(t-\frac{1}{2}\right)^2$

$\phi(t)_{max}=\frac{8}{12}=\frac{2}{3}=e \Rightarrow e^2=1-\frac{b^2}{a^2}=\frac{4}{9}$

... (ii)

$\Rightarrow\ a^2 = 81$ (from (i) & (ii))

So, $a^2 + b^2 = 81 + 45 = 126$

7. Official Ans. by NTA (2)

Sol. Ellipse : $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

directrix : $x = \frac{a}{e}=4$ & $e = \frac{1}{2}$

$\Rightarrow a = 2\ \&\ b^2 = a^2\ (1-e^2) = 3$

$\Rightarrow$ Ellipse is $\frac{x^2}{4}+\frac{y^2}{3}=1$

P is $\left(1,\frac{3}{2}\right)$

Normal is : $\frac{4x}{1}-\frac{3y}{3/2}=4-3$

$\Rightarrow 4x - 2y = 1$

8. Official Ans. by NTA (2)

Sol. Given ellipse is $\frac{x^2}{5}+\frac{y^2}{4}=1$

Let point P is $(\sqrt{5}\cos\theta, 2\sin\theta)$

$(PQ)^2 = 5\cos^2\theta + 4\ (\sin\theta + 2)^2$

$(PQ)^2 = \cos^2\theta + 16\sin\theta + 20$

$(PQ)^2 = -\sin^2\theta + 16\sin\theta + 21$

$= 85 - (\sin\theta - 8)^2$

will be maximum when $\sin\theta = 1$

$\Rightarrow (PQ)^2_{max} = 85 - 49 = 36$

9. Official Ans. by NTA (1)

Sol. $\frac{x^2}{16}+\frac{y^2}{9}=1$

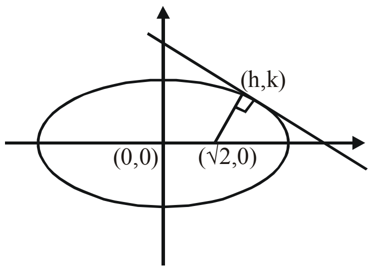
$a = 4;\ b = 3;\ e = \sqrt{\frac{16-9}{16}}=\frac{\sqrt{7}}{4}$

A and B are foci

$\Rightarrow PA + PB = 2a = 2 \times 4 = 8$

10. Official Ans. by NTA (1)

Sol. Let foot of perpendicular is (h,k)



$$\frac{x^2}{4}+\frac{y^2}{2}=1 \text{ (Given)}$$

a =2 , $b=\sqrt{2}$, $e=\sqrt{1-\frac{2}{4}}=\frac{1}{\sqrt{2}}$

$\therefore$ Focus (ae,0) $=\left(\sqrt{2},0\right)$

Equation of tangent

$$y=mx+\sqrt{a^2m^2+b^2}$$

$$y=mx+\sqrt{4m^2+2}$$

Passes throguh (h,k)

$(k-mh)^2 = 4m^2 + 2$ ...(1)

line perpendicular to tangent will have slope

$-\frac{1}{m}$

$$y-0=-\frac{1}{m}\left(x-\sqrt{2}\right)$$

$$my=-x+\sqrt{2}$$

$(h + mk)^2 = 2$ ...(2)

Add equaiton (1) and (2)

$k^2(1 + m^2) + h^2(1 + m^2) = 4(1 + m^2)$

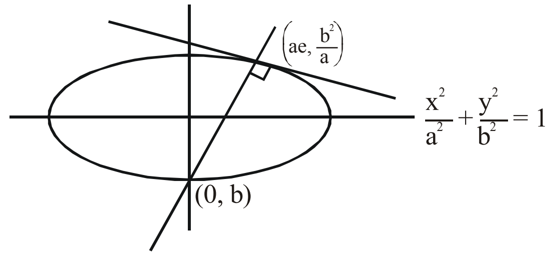
$h^2 + k^2 = 4$

$x^2 + y^2 = 4$ (Auxilary circle)

$\therefore \left(-1,\sqrt{3}\right)$ lies on the locus.

11. Official Ans. by NTA (4)

Sol. $\frac{a^2x}{x_1}-\frac{b^2y}{y_1}=a^2e^2$



$$\frac{a^2x}{ae}-\frac{b^2y}{b^2}.a=a^2e^2$$

$$\frac{ax}{e}-ay=a^2e^2 \Rightarrow \frac{x}{e}-y=ae^2$$

passes through (0, b)

$-b = ae^2 \Rightarrow b^2 = a^2e^4$

$a^2(1-e^2) = a^2e^4 \Rightarrow e^4 + e^2 = 1$

HYPERBOLA

1. NTA Ans. (3)

Sol. $\frac{x^2}{36}-\frac{y^2}{b^2}=1$

...(i)

P(10,16) lies on (i) get $b^2 = 144$

$$\frac{x^2}{36}-\frac{y^2}{144}=1$$

Equation of normal is

$$\frac{a^2x}{x_1}+\frac{b^2y}{y_1}=a^2e^2$$

$2x + 5y = 100$

(3) Option

2. NTA Ans. (4)

Sol. $e_1=\sqrt{1-\frac{4}{18}}=\frac{\sqrt{7}}{3}$

$$e_2=\sqrt{1+\frac{4}{9}}=\frac{\sqrt{13}}{3}$$

$\because$ (e_1, e_2) lies on $15x^2 + 3y^2 = k$

$\Rightarrow 15e_1^2+3e_2^2=k$

$\Rightarrow k = 16$

3. Official Ans. by NTA (2)

Sol. Slope of tangent is 2, Tangent of hyperbola

$\frac{x^2}{4}-\frac{y^2}{2}=1$ at the point (x_1, y_1) is

$\frac{xx_1}{4}-\frac{yy_1}{2}=1$ (T = 0)

Slope : $\frac{1}{2}\frac{x_1}{y_1}=2 \Rightarrow \boxed{x_1=4y_1}$ ...(1)

(x_1, y_1) lies on hyperbola

$\Rightarrow \boxed{\frac{x_1^2}{4}-\frac{y_1^2}{2}=1}$ ...(2)

From (1) & (2)

$\frac{(4y_1)^2}{4}-\frac{y_1^2}{2}=1 \Rightarrow 4y_1^2-\frac{y_1^2}{2}=1$

$\Rightarrow 7y_1^2=2 \Rightarrow \boxed{y_1^2=2/7}$

Now $x_1^2 + 5y_1^2 = (4y_1)^2 + 5y_1^2$

$= (21)y_1^2 = 21 \times \frac{2}{7}=6$

4. Official Ans. by NTA (2)

Sol. Given $\theta \in \left(0,\frac{\pi}{2}\right)$

equation of hyperbola $\Rightarrow x^2 - y^2\sec^2\theta = 10$

$\Rightarrow \frac{x^2}{10}-\frac{y^2}{10\cos^2\theta}=1$

Hence eccentricity of hyperbola

$(e_H)=\sqrt{1+\frac{10\cos^2\theta}{10}}$ ...(1)

$\left\{e=\sqrt{1+\frac{b^2}{a^2}}\right\}$

Now equation of ellipse $\Rightarrow x^2\sec^2\theta + y^2 = 5$

$\Rightarrow \frac{x^2}{5\cos^2\theta}+\frac{y^2}{5}=1$ $\left\{e=\sqrt{1-\frac{a^2}{b^2}}\right\}$

Hence eccenticity of ellipse

$(e_E)=\sqrt{1-\frac{5\cos^2\theta}{5}}$

$(e_E)=\sqrt{1-\cos^2\theta}=|\sin\theta|=\sin\theta$ ...(2)

$\left\{\because \theta \in \left(0,\frac{\pi}{2}\right)\right\}$

given $\Rightarrow e_H = \sqrt{5}\, e_e$

Hence $1 + \cos^2\theta = 5\sin^2\theta$

$1 + \cos^2\theta = 5(1 - \cos^2\theta)$

$1 + \cos^2\theta = 5 - 5\cos^2\theta$

$6\cos^2\theta = 4$

$\cos^2\theta = \frac{2}{3}$ ...(3)

Now length of latus rectum of ellipse

$=\frac{2a^2}{b}=\frac{10\cos^2\theta}{\sqrt{5}}=\frac{20}{3\sqrt{5}}=\frac{4\sqrt{5}}{3}$

5. Official Ans. by NTA (2)

Sol. Ellipse : $\frac{x^2}{4}+\frac{y^2}{3}=1$

eccentricity $=\sqrt{1-\frac{3}{4}}=\frac{1}{2}$

$\therefore$ foci = $(\pm 1, 0)$

for hyperbola, given $2a = \sqrt{2} \Rightarrow a = \frac{1}{\sqrt{2}}$

$\therefore$ hyperbola will be

$\frac{x^2}{1/2}-\frac{y^2}{b^2}=1$

eccentricity $=\sqrt{1+2b^2}$

$\therefore$ foci $=\left(\pm\sqrt{\frac{1+2b^2}{2}}, 0\right)$

$\because$ Ellipse and hyperbola have same foci

$\Rightarrow \sqrt{\frac{1+2b^2}{2}}=1$

$\Rightarrow b^2=\frac{1}{2}$

$\therefore$ Equation of hyperbola : $\frac{x^2}{1/2}-\frac{y^2}{1/2}=1$

$\Rightarrow x^2 - y^2 = \frac{1}{2}$

Clearly $\left(\sqrt{\frac{3}{2}}, \frac{1}{\sqrt{2}}\right)$ does not lie on it.

6. Official Ans. by NTA (1)

Sol. Since, (3, 3) lies on $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

$\frac{9}{a^2}-\frac{9}{b^2}=1$
....(1)

Now, normal at (3, 3) is $y-3=-\frac{a^2}{b^2}(x-3)$,

which passes through (9, 0) $\Rightarrow$ $b^2 = 2a^2$
....(2)

So, $e^2 = 1+\frac{b^2}{a^2} = 3$

Also, $a^2 = \frac{9}{2}$ (from (i) & (ii))

Thus, $(a^2, e^2) = \left(\frac{9}{2}, 3\right)$

7. Official Ans. by NTA (2)

Sol. y = mx + c is tangent to

$\frac{x^2}{100}-\frac{y^2}{64}=1$ and $x^2 + y^2 = 36$

$c^2 = 100m^2 - 64 \mid c^2 = 36(1+m^2)$

$\Rightarrow 100m^2 - 64 = 36 + 36m^2$

$m^2 = \frac{100}{64} \Rightarrow m = \pm\frac{10}{8}$

$c^2 = 36\left(1+\frac{100}{64}\right) = \frac{36\times164}{64}$

$4c^2 = 369$

COMPLEX NUMBER

1. NTA Ans. (3)

Sol. $\frac{3+i\sin\theta}{4-i\cos\theta}$ is a real number

$\Rightarrow 3\cos\theta + 4\sin\theta = 0$

$\Rightarrow \tan\theta = \frac{-3}{4}$

argument of $\sin\theta + i\cos\theta = \pi-\tan^{-1}\frac{4}{3}$

2. NTA Ans. (2)

Sol. $\text{Re}\left(\frac{z-1}{2z+i}\right)=1$

Put z = x + iy

$\text{Re}\left(\frac{(x+iy)-1}{2(x+iy)+i}\right)=1$

$\text{Re}\left(\left(\frac{(x-1)+iy}{2x+i(2y+1)}\right)\left(\frac{2x-i(2y+1)}{2x-i(2y+1)}\right)\right)=1$

$\Rightarrow 2x^2 + 2y^2 + 2x + 3y + 1 = 0$

$x^2 + y^2 + x + \frac{3}{2}y + \frac{1}{2} = 0$

$\Rightarrow$ locus is a circle whose

Centre is $\left(-\frac{1}{2}, -\frac{3}{4}\right)$ and radius $\frac{\sqrt{5}}{4}$

$\Rightarrow$ diameter $= \frac{\sqrt{5}}{2}$

3. NTA Ans. (1)

Sol. $\alpha = \omega$

$a = (1+\omega)(1+\omega^2+\omega^4+\ldots\ldots+\omega^{200})$

$a=(1+\omega)\frac{\left(1-(\omega^2)^{101}\right)}{1-\omega^2}=1$

$b = 1+\omega^3+\omega^6+\ldots\ldots+\omega^{300} = 101$

$x^2 - 102x + 101 = 0$

(1) Option

4. NTA Ans. (4)

Sol. Assuming z is a root of the given equation,

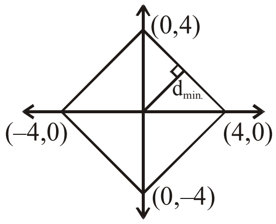
$z=\frac{-b\pm i\sqrt{180-b^2}}{2}$

so, $\left(1-\frac{b}{2}\right)^2+\frac{180-b^2}{4}=40$

$\Rightarrow -4b + 184 = 160 \Rightarrow b = 6$

5. NTA Ans. (4)

Sol. z = x + iy



|x| + |y| = 4

$|z| = \sqrt{x^2 + y^2} \Rightarrow |z|_{min} = \sqrt{8}$ & $|z|_{max} = 4 = \sqrt{16}$

So |z| cannot be $\sqrt{7}$

6. NTA Ans. (3)

Sol. $\left|\frac{z-i}{z+2i}\right| = 1$

$\Rightarrow |z - i| = |z + 2i|$

$\Rightarrow$ z lies on perpendicular bisector of (0, 1) and (0, –2).

$\Rightarrow \text{Im} z = -\frac{1}{2}$

Let $z = x - \frac{i}{2}$

$\because |z| = \frac{5}{2} \Rightarrow x^2 = 6$

$\therefore |z + 3i| = \left|x + \frac{5i}{2}\right| = \sqrt{x^2 + \frac{25}{4}}$

$= \sqrt{6 + \frac{25}{4}} = \frac{7}{2}$

7. Official Ans. by NTA (2)

Sol. The value of $\left(\frac{1 + \sin 2\pi/9 + i\cos 2\pi/9}{1 + \sin\frac{2\pi}{9} - i\cos\frac{2\pi}{9}}\right)$

$= \left(\frac{1 + \sin\left(\frac{\pi}{2} - \frac{5\pi}{18}\right) + i\cos\left(\frac{\pi}{2} - \frac{5\pi}{18}\right)}{1 + \sin\left(\frac{\pi}{2} - \frac{5\pi}{18}\right) - i\cos\left(\frac{\pi}{2} - \frac{5\pi}{18}\right)}\right)^3$

$= \left(\frac{1 + \cos\frac{5\pi}{18} + i\sin\frac{5\pi}{18}}{1 + \cos\frac{5\pi}{18} - i\sin\frac{5\pi}{18}}\right)^3$

$= \left(\frac{2\cos^2\frac{5\pi}{36} + 2i\sin\frac{5\pi}{36}\cos\frac{5\pi}{36}}{2\cos^2\frac{5\pi}{36} - 2i\sin\frac{5\pi}{36}.\cos\frac{5\pi}{36}}\right)^3$

$= \left(\frac{\cos\frac{5\pi}{36} + i\sin\frac{5\pi}{36}}{\cos\frac{5\pi}{36} - i\sin\frac{5\pi}{36}}\right)^3$

$= \left(\frac{e^{i5\pi/36}}{e^{-i5\pi/36}}\right)^3 = \left(e^{i5\pi/18}\right)^3$

$= \cos\frac{5\pi}{6} + i\sin 5\pi/6$

$= -\frac{\sqrt{3}}{2} + i/2$

8. Official Ans. by NTA (1)

Sol. $\left(3 + 2\sqrt{-54}\right) = 3 + 2 \times 3 \times \sqrt{6}\,i$

$= \left(3 + \sqrt{6}\,i\right)^2$

$\left(3 - 2\sqrt{54}\right) = \left(3 - \sqrt{6}\,i\right)^2$

$\left(3 + 2\sqrt{-54}\right)^{1/2} + \left(3 - 2\sqrt{-54}\right)^{1/2}$

$= \pm\left(3 + \sqrt{6}\,i\right) \pm \left(3 - \sqrt{6}\,i\right)$

$= 6, -6, 2\sqrt{6}i, -2\sqrt{6}i,$

9. Official Ans. by NTA (4)

Sol. $\left(\frac{1+i}{1-i}\right)^{m/2} = \left(\frac{1+i}{i-1}\right)^{n/3} = 1$

$\Rightarrow \left(\frac{(1+i)^2}{2}\right)^{m/2} = \left(\frac{(1+i)^2}{-2}\right)^{n/3} = 1$

$\Rightarrow (i)^{m/2} = (-i)^{n/3} = 1$

$\Rightarrow \frac{m}{2} = 4k_1$ and $\frac{n}{3} = 4k_2$

$\Rightarrow m = 8k_1$ and $n = 12k_2$

Least value of m = 8 and n = 12.

$\therefore$ GCD = 4

10. Official Ans. by NTA (4)

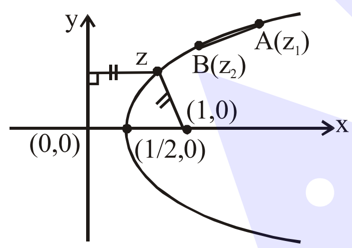
Sol. $Re(z) = |z - 1|$

$\Rightarrow x = \sqrt{(x-1)^2 + (y-0)^2}$ $(x > 0)$

$\Rightarrow y^2 = 2x - 1 = 4.\frac{1}{2}\left(x - \frac{1}{2}\right)$

$\Rightarrow$ a parabola with focus (1, 0) & directrix as imaginary axis.

$\therefore$ Vertex $= \left(\frac{1}{2}, 0\right)$



$A(z_1)$ & $B(z_2)$ are two points on it such that

slope of AB = $\tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$

$(\arg(z_1 - z_2) = \frac{\pi}{6})$

for $y^2 = 4ax$

Let $A(at_1^2, 2at_1)$ & $B(at_2^2, 2at_2)$

$m_{AB} = \frac{2}{t_1 + t_2} = \frac{4a}{y_1 + y_2} = \frac{1}{\sqrt{3}}$

$\left(\text{Here } a = \frac{1}{2}\right)$

$\Rightarrow y_1 + y_2 = 4a\sqrt{3} = 2\sqrt{3}$

11. Official Ans. by NTA (3)

Sol. $A^2 = \begin{pmatrix} \cos 2\theta & i\sin 2\theta \\ i\sin 2\theta & \cos 2\theta \end{pmatrix}$

Similarly, $A^5 = \begin{pmatrix} \cos 5\theta & i\sin 5\theta \\ i\sin 5\theta & \cos 5\theta \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

(1) $a^2 + b^2 = \cos^2 5\theta - \sin^2 5\theta = \cos 10\theta = \cos 75°$

(2) $a^2 - d^2 = \cos^2 5\theta - \cos^2 5\theta = 0$

(3) $a^2 - b^2 = \cos^2 5\theta + \sin^2 5\theta = 1$

(4) $a^2 - c^2 = \cos^2 5\theta + \sin^2 5\theta = 1$

12. Official Ans. by NTA (3)

Sol. $u = \frac{2z + i}{z - ki}$

$= \frac{2x^2 + (2y+1)(y-k)}{x^2 + (y-k)^2} + i\frac{(x(2y+1) - 2x(y-k))}{x^2 + (y-k)^2}$

Since $Re(u) + Im(u) = 1$

$\Rightarrow 2x^2 + (2y+1)(y-k) + x(2y+1) - 2x(y-k) = x^2 + (y-k)^2$

$\left.\begin{matrix} P(0, y_1) \\ Q(0, y_2) \end{matrix}\right\rangle \Rightarrow y^2 + y - k - k^2 = 0 \left\langle\begin{matrix} y_1 + y_2 = -1 \\ y_1 y_2 = -k - k^2 \end{matrix}\right.$

$\because$ PQ = 5

$\Rightarrow |y_1 - y_2| = 5 \Rightarrow k^2 + k - 6 = 0$

$\Rightarrow k = -3, 2$

So, k = 2 (k > 0)

13. Official Ans. by NTA (4)

Sol. $\alpha = \omega \quad (\omega^3 = 1)$

$\Rightarrow (2 + \omega)^4 = a + b\omega$

$\Rightarrow 2^4 + 4.2^3\omega + 6.2^2\omega^3 + 4.2 . \omega^3 + \omega^4 = a + b\omega$

$\Rightarrow 16 + 32\omega + 24\omega^2 + 8 + \omega = a + b\omega$

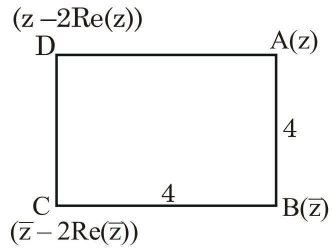
$\Rightarrow 24 + 24\omega^2 + 33\omega = a + b\omega$

$\Rightarrow -24\omega + 33\omega = a + b\omega$

$\Rightarrow a = 0, b = 9$

14. Official Ans. by NTA (4)

Sol. Let z = x + iy



Length of side = 4

AB = 4

$|z-\overline{z}|=4$

|2y| = 4 ; |y| = 2

BC = 4

$\left|\overline{z}-(\overline{z}-2\operatorname{Re}(\overline{z})\right|=4$

|2x| = 4 ; |x| = 2

$|z| = \sqrt{x^2+y^2} = \sqrt{4+4} = 2\sqrt{2}$

15. Official Ans. by NTA (3)

Sol. $\left(\frac{-1+i\sqrt{3}}{1-i}\right)^{30} = \left(\frac{2\omega}{1-i}\right)^{30}$

$= \frac{2^{30}\cdot\omega^{30}}{\left((1-i)^2\right)^{30}}$

$= \frac{2^{30}\cdot 1}{(1+i^2-2i)^{15}}$

$= \frac{2^{30}}{-2^{15}\cdot i^{15}}$

$= -2^{15}i$

16. Official Ans. by NTA (4)

Sol. z = x + iy

$|z| - ke(z) \le 1$

$\Rightarrow \sqrt{x^2+y^2} - x \le 1$

$\Rightarrow \sqrt{x^2+y^2} \le 1 + x$

$\Rightarrow x^2 + y^2 \le 1 + 2x + x^2$

$\Rightarrow y^2 \le 2x + 1$

$\Rightarrow y^2 \le 2\left(x+\frac{1}{2}\right)$

17. Official Ans. by NTA (3)

Sol. z = x + iy

$z^2 = i|z|^2$

$(x + iy)^2 = i(x^2 + y^2)$

$(x^2 - y^2) - i(x^2 + y^2 - 2xy) = 0$

$(x - y)(x + y) - i(x - y)^2 = 0$

$(x - y)((x + y) - i(x - y)) = 0$

$\Rightarrow x = y$

z lies on y = x

PROBABILITY

1. NTA Ans. (3)

Sol. Probability that at most 2 machines are out of service

$= {}^5C_0\left(\frac{3}{4}\right)^5 + {}^5C_1\left(\frac{3}{4}\right)^4\left(\frac{1}{4}\right) + {}^5C_2\left(\frac{3}{4}\right)^3\left(\frac{1}{4}\right)^2$

$= \left(\frac{3}{4}\right)^4 \times \frac{17}{8} \Rightarrow k = \frac{17}{8}$

2. NTA Ans. (3)

Sol.

k	0	1	2	3	4	5
P(k)	$\frac{1}{32}$	$\frac{12}{32}$	$\frac{11}{32}$	$\frac{5}{32}$	$\frac{2}{32}$	$\frac{1}{32}$

Expected value = $\sum XP(k)$

$-\frac{1}{32}-\frac{12}{32}-\frac{11}{32}+\frac{15}{32}+\frac{8}{32}+\frac{5}{32}$

$= \frac{28-24}{32} = \frac{4}{32} = \frac{1}{8}$

3. NTA Ans. (4)

Sol. $P(A) + P(B) - 2P(A \cap B) = \frac{2}{5}$

$P(A) + P(B) - P(A \cap B) = \frac{1}{2}$

$P(A \cap B) = \frac{1}{10}$

(4) Option

4. NTA Ans. (3)

Sol. (1) $P(A/B) = P(A) = \dfrac{1}{3}$

(2) $P(A/(A \cup B)) = \dfrac{P(A \cap (A \cup B))}{P(A \cup B)} = \dfrac{P(A)}{P(A \cup B)}$

$$= \frac{\frac{1}{3}}{\frac{1}{3} + \frac{1}{6} - \frac{1}{18}} = \frac{3}{4}$$

(3) $P(A/B') = P(A) = \dfrac{1}{3}$

(4) $P(A'/B') = P(A') = \dfrac{2}{3}$

5. NTA Ans. (2)

Sol. $\sum P(X) = 1 \Rightarrow K^2 + 2K + K + 2K + 5K^2 = 1$

$\Rightarrow 6K^2 + 5K - 1 = 0 \Rightarrow (6K - 1)(K + 1) = 0$

$\Rightarrow K = -1$ (rejected) $\Rightarrow K = \dfrac{1}{6}$

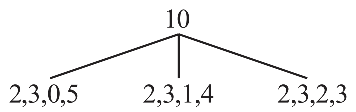
$P(X > 2) = K + 2K + 5K^2 = \dfrac{23}{36}$

6. NTA Ans. (3)

ALLEN Ans. (BONUS)

Note: Interpretating the given question, we find an answer that does not match with any of the given options. So, it should be bonus, but NTA retained the answer as option(3).

Sol. 10 different balls in 4 different boxes.



$$\frac{1}{4^{10}}\left(4! \times \frac{10!}{2! \times 3! \times 0! \times 5!} + 4! \times \frac{10!}{2! \times 3! \times 1! \times 4!} + 4! \times \frac{10!}{(2!)^2 \times 2! \times (3!)^2 \times 2!}\right)$$

$$= \frac{17 \times 945}{2^{15}}$$

7. NTA Ans. (1)

Sol. A : Event when card A is drawn

B : Event when card B is drawn.

$$P(A) = P(B) = \frac{1}{2}$$

Required probability = P(AA or (AB)A or (BA)A or (ABB)A or (BAB)A or (BBA)A)

$$= \frac{1}{2} \times \frac{1}{2} + \left(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}\right) \times 2 + \left(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}\right) \times 3$$

$$= \frac{1}{4} + \frac{1}{4} + \frac{3}{16} = \frac{11}{16}$$

8. Official Ans. by NTA (1)

Sol. Let B_1 be the event where Box–I is selected.

& $B_2 \rightarrow$ where box-II selected

$P(B_1) = P(B_2) = \dfrac{1}{2}$

Let E be the event where selected card is non prime.

For B_1 : Prime numbers :

{2, 3, 5, 7, 11, 13, 17, 19, 23, 29}

For B_2 : Prime numbers :

{31, 37, 41, 43, 47}

$$P(E) = P(B_1) \times P\left(\frac{E}{B_1}\right) + P(B_2)P\left(\frac{E}{B_2}\right)$$

$$= \frac{1}{2} \times \frac{20}{30} + \frac{1}{2} \times \frac{15}{20}$$

Required probability :

$$P\left(\frac{B_1}{E}\right) = \frac{\frac{1}{2} \times \frac{20}{30}}{\frac{1}{2} \times \frac{20}{30} + \frac{1}{2} \times \frac{15}{20}} = \frac{\frac{2}{3}}{\frac{2}{3} + \frac{3}{4}} = \frac{8}{17}$$

9. Official Ans. by NTA (1)

Sol. Given E_1, E_2, E_3 are pairwise indepedent events so $P(E_1 \cap E_2) = P(E_1).P(E_2)$

and $P(E_2 \cap E_3) = P(E_2).P(E_3)$

and $P(E_3 \cap E_1) = P(E_3).P(E_1)$

& $P(E_1 \cap E_2 \cap E_3) = 0$

Now $P\left(\frac{\bar{E}_2 \cap \bar{E}_3}{E_1}\right) = \frac{P\left[E_1 \cap (\bar{E}_2 \cap \bar{E}_3)\right]}{P(E_1)}$

$$= \frac{P(E_1) - [P(E_1 \cap E_2) + P(E_1 \cap E_3) - P(E_1 \cap E_2 \cap E_3)]}{P(E_1)}$$

$$= \frac{P(E_1) - P(E_1).P(E_2) - P(E_1)P(E_3) - 0}{P(E_1)}$$

$= 1 - P(E_2) - P(E_3)$

$= [1 - P(E_3)] - P(E_2)$

$= P(E_3^C) - P(E_2)$

10. Official Ans. by NTA (2)

Sol. A : Sum obtained is a multiple of 4.

A = {(1, 3), (2, 2), (3, 1), (2, 6), (3, 5), (4, 4), (5, 3), (6, 2), (6, 6)}

B : Score of 4 has appeared at least once.

B = {(1, 4), (2, 4), (3, 4), (4, 4), (5, 4), (6, 4), (4, 1), (4, 2), (4, 3), (4, 5), (4, 6)}

Required probability $= P\left(\frac{B}{A}\right) = \frac{P(B \cap A)}{P(A)}$

$$= \frac{1/36}{9/36} = \frac{1}{9}$$

11. Official Ans. by NTA (3)

Sol. First Case: Choose two non-zero digits ${}^9C_2$

Second Case : Number of 5-digit numbers containing both digits $= 2^5 - 2$

Choose one non-zero & one zero as digit $= {}^9C_1$

Number of 5-digit numbers containg one non zero and one zero both $= (2^4 - 1)$

$\therefore$ Required prob.

$$= \frac{\left({}^9C_2 \times (2^5 - 2) + {}^9C_1 \times (2^4 - 1)\right)}{9 \times 10^4}$$

$$= \frac{36 \times (32 - 2) + 9 \times (16 - 1)}{9 \times 10^4}$$

$$= \frac{4 \times 30 + 15}{10^4} = \frac{135}{10^4}$$

12. Official Ans. by NTA (3)

Sol. We have, 1 – (probability of all shots result in failure) $> \frac{1}{4}$

$$\Rightarrow 1 - \left(\frac{9}{10}\right)^n > \frac{1}{4}$$

$$\Rightarrow \frac{3}{4} > \left(\frac{9}{10}\right)^n \Rightarrow n \geq 3$$

13. Official Ans. by NTA (4)

Sol. $P(6) = \frac{1}{6}$, $P(7) = \frac{5}{36}$

P(A) = W + FFW + FFFFW + ...

$$= \frac{1}{6} + \frac{5}{6} \times \frac{31}{36} \times \frac{1}{6} + \left(\frac{5}{6}\right)^2 \left(\frac{31}{36}\right)^2 \frac{1}{6} + \ldots$$

$$= \frac{\frac{1}{6}}{1 - \frac{155}{216}} = \frac{36}{61}$$

14. Official Ans. by NTA (11)

Sol. 4 dice are independently thrown. Each die has probability to show 3 or 5 is

$$p = \frac{2}{6} = \frac{1}{3}$$

$$\therefore q = 1 - \frac{1}{3} = \frac{2}{3} \text{ (not showing 3 or 5)}$$

Experiment is performed with 4 dices independently.

$\therefore$ Their binomial distribution is

$(q + p)^4 = (q)^4 + {}^4C_1 q^3p + {}^4C_2 q^2p^2 + {}^4C_3 qp^3 + {}^4C_4p^4$

$\therefore$ In one throw of each dice probability of showing 3 or 5 at least twice is

$$= p^4 + {}^4C_3\, qp^3 + {}^4C_2q^2p^2 = \frac{33}{81}$$

$\therefore$ Such experiment performed 27 times

$\therefore$ so expected out comes = np

$$= \frac{33}{81} \times 27$$

= 11

15. Official Ans. by NTA (11.00)

Sol. $P(H) = \frac{1}{2}$

$$P(\bar{H}) = \frac{1}{2}$$

Let total 'n' bomb are required to destroy the target

$$1 - {}^nC_n\left(\frac{1}{2}\right)^n - {}^nC_1\left(\frac{1}{2}\right)^n \geq \frac{99}{100}$$

$$1 - \frac{1}{2^n} - \frac{n}{2^n} \geq \frac{99}{100}$$

$$\frac{1}{100} \geq \frac{n+1}{2^n}$$

Now check for value of n

$\boxed{n = 11}$

16. Official Ans. by NTA (2)

Sol. Total numbers in three familes = 3 + 3 + 4 = 10

so total arrangement = 10!

Family 1	Family 2	Family 3
3	3	4

Favourable cases

$$= \underset{\text{Arrangment of 3 Families}}{3!} \quad \underset{\text{Interval Arrangment of families members}}{3! \times 3! \times 4!}$$

$\therefore$ Probability of same family memebers are together $= \frac{3!\,3!\,3!\,4!}{10!} = \frac{1}{700}$

so option(2) is correct.

17. Official Ans. by NTA (3)

Sol. Out of 11 consecutive natural numbers either 6 even and 5 odd numbers or 5 even and 6 odd numbers

when 3 numbers are selected at random then total cases = ${}^{11}C_3$

Since these 3 numbers are in A.P. Let no's are a,b,c

$2b \Rightarrow$ even number

$$a + c \Rightarrow \begin{pmatrix} \text{even} + \text{even} \\ \text{odd} + \text{odd} \end{pmatrix}$$

so favourable cases = ${}^6C_2 + {}^5C_2$

= 15 + 10 = 25

$$P(\text{3 numbers are in A.P.} = \frac{25}{{}^{11}C_3} = \frac{25}{165} = \frac{5}{33})$$

18. Official Ans. by NTA (3)

Sol. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$0.8 = 0.6 + 0.4 - P(A \cap B)$

$P(A \cap B) = 0.2$

$P(A \cup B \cup C) = \Sigma P(A) - \Sigma P(A \cap B) + P(A \cap B \cap C)$

$\alpha = 1.5 - (0.2 + 0.3 + \beta) + 0.2$

$\alpha = 1.2 - \beta \in [0.85, 0.95]$

(where $\alpha \in [0.85, 0.95]$)

$\beta \in [0.25, 0.35]$

STATISTICS

1. NTA Ans. (54.00)

Sol. $\frac{3+7+9+12+13+20+x+y}{8}=10$

$x + y = 16$

$\frac{\Sigma x^2}{n}-\left(\frac{\Sigma x}{n}\right)^2=25$

$3^2 + 7^2 + 9^2 + 12^2 + 13^2 + 20^2 + x^2 + y^2 = 1000$

$x^2 + y^2 = 148$

$xy = 54$

2. NTA Ans. (18)

Sol. Variance of first 'n' natural numbers $=\frac{n^2-1}{12}=10$

$\Rightarrow n = 11$

and variance of first 'm' even natural numbers

$= 4\left(\frac{m^2-1}{12}\right)\Rightarrow\frac{m^2-1}{3}=16\Rightarrow m=7$

$m + n = 18$

3. NTA Ans. (1)

Sol. $\frac{\sum x_i}{20}=10 \Rightarrow \Sigma x_i = 200$ ...(i)

$\frac{\sum x_i^2}{20}-100=4 \Rightarrow \Sigma x_i^2 = 2080$ ...(ii)

Actual mean $=\frac{200-9+11}{20}=\frac{202}{20}$

Variance $=\frac{2080-81+121}{20}-\left(\frac{202}{20}\right)^2=3.99$

(1) Option

4. NTA Ans. (1)

Sol. $20p - q = 10$ ...(i)

and $2|p| = 1 \Rightarrow p=\pm\frac{1}{2}$ ...(ii)

so, $p=-\frac{1}{2}$ and $q=-20$

5. NTA Ans. (4)

Sol. $\sum_{i=1}^{10}(x_i-5)=10$

$\Rightarrow$ Mean of observation $x_i-5=\frac{1}{10}\sum_{i=1}^{3}(x_i-5)=1$

$\Rightarrow \mu$ = mean of observation $(x_i - 3)$

= (mean of observation $(x_i - 5)$) + 2

= 1 + 2 = 3

Variance of observation

$x_i-5=\frac{1}{10}\sum_{i=1}^{10}(x_i-5)^2$

– (Mean of $(x_i - 5))^2 = 3$

$\Rightarrow \lambda$ = variance of observation $(x_i - 3)$

= variance of observation $(x_i - 5) = 3$

$\therefore (\mu, \lambda) = (3, 3)$

6. Official Ans. by NTA (1)

Sol. σ^2 = variance

μ = mean

$\sigma^2=\frac{\sum_{i=1}^{n}(x_i-\mu)^2}{n}$

$\mu = 17$

$\Rightarrow \frac{\sum_{x=1}^{17}(ax+b)}{17}=17$

$\Rightarrow 9a + b = 17$ ....(1)

$\sigma^2 = 216$

$\Rightarrow \frac{\sum_{x=1}^{17}(ax+b-17)^2}{17}=216$

$\Rightarrow \frac{\sum_{x=1}^{17}a^2(x-9)^2}{17}=216$

$\Rightarrow a^281 - 18 \times 9a^2 + a^2\, 3 \times (35) = 216$

$\Rightarrow a^2=\frac{216}{24}=9 \Rightarrow a = 3\ (a > 0)$

$\Rightarrow$ From (1), $b = -10$

So, $a + b = -7$

7. Official Ans. by NTA (3.00)

Sol. Let a be the first term and d be the common difference of the given A.P. Where d > 0

$$\overline{X} = a + \frac{0 + d + 2d + ... + 10d}{11}$$

$= a + 5d$

$\Rightarrow$ varience $= \frac{\Sigma(\overline{X} - x_i)^2}{11}$

$\Rightarrow 90 \times 11 = (25d^2 + 16d^2 + 9d^2 + 4d^2) \times 2$

$\Rightarrow d = \pm 3 \Rightarrow d = 3$

8. Official Ans. by NTA (4)

Sol. $\because \sigma^2 \le \frac{1}{4}(M - m)^2$

Where M and m are upper and lower bounds of values of any random variable.

$\therefore\ \sigma^2 < \frac{1}{4}(10 - 0)^2$

$\Rightarrow 0 < \sigma < 5$

$\therefore\ \sigma \ne 6.$

9. Official Ans. by NTA (3)

Sol. Variance $= \frac{\Sigma(x_i - p)^2}{n} - \left(\frac{\Sigma(x_i - p)}{n}\right)^2$

$= \frac{9}{10} - \left(\frac{3}{10}\right)^2 = \frac{81}{100}$

S.D. $= \frac{9}{10}$

10. Official Ans. by NTA (1)

Sol. $\overline{x} = 10$

$\Rightarrow \overline{x} = \frac{63 + a + b}{8} = 10 \Rightarrow a + b = 17$ ...(1)

Since, variance is independent of origin.

So, we subtract 10 from each observation.

So, $\sigma^2 = 13.5 = \frac{79 + (a-10)^2 + (b-10)^2}{8} - (10-10)^2$

$\Rightarrow a^2 + b^2 - 20(a + b) = -171$

$\Rightarrow a^2 + b^2 = 169$ ...(2)

From (i) & (ii) ; a = 12 & b = 5

11. Official Ans. by NTA (4)

Sol. $\because$ Variance is independent of shifting of origin

$\Rightarrow x_i$:	15	25	35	or	–10	0	10
f_i :	2	x	2		2	x	2

$\Rightarrow$ Variance $(\sigma^2) = \frac{\Sigma x_i^2 f_i}{\Sigma f_i} - (\overline{x})^2$

$\Rightarrow 50 = \frac{200 + 0 + 200}{x + 4} - 0 \quad \{\overline{x} = 0\}$

$\Rightarrow 200 + 50x = 200 + 200$

$\Rightarrow x = 4$

12. Official Ans. by NTA (1)

Sol. $\overline{x} = \frac{2 + 4 + 10 + 12 + 14 + x + y}{7} = 8$

$x + y = 14$
.....(i)

$(\sigma)^2 = \frac{\sum(x_i)^2}{n} - \left(\frac{\sum x_i}{n}\right)^2$

$16 = \frac{4 + 16 + 100 + 144 + 196 + x^2 + y^2}{7} - 8^2$

$16 + 64 = \frac{460 + x^2 + y^2}{7}$

$560 = 460 + x^2 + y^2$

$x^2 + y^2 = 100$ ....(ii)

Clearly by (i) and (ii), $|x - y| = 2$

Ans. 1

13. Official Ans. by NTA (2)

Sol. Mean = 5

$\frac{3 + 5 + 7 + a + b}{5} = 5$

$a + b = 10$ ....(i)

S.d. $= 2 \Rightarrow \sqrt{\frac{\sum_{i=1}^{5}(x_i - \overline{x})^2}{5}} = 2$

$(3–5)^2 + (5–5)^2 + (7–5)^2 + (a–5)^2 + (b–5)^2 = 20$

$\Rightarrow 4 + 0 + 4 + (a - 5)^2 + (b - 5)^2 = 20$

$a^2 + b^2 - 10(a + b) + 50 = 12$

$(a + b)^2 - 2ab - 100 + 50 = 12$

$ab = 19$ ....(ii)

Equation is $x^2 - 10x + 19 = 0$

14. Official Ans. by NTA (2)

Sol. $$S.D = \sqrt{\frac{\sum_{i=1}^{n}(x_i-a)}{n} - \left(\frac{\sum_{i=1}^{n}(x_i-a)}{n}\right)^2}$$

$$= \sqrt{\frac{na}{n} - \left(\frac{n}{n}\right)^2}$$

$\{$Given $\sum_{i=1}^{n}(x_i-a) = n \quad \sum_{i=1}^{n}(x_i-a)^2 = na\}$

$= \sqrt{a-1}$

15. Official Ans. by NTA (6.00)

Sol.

x	0	2	4	8		2^n
f	${}^nC_0$	${}^nC_1$	${}^nC_2$	${}^nC_3$		${}^nC_n$

$$\text{Mean} = \frac{\sum x_i f_i}{\sum f_i} = \frac{\sum_{r=1}^{n} 2^r\ {}^nC_r}{\sum_{r=0}^{n} {}^nC_r}$$

$$\text{Mean} = \frac{(1+2)^n - {}^nC_0}{2^n} = \frac{728}{2^n}$$

$$\Rightarrow \frac{3^n-1}{2^n} = \frac{728}{2^n}$$

$\Rightarrow 3^n = 729 \Rightarrow n = 6$

MATHEMATICAL REASONING

1. NTA Ans. (2)

Sol. Contrapositive of $p \to q$ is $\sim q \to \sim p$

$(A \subseteq B) \wedge (B \subseteq D) \longrightarrow (A \subseteq C)$

Contrapositive is

$\sim(A \subseteq C) \longrightarrow \sim(A \subseteq B) \vee \sim(B \subseteq D)$

$A \not\subseteq C \to (A \not\subseteq B) \vee (B \not\subseteq D)$

2. NTA Ans. (3)

Sol. $(p \to q) \wedge (q \to \sim p)$

$\equiv (\sim p \vee q) \wedge (\sim q \vee \sim p)$

$\equiv \sim p \vee (q \wedge \sim q)$

$\equiv \sim p \vee C \equiv \sim p$

3. NTA Ans. (1)

Sol. $\sim(p \vee \sim q) \to p \vee q$

$(\sim p \wedge q) \to p \vee q$

$\sim\{(\sim p \wedge q) \wedge (\sim p \wedge \sim q)\}$

$\sim(\sim p \wedge f)$

(1) Option

4. NTA Ans. (4)

Sol. (1) $P \wedge (P \vee Q) \equiv P$

(2) $P \vee (P \wedge Q) \equiv P$

(3) $Q \to (P \wedge (P \to Q))$

$\equiv Q \to (P \wedge (\sim P \vee Q)) \equiv Q \to (P \wedge Q)$

$\equiv (\sim Q) \vee (P \wedge Q) \equiv (P \vee (\sim Q))$

(4) $(P \wedge (P \to Q)) \to Q$

$\equiv (P \wedge (\sim P \vee Q)) \to Q \equiv (P \wedge Q) \to Q$

$\equiv ((\sim P) \vee (\sim Q)) \vee Q \equiv (\sim P) \vee t \equiv t$

5. NTA Ans. (2)

Sol. $p \to (p \wedge \sim q)$ is F $\Rightarrow$ p is T & $p \wedge \sim q$ is F $\Rightarrow$ q is T

$\therefore$ p is T, q is T

6. NTA Ans. (2)

Sol. $p = \sqrt{5}$ is an integer.

q : 5 is irrational

$\sim(p \vee q) \equiv \sim p \wedge \sim q$

$= \sqrt{5}$ is not an integer and 5 is not irrational.

7. Official Ans. by NTA (3)

Sol. Let p denotes statement

p : I reach the station in time.

q : I will catch the train.

Contrapositive of $p \to q$

is $\sim q \to \sim p$

$\sim q \to \sim p$: I will not catch the train, then I do not reach the station in time.

8. Official Ans. by NTA (1)

Sol. Option (1) is

$\sim p\wedge(p\vee q) \to q$

$\equiv (\sim p\wedge p)\vee(\sim p\wedge q) \to q$

$\equiv C\vee(\sim p\wedge q) \to q$

$\equiv (\sim p\wedge q)\to q$

$\equiv \sim(\sim p\wedge q)\vee q$

$\equiv (p\vee \sim q)\vee q$

$\equiv (p\vee q)\vee(\sim q\vee q)$

$\equiv (p\vee q) \vee t$

so $\sim p\wedge(p\vee q) \to q$ is a tautology

9. Official Ans. by NTA (1)

Sol. $p \to \sim(p\wedge \sim q)$

$= \sim p\vee \sim(p\wedge \sim q)$

$= \sim p \vee \sim p \vee q$

$= \sim(p\wedge q)\vee q$

$= \sim p\vee q$

10. Official Ans. by NTA (3)

Sol. $(p \wedge q) \to (\sim q \vee r)$ = false

when $(p \wedge q) = T$

and $(\sim q \vee r) = F$

So $(p \wedge q) = T$ is possible when $p = q$ = true

$\therefore \sim q$ = False (q = true)

So $(\sim q \vee r)$ = False is possible if r is false

$\therefore p = T, q = T, r = F$

11. Official Ans. by NTA (3)

Sol. Let TV(r) denotes truth value of a statement r.

Now, if TV(p) = TV(q) = T

$\Rightarrow TV(S_1) = F$

Also, if TV(p) = T & TV(q) = F

$\Rightarrow TV(S_2) = T$

12. Official Ans. by NTA (3)

Sol. p = function is differantiable at a

q = function is continuous at a

contrapositive of statement $p \to q$ is

$\sim q \to \sim p$

13. Official Ans. by NTA (3)

Sol. $p \leftrightarrow q \equiv (p\to q)\wedge(q\to p)$

$x \leftrightarrow \sim y \equiv (x\to\sim y)\wedge(\sim y\to x)$

$\because (p\to q \equiv \sim p\vee q)$

$x \leftrightarrow \sim y \equiv (\sim x\vee \sim y)\wedge(y\vee x)$

$\sim(x\leftrightarrow \sim y) \equiv (x\wedge y)\vee(\sim x\wedge \sim y)$

14. Official Ans. by NTA (3)

Sol.

p	q	$q\to p$	$p\to(q\to p)$	$p\vee q$	$p\to p\vee q$	$(p\to(q\to p))\to(p\to(p\vee q))$
T	T	T	T	T	T	T
T	F	T	T	T	T	T
F	T	F	T	T	T	T
F	F	T	T	F	T	T

15. Official Ans. by NTA (3)

Sol. Negation of $\phi\vee(\sim p\wedge q)$

$p\vee(\sim p\wedge q) = (p\vee\sim p)\wedge(p\vee q)$

$=(T)\wedge(p\vee q)$

$=(p\vee q)$

now negation of $(p\vee q)$ is

$\sim(p\vee q) = \sim p\wedge\sim q$

16. Official Ans. by NTA (2)

Sol. Contrapositive of $(p \to q)$ is $\sim q \to \sim p$

For an integer n, if n is even then $(n^3 - 1)$ is odd