



Chapter Contents

03

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MATHEMATICS

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7. Let α and β be the roots of the equation $5x^2 + 6x - 2 = 0$. If $S_n = \alpha^n + \beta^n$, $n = 1, 2, 3, \dots$, then :
- (1) $5S_6 + 6S_5 = 2S_4$
 (2) $5S_6 + 6S_5 + 2S_4 = 0$
 (3) $6S_6 + 5S_5 + 2S_4 = 0$
 (4) $6S_6 + 5S_5 = 2S_4$
8. Let $f(x)$ be a quadratic polynomial such that $f(-1) + f(2) = 0$. If one of the roots of $f(x) = 0$ is 3, then its other root lies in :
- (1) $(-3, -1)$ (2) $(1, 3)$
 (3) $(-1, 0)$ (4) $(0, 1)$
9. If α and β are the roots of the equation $x^2 + px + 2 = 0$ and $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ are the roots of the equation $2x^2 + 2qx + 1 = 0$, then $\left(\alpha - \frac{1}{\alpha}\right)\left(\beta - \frac{1}{\beta}\right)\left(\alpha + \frac{1}{\beta}\right)\left(\beta + \frac{1}{\alpha}\right)$ is equal to :
- (1) $\frac{9}{4}(9 + p^2)$ (2) $\frac{9}{4}(9 - q^2)$
 (3) $\frac{9}{4}(9 - p^2)$ (4) $\frac{9}{4}(9 + q^2)$
10. The set of all real values of λ for which the quadratic equations, $(\lambda^2 + 1)x^2 - 4\lambda x + 2 = 0$ always have exactly one root in the interval $(0, 1)$ is :
- (1) $(-3, -1)$ (2) $(1, 3]$
 (3) $(0, 2)$ (4) $(2, 4]$
11. Let α and β be the roots of $x^2 - 3x + p = 0$ and γ and δ be the roots of $x^2 - 6x + q = 0$. If $\alpha, \beta, \gamma, \delta$ form a geometric progression. Then ratio $(2q + p) : (2q - p)$ is :
- (1) $3 : 1$ (2) $33 : 31$
 (3) $9 : 7$ (4) $5 : 3$
12. Let $\lambda \neq 0$ be in \mathbb{R} . If α and β are the roots of the equation, $x^2 - x + 2\lambda = 0$ and α and γ are the roots of the equation, $3x^2 - 10x + 27\lambda = 0$, then $\frac{\beta\gamma}{\lambda}$ is equal to :
- (1) 36 (2) 27
 (3) 9 (4) 18
13. The product of the roots of the equation $9x^2 - 18|x| + 5 = 0$, is
- (1) $\frac{25}{9}$ (2) $\frac{25}{81}$
 (3) $\frac{5}{27}$ (4) $\frac{5}{9}$
14. If α and β are the roots of the equation, $7x^2 - 3x - 2 = 0$, then the value of $\frac{\alpha}{1 - \alpha^2} + \frac{\beta}{1 - \beta^2}$ is equal to:
- (1) $\frac{27}{16}$ (2) $\frac{1}{24}$
 (3) $\frac{27}{32}$ (4) $\frac{3}{8}$
15. If α and β be two roots of the equation $x^2 - 64x + 256 = 0$. Then the value of $\left(\frac{\alpha^3}{\beta^5}\right)^{\frac{1}{8}} + \left(\frac{\beta^3}{\alpha^5}\right)^{\frac{1}{8}}$ is
- (1) 1 (2) 3
 (3) 4 (4) 2
16. If α and β are the roots of the equation $2x(2x + 1) = 1$, then β is equal to :
- (1) $2\alpha^2$ (2) $2\alpha(\alpha + 1)$
 (3) $-2\alpha(\alpha + 1)$ (4) $2\alpha(\alpha - 1)$

SEQUENCE & PROGRESSION

1. If the sum of the first 40 terms of the series, $3 + 4 + 8 + 9 + 13 + 14 + 18 + 19 + \dots$ is $(102)m$, then m is equal to :
- (1) 20 (2) 5
 (3) 10 (4) 25

2. Let a_1, a_2, a_3, \dots be a G.P. such that $a_1 < 0$, $a_1 + a_2 = 4$ and $a_3 + a_4 = 16$. If $\sum_{i=1}^9 a_i = 4\lambda$, then λ is equal to :
- (1) -171 (2) 171
 (3) $\frac{511}{3}$ (4) -513
3. Five numbers are in A.P., whose sum is 25 and product is 2520. If one of these five numbers is $-\frac{1}{2}$, then the greatest number amongst them is :
- (1) $\frac{21}{2}$ (2) 27
 (3) 16 (4) 7
4. The greatest positive integer k , for which $49^k + 1$ is a factor of the sum $49^{125} + 49^{124} + \dots + 49^2 + 49 + 1$, is :
- (1) 32 (2) 60
 (3) 63 (4) 65
5. If the 10th term of an A.P. is $\frac{1}{20}$ and its 20th term is $\frac{1}{10}$, then the sum of its first 200 terms is
- (1) $50\frac{1}{4}$ (2) $100\frac{1}{2}$
 (3) 50 (4) 100
6. The sum, $\sum_{n=1}^7 \frac{n(n+1)(2n+1)}{4}$ is equal to _____.
7. The sum $\sum_{k=1}^{20} (1+2+3+\dots+k)$ is
8. If $x = \sum_{n=0}^{\infty} (-1)^n \tan^{2n} \theta$ and $y = \sum_{n=0}^{\infty} \cos^{2n} \theta$, for $0 < \theta < \frac{\pi}{4}$, then :
- (1) $y(1+x) = 1$ (2) $x(1+y) = 1$
 (3) $y(1-x) = 1$ (4) $x(1-y) = 1$

9. Let a_n be the n^{th} term of a G.P. of positive terms. If $\sum_{n=1}^{100} a_{2n+1} = 200$ and $\sum_{n=1}^{100} a_{2n} = 100$, then $\sum_{n=1}^{200} a_n$ is equal to :
- (1) 225 (2) 175
 (3) 300 (4) 150
10. The number of terms common to the two A.P.'s 3, 7, 11,, 407 and 2, 9, 16,, 709 is _____.
11. The product $2^{\frac{1}{4}} \cdot 4^{\frac{1}{16}} \cdot 8^{\frac{1}{48}} \cdot 16^{\frac{1}{128}} \cdot \dots$ to ∞ is equal to :
- (1) $2^{\frac{1}{2}}$ (2) $2^{\frac{1}{4}}$
 (3) 2 (4) 1
12. If $|x| < 1$, $|y| < 1$ and $x \neq y$, then the sum to infinity of the following series $(x+y) + (x^2+xy+y^2) + (x^3+x^2y+xy^2+y^3)+\dots$
- (1) $\frac{x+y-xy}{(1-x)(1-y)}$ (2) $\frac{x+y-xy}{(1+x)(1+y)}$
 (3) $\frac{x+y+xy}{(1+x)(1+y)}$ (4) $\frac{x+y+xy}{(1-x)(1-y)}$
13. The sum of the first three terms of a G.P. is S and their product is 27. Then all such S lie in:
- (1) $[-3, \infty)$ (2) $(-\infty, 9]$
 (3) $(-\infty, -9] \cup [3, \infty)$ (4) $(-\infty, -3] \cup [9, \infty)$
14. If the sum of first 11 terms of an A.P., a_1, a_2, a_3, \dots is 0 ($a_1 \neq 0$), then the sum of the A.P., $a_1, a_3, a_5, \dots, a_{23}$ is ka_1 , where k is equal to :
- (1) $\frac{121}{10}$ (2) $-\frac{72}{5}$
 (3) $\frac{72}{5}$ (4) $-\frac{121}{10}$
15. Let S be the sum of the first 9 terms of the series: $\{x + ka\} + \{x^2 + (k+2)a\} + \{x^3 + (k+4)a\} + \{x^4 + (k+6)a\} + \dots$ where $a \neq 0$ and $x \neq 1$. If $S = \frac{x^{10} - x + 45a(x-1)}{x-1}$, then k is equal to :
- (1) -5 (2) 1
 (3) -3 (4) 3

16. If the first term of an A.P. is 3 and the sum of its first 25 terms is equal to the sum of its next 15 terms, then the common difference of this A.P. is :
- (1) $\frac{1}{4}$ (2) $\frac{1}{5}$
 (3) $\frac{1}{7}$ (4) $\frac{1}{6}$
17. If the sum of the series $20 + 19\frac{3}{5} + 19\frac{1}{5} + 18\frac{4}{5} + \dots$ upto n^{th} term is 488 and the n^{th} term is negative, then :
- (1) n^{th} term is $-4\frac{2}{5}$ (2) $n = 41$
 (3) n^{th} term is -4 (4) $n = 60$
18. If m arithmetic means (A.Ms) and three geometric means (G.Ms) are inserted between 3 and 243 such that 4th A.M. is equal to 2nd G.M., then m is equal to _____.
19. If $1 + (1 - 2^2 \cdot 1) + (1 - 4^2 \cdot 3) + (1 - 6^2 \cdot 5) + \dots + (1 - 20^2 \cdot 19) = \alpha - 220\beta$, then an ordered pair (α, β) is equal to:
- (1) (10, 97) (2) (11, 103)
 (3) (10, 103) (4) (11, 97)
20. Let a_1, a_2, \dots, a_n be a given A.P. whose common difference is an integer and $S_n = a_1 + a_2 + \dots + a_n$. If $a_1 = 1$, $a_n = 300$ and $15 \leq n \leq 50$, then the ordered pair (S_{n-4}, a_{n-4}) is equal to :
- (1) (2480, 249) (2) (2490, 249)
 (3) (2490, 248) (4) (2480, 248)
21. The minimum value of $2^{\sin x} + 2^{\cos x}$ is :-
- (1) $2^{1-\frac{1}{\sqrt{2}}}$ (2) $2^{-1+\sqrt{2}}$
 (3) $2^{1-\sqrt{2}}$ (4) $2^{-1+\frac{1}{\sqrt{2}}}$
22. If $3^{2 \sin 2\alpha - 1}$, 14 and $3^{4 - 2 \sin 2\alpha}$ are the first three terms of an A.P. for some α , then the sixth term of this A.P. is :
- (1) 66 (2) 65
 (3) 81 (4) 78
23. If $2^{10} + 2^9 \cdot 3^1 + 2^8 \cdot 3^2 + \dots + 2 \cdot 3^9 + 3^{10} = S - 2^{11}$, then S is equal to :
- (1) $\frac{3^{11}}{2} + 2^{10}$ (2) $3^{11} - 2^{12}$
 (3) 3^{11} (4) $2 \cdot 3^{11}$
24. If the sum of the first 20 terms of the series $\log_{(7^{1/2})} x + \log_{(7^{1/3})} x + \log_{(7^{1/4})} x + \dots$ is 460, then x is equal to:
- (1) $7^{46/21}$ (2) $7^{1/2}$
 (3) e^2 (4) 7^2
25. If the sum of the second, third and fourth terms of a positive term G.P. is 3 and the sum of its sixth, seventh and eighth terms is 243, then the sum of the first 50 terms of this G.P. is :
- (1) $\frac{2}{13}(3^{50} - 1)$ (2) $\frac{1}{26}(3^{50} - 1)$
 (3) $\frac{1}{13}(3^{50} - 1)$ (4) $\frac{1}{26}(3^{49} - 1)$
26. If $f(x+y) = f(x)f(y)$ and $\sum_{x=1}^{\infty} f(x) = 2, x, y \in \mathbb{N}$, where \mathbb{N} is the set of all natural numbers, then the value of $\frac{f(4)}{f(2)}$ is
- (1) $\frac{1}{9}$ (2) $\frac{4}{9}$
 (3) $\frac{1}{3}$ (4) $\frac{2}{3}$
27. Let a, b, c, d and p be any non zero distinct real numbers such that $(a^2 + b^2 + c^2)p^2 - 2(ab + bc + cd)p + (b^2 + c^2 + d^2) = 0$. Then :
- (1) a, c, p are in G.P.
 (2) a, c, p are in A.P.
 (3) a, b, c, d are in G.P.
 (4) a, b, c, d are in A.P.

28. The common difference of the A.P. b_1, b_2, \dots, b_m is 2 more than the common difference of A.P. a_1, a_2, \dots, a_n . If $a_{40} = -159$, $a_{100} = -399$ and $b_{100} = a_{70}$, then b_1 is equal to :
- (1) -127 (2) -81
(3) 81 (4) 127

TRIGONOMETRIC EQUATION

1. The number of distinct solutions of the equation $\log_{\frac{1}{2}} |\sin x| = 2 - \log_{\frac{1}{2}} |\cos x|$ in the interval $[0, 2\pi]$, is _____.
2. If the equation $\cos^4\theta + \sin^4\theta + \lambda = 0$ has real solutions for θ , then λ lies in the interval :
- (1) $\left[-\frac{3}{2}, -\frac{5}{4}\right]$ (2) $\left[-\frac{1}{2}, -\frac{1}{4}\right]$
(3) $\left[-\frac{5}{4}, -1\right]$ (4) $\left[-1, -\frac{1}{2}\right]$
3. Let $a, b, c \in \mathbb{R}$ be such that $a^2 + b^2 + c^2 = 1$. If $a \cos \theta = b \cos \left(\theta + \frac{2\pi}{3}\right) = c \cos \left(\theta + \frac{4\pi}{3}\right)$, where $\theta = \frac{\pi}{9}$, then the angle between the vectors $a\hat{i} + b\hat{j} + c\hat{k}$ and $b\hat{i} + c\hat{j} + a\hat{k}$ is :
- (1) $\frac{\pi}{2}$ (2) 0
(3) $\frac{\pi}{9}$ (4) $\frac{2\pi}{3}$

SOLUTION OF TRIANGLE

1. If a ΔABC has vertices $A(-1, 7)$, $B(-7, 1)$ and $C(5, -5)$, then its orthocentre has coordinates:
- (1) (3, -3) (2) $\left(-\frac{3}{5}, \frac{3}{5}\right)$
(3) (-3, 3) (4) $\left(\frac{3}{5}, -\frac{3}{5}\right)$

HEIGHT & DISTANCE

1. Two vertical poles $AB = 15$ m and $CD = 10$ m are standing apart on a horizontal ground with points A and C on the ground. If P is the point of intersection of BC and AD, then the height of P (in m) above the line AC is :
- (1) $\frac{20}{3}$ (2) 5
(3) $\frac{10}{3}$ (4) 6
2. The angle of elevation of a cloud C from a point P, 200 m above a still lake is 30° . If the angle of depression of the image of C in the lake from the point P is 60° , then PC (in m) is equal to :
- (1) 400 (2) $400\sqrt{3}$
(3) 100 (4) $200\sqrt{3}$
3. The angle of elevation of the top of a hill from a point on the horizontal plane passing through the foot of the hill is found to be 45° . After walking a distance of 80 meters towards the top, up a slope inclined at an angle of 30° to the horizontal plane, the angle of elevation of the top of the hill becomes 75° . Then the height of the hill (in meters) is_.
4. The angle of elevation of the summit of a mountain from a point on the ground is 45° . After climbing up one km towards the summit at an inclination of 30° from the ground, the angle of elevation of the summit is found to be 60° . Then the height (in km) of the summit from the ground is :
- (1) $\frac{1}{\sqrt{3}-1}$ (2) $\frac{1}{\sqrt{3}+1}$
(3) $\frac{\sqrt{3}-1}{\sqrt{3}+1}$ (4) $\frac{\sqrt{3}+1}{\sqrt{3}-1}$

DETERMINANT

1. If the system of linear equations,
- $$\begin{aligned}x + y + z &= 6 \\x + 2y + 3z &= 10 \\3x + 2y + \lambda z &= \mu\end{aligned}$$
- has more two solutions, then $\mu - \lambda^2$ is equal to _____
2. If the system of linear equations
- $$\begin{aligned}2x + 2ay + az &= 0 \\2x + 3by + bz &= 0 \\2x + 4cy + cz &= 0,\end{aligned}$$
- where $a, b, c \in \mathbb{R}$ are non-zero and distinct; has a non-zero solution, then :
- (1) a, b, c are in A.P.
 - (2) $a + b + c = 0$
 - (3) a, b, c are in G.P.
 - (4) $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P.
3. The system of linear equations
- $$\begin{aligned}\lambda x + 2y + 2z &= 5 \\2\lambda x + 3y + 5z &= 8 \\4x + \lambda y + 6z &= 10\end{aligned}$$
- has
- (1) infinitely many solutions when $\lambda = 2$
 - (2) a unique solution when $\lambda = -8$
 - (3) no solution when $\lambda = 8$
 - (4) no solution when $\lambda = 2$
4. For which of the following ordered pairs (μ, δ) , the system of linear equations
- $$\begin{aligned}x + 2y + 3z &= 1 \\3x + 4y + 5z &= \mu \\4x + 4y + 4z &= \delta\end{aligned}$$
- is inconsistent ?
- (1) (1,0)
 - (2) (4,6)
 - (3) (3,4)
 - (4) (4,3)
5. Let $a - 2b + c = 1$. If
- $$f(x) = \begin{vmatrix} x+a & x+2 & x+1 \\ x+b & x+3 & x+2 \\ x+c & x+4 & x+3 \end{vmatrix}, \text{ then :}$$
- (1) $f(-50) = 501$
 - (2) $f(-50) = -1$
 - (3) $f(50) = 1$
 - (4) $f(50) = -501$
6. The following system of linear equations
- $$\begin{aligned}7x + 6y - 2z &= 0 \\3x + 4y + 2z &= 0 \\x - 2y - 6z &= 0,\end{aligned}$$
- has
- (1) infinitely many solutions, (x, y, z) satisfying $x = 2z$
 - (2) no solution
 - (3) only the trivial solution
 - (4) infinitely many solutions, (x, y, z) satisfying $y = 2z$
7. Let S be the set of all $\lambda \in \mathbb{R}$ for which the system of linear equations
- $$\begin{aligned}2x - y + 2z &= 2 \\x - 2y + \lambda z &= -4 \\x + \lambda y + z &= 4\end{aligned}$$
- has no solution. Then the set S
- (1) contains more than two elements.
 - (2) is a singleton.
 - (3) contains exactly two elements.
 - (4) is an empty set.
8. Let S be the set of all integer solutions, (x, y, z) , of the system of equations
- $$\begin{aligned}x - 2y + 5z &= 0 \\-2x + 4y + z &= 0 \\-7x + 14y + 9z &= 0\end{aligned}$$
- such that $15 \leq x^2 + y^2 + z^2 \leq 150$. Then, the number of elements in the set S is equal to _____.
9. If $\Delta = \begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ 2x-3 & 3x-4 & 4x-5 \\ 3x-5 & 5x-8 & 10x-17 \end{vmatrix} = Ax^3 + Bx^2 + Cx + D$, then $B + C$ is equal to :
- (1) -1
 - (2) 1
 - (3) -3
 - (4) 9
10. If the system of equations
- $$\begin{aligned}x - 2y + 3z &= 9 \\2x + y + z &= b \\x - 7y + az &= 24,\end{aligned}$$
- has infinitely many solutions, then $a - b$ is equal to _____.

11. If the system of equations
 $x + y + z = 2$
 $2x + 4y - z = 6$
 $3x + 2y + \lambda z = \mu$
 has infinitely many solutions, then :
 (1) $\lambda - 2\mu = -5$ (2) $2\lambda - \mu = 5$
 (3) $2\lambda + \mu = 14$ (4) $\lambda + 2\mu = 14$
12. If the minimum and the maximum values of the function $f : \left[\frac{\pi}{4}, \frac{\pi}{2}\right] \rightarrow \mathbb{R}$, defined by :
- $$f(\theta) = \begin{vmatrix} -\sin^2 \theta & -1 - \sin^2 \theta & 1 \\ -\cos^2 \theta & -1 - \cos^2 \theta & 1 \\ 12 & 10 & -2 \end{vmatrix}$$
- are m and M respectively, then the ordered pair (m, M) is equal to:
 (1) $(0, 4)$ (2) $(-4, 4)$
 (3) $(0, 2\sqrt{2})$ (4) $(-4, 0)$
13. Let $\lambda \in \mathbb{R}$. The system of linear equations
 $2x_1 - 4x_2 + \lambda x_3 = 1$
 $x_1 - 6x_2 + x_3 = 2$
 $\lambda x_1 - 10x_2 + 4x_3 = 3$
 is inconsistent for :
 (1) exactly one negative value of λ .
 (2) exactly one positive value of λ .
 (3) every value of λ .
 (4) exactly two values of λ .
14. If the system of linear equations
 $x + y + 3z = 0$
 $x + 3y + k^2z = 0$
 $3x + y + 3z = 0$
 has a non-zero solution (x, y, z) for some $k \in \mathbb{R}$, then $x + \left(\frac{y}{z}\right)$ is equal to :
 (1) 9 (2) -3
 (3) -9 (4) 3

15. If $a + x = b + y = c + z + 1$, where a, b, c, x, y, z are non-zero distinct real numbers, then
 $\begin{vmatrix} x & a+y & x+a \\ y & b+y & y+b \\ z & c+y & z+c \end{vmatrix}$ is equal to :
 (1) 0 (2) $y(a - b)$
 (3) $y(b - a)$ (4) $y(a - c)$
16. The values of λ and μ for which the system of linear equations
 $x + y + z = 2$
 $x + 2y + 3z = 5$
 $x + 3y + \lambda z = \mu$
 has infinitely many solutions are, respectively
 (1) 5 and 7 (2) 6 and 8
 (3) 4 and 9 (4) 5 and 8
17. Let m and M be respectively the minimum and maximum values of
 $\begin{vmatrix} \cos^2 x & 1 + \sin^2 x & \sin 2x \\ 1 + \cos^2 x & \sin^2 x & \sin 2x \\ \cos^2 x & \sin^2 x & 1 + \sin 2x \end{vmatrix}$. Then the ordered pair (m, M) is equal to
 (1) $(-3, -1)$ (2) $(-4, -1)$
 (3) $(1, 3)$ (4) $(-3, 3)$
18. The sum of distinct values of λ for which the system of equations
 $(\lambda - 1)x + (3\lambda + 1)y + 2\lambda z = 0$
 $(\lambda - 1)x + (4\lambda - 2)y + (\lambda + 3)z = 0$
 $2x + (3\lambda + 1)y + 3(\lambda - 1)z = 0$,
 has non-zero solutions, is _____.

STRAIGHT LINE

1. The locus of the mid-points of the perpendiculars drawn from points on the line, $x = 2y$ to the line $x = y$ is :
 (1) $2x - 3y = 0$ (2) $7x - 5y = 0$
 (3) $5x - 7y = 0$ (4) $3x - 2y = 0$
2. Let $A(1, 0)$, $B(6, 2)$ and $C\left(\frac{3}{2}, 6\right)$ be the vertices of a triangle ABC . If P is a point inside the triangle ABC such that the triangles APC , APB and BPC have equal areas, then the length of the line segment PQ , where Q is the point $\left(-\frac{7}{6}, -\frac{1}{3}\right)$, is _____.

3. Let two points be A(1,-1) and B(0,2). If a point P(x',y') be such that the area of $\Delta PAB = 5$ sq. units and it lies on the line, $3x + y - 4\lambda = 0$, then a value of λ is
 (1) 1 (2) 4
 (3) 3 (4) -3
4. Let C be the centroid of the triangle with vertices (3, -1), (1, 3) and (2, 4). Let P be the point of intersection of the lines $x + 3y - 1 = 0$ and $3x - y + 1 = 0$. Then the line passing through the points C and P also passes through the point :
 (1) (7, 6) (2) (-9, -6)
 (3) (-9, -7) (4) (9, 7)
5. The set of all possible values of θ in the interval $(0, \pi)$ for which the points (1, 2) and $(\sin \theta, \cos \theta)$ lie on the same side of the line $x + y = 1$ is :
 (1) $\left(0, \frac{\pi}{4}\right)$ (2) $\left(0, \frac{3\pi}{4}\right)$
 (3) $\left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$ (4) $\left(0, \frac{\pi}{2}\right)$
6. A triangle ABC lying in the first quadrant has two vertices as A(1, 2) and B(3, 1). If $\angle BAC = 90^\circ$, and $\text{ar}(\Delta ABC) = 5\sqrt{5}$ sq. units, then the abscissa of the vertex C is :
 (1) $2 + \sqrt{5}$ (2) $1 + \sqrt{5}$
 (3) $1 + 2\sqrt{5}$ (4) $2\sqrt{5} - 1$
7. If the perpendicular bisector of the line segment joining the points P (1, 4) and Q (k, 3) has y-intercept equal to -4, then a value of k is :-
 (1) $\sqrt{15}$ (2) -2
 (3) $\sqrt{14}$ (4) -4
8. If the line, $2x - y + 3 = 0$ is at a distance $\frac{1}{\sqrt{5}}$ and $\frac{2}{\sqrt{5}}$ from the lines $4x - 2y + \alpha = 0$ and $6x - 3y + \beta = 0$, respectively, then the sum of all possible values of α and β is _____

9. A ray of light coming from the point $(2, 2\sqrt{3})$ is incident at an angle 30° on the line $x=1$ at the point A. The ray gets reflected on the line $x=1$ and meets x-axis at the point B. Then, the line AB passes through the point:
 (1) $\left(3, -\frac{1}{\sqrt{3}}\right)$ (2) $(3, -\sqrt{3})$
 (3) $\left(4, -\frac{\sqrt{3}}{2}\right)$ (4) $(4, -\sqrt{3})$
10. Let L denote the line in the xy-plane with x and y intercepts as 3 and 1 respectively. Then the image of the point $(-1, -4)$ in this line is :
 (1) $\left(\frac{8}{5}, \frac{29}{5}\right)$ (2) $\left(\frac{29}{5}, \frac{11}{5}\right)$
 (3) $\left(\frac{11}{5}, \frac{28}{5}\right)$ (4) $\left(\frac{29}{5}, \frac{8}{5}\right)$

CIRCLE

1. Let the tangents drawn from the origin to the circle, $x^2 + y^2 - 8x - 4y + 16 = 0$ touch it at the points A and B. The $(AB)^2$ is equal to :
 (1) $\frac{52}{5}$ (2) $\frac{32}{5}$
 (3) $\frac{56}{5}$ (4) $\frac{64}{5}$
2. If a line, $y = mx + c$ is a tangent to the circle, $(x - 3)^2 + y^2 = 1$ and it is perpendicular to a line L_1 , where L_1 is the tangent to the circle, $x^2 + y^2 = 1$ at the point $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$, then
 (1) $c^2 - 6c + 7 = 0$ (2) $c^2 + 6c + 7 = 0$
 (3) $c^2 + 7c + 6 = 0$ (4) $c^2 - 7c + 6 = 0$
3. If the curves, $x^2 - 6x + y^2 + 8 = 0$ and $x^2 - 8y + y^2 + 16 - k = 0$, ($k > 0$) touch each other at a point, then the largest value of k is _____.
4. The number of integral values of k for which the line, $3x + 4y = k$ intersects the circle, $x^2 + y^2 - 2x - 4y + 4 = 0$ at two distinct points is _____.

5. The diameter of the circle, whose centre lies on the line $x + y = 2$ in the first quadrant and which touches both the lines $x = 3$ and $y = 2$, is _____ .
6. The circle passing through the intersection of the circles, $x^2 + y^2 - 6x = 0$ and $x^2 + y^2 - 4y = 0$, having its centre on the line, $2x - 3y + 12 = 0$, also passes through the point :
- (1) $(1, -3)$ (2) $(-1, 3)$
 (3) $(-3, 1)$ (4) $(-3, 6)$
7. Let PQ be a diameter of the circle $x^2 + y^2 = 9$. If α and β are the lengths of the perpendiculars from P and Q on the straight line, $x + y = 2$ respectively, then the maximum value of $\alpha\beta$ is _____
8. If the length of the chord of the circle, $x^2 + y^2 = r^2$ ($r > 0$) along the line, $y - 2x = 3$ is r , then r^2 is equal to:
- (1) $\frac{9}{5}$ (2) $\frac{12}{5}$
 (3) 12 (4) $\frac{24}{5}$

PERMUTATION & COMBINATION

1. Total number of 6-digit numbers in which only and all the five digits 1, 3, 5, 7 and 9 appear, is:
- (1) $\frac{5}{2}(6!)$ (2) 5^6
 (3) $\frac{1}{2}(6!)$ (4) $6!$
2. The number of 4 letter words (with or without meaning) that can be formed from the eleven letters of the word 'EXAMINATION' is _____.
3. If a, b and c are the greatest value of ${}^{19}C_p$, ${}^{20}C_q$ and ${}^{21}C_r$ respectively, then
- (1) $\frac{a}{11} = \frac{b}{22} = \frac{c}{21}$ (2) $\frac{a}{10} = \frac{b}{11} = \frac{c}{21}$
 (3) $\frac{a}{10} = \frac{b}{11} = \frac{c}{42}$ (4) $\frac{a}{11} = \frac{b}{22} = \frac{c}{42}$
4. An urn contains 5 red marbles, 4 black marbles and 3 white marbles. Then the number of ways in which 4 marbles can be drawn so that at the most three of them are red is
5. If the number of five digit numbers with distinct digits and 2 at the 10th place is 336 k, then k is equal to :
- (1) 8 (2) 6
 (3) 4 (4) 7
6. If the letters of the word 'MOTHER' be permuted and all the words so formed (with or without meaning) be listed as in a dictionary, then the position of the word 'MOTHER' is _____.
7. Let $n > 2$ be an integer. Suppose that there are n Metro stations in a city located along a circular path. Each pair of stations is connected by a straight track only. Further, each pair of nearest stations is connected by blue line, whereas all remaining pairs of stations are connected by red line. If the number of red lines is 99 times the number of blue lines, then the value of n is :-
- (1) 199 (2) 101
 (3) 201 (4) 200
8. The value of $(2 \cdot {}^1P_0 - 3 \cdot {}^2P_1 + 4 \cdot {}^3P_2 - \dots$ up to 51th term) + $(1! - 2! + 3! - \dots$ up to 51th term) is equal to :
- (1) $1 + (51)!$ (2) $1 - 51(51)!$
 (3) $1 + (52)!$ (4) 1
9. The total number of 3-digit numbers, whose sum of digits is 10, is _____.
10. A test consists of 6 multiple choice questions, each having 4 alternative answers of which only one is correct. The number of ways, in which a candidate answers all six questions such that exactly four of the answers are correct, is _____
11. The number of words, with or without meaning, that can be formed by taking 4 letters at a time from the letters of the word 'SYLLABUS' such that two letters are distinct and two letters are alike, is _____.

12. There are 3 sections in a question paper and each section contains 5 questions. A candidate has to answer a total of 5 questions, choosing at least one question from each section. Then the number of ways, in which the candidate can choose the questions, is :
- (1) 1500 (2) 2255
(3) 3000 (4) 2250
13. The number of words (with or without meaning) that can be formed from all the letters of the word "LETTER" in which vowels never come together is _____.

BINOMIAL THEOREM

1. The number of ordered pairs (r, k) for which $6 \cdot {}^{35}C_r = (k^2 - 3) \cdot {}^{36}C_{r+1}$, where k is an integer, is:
- (1) 3 (2) 2
(3) 4 (4) 6
2. The coefficient of x^7 in the expression $(1+x)^{10} + x(1+x)^9 + x^2(1+x)^8 + \dots + x^{10}$ is :
- (1) 120 (2) 330
(3) 210 (4) 420
3. If the sum of the coefficients of all even powers of x in the product $(1+x+x^2+\dots+x^{2n})(1-x+x^2-x^3+\dots+x^{2n})$ is 61, then n is equal to _____.
4. If α and β be the coefficients of x^4 and x^2 respectively in the expansion of $(x+\sqrt{x^2-1})^6 + (x-\sqrt{x^2-1})^6$, then
- (1) $\alpha + \beta = 60$ (2) $\alpha + \beta = -30$
(3) $\alpha - \beta = -132$ (4) $\alpha - \beta = 60$
5. In the expansion of $\left(\frac{x}{\cos\theta} + \frac{1}{x\sin\theta}\right)^{16}$, if ℓ_1 is the least value of the term independent of x when $\frac{\pi}{8} \leq \theta \leq \frac{\pi}{4}$ and ℓ_2 is the least value of the term independent of x when $\frac{\pi}{16} \leq \theta \leq \frac{\pi}{8}$, then the ratio $\ell_2 : \ell_1$ is equal to :
- (1) 1 : 8 (2) 1 : 16
(3) 8 : 1 (4) 16 : 1

6. If $C_r \equiv {}^{25}C_r$ and $C_0 + 5.C_1 + 9.C_2 + \dots + (101).C_{25} = 2^{25}.k$, then k is equal to _____.
7. The coefficient of x^4 is the expansion of $(1+x+x^2)^{10}$ is _____.
8. Let $\alpha > 0, \beta > 0$ be such that $\alpha^3 + \beta^2 = 4$. If the maximum value of the term independent of x in the binomial expansion of $(\alpha x^{\frac{1}{3}} + \beta x^{-\frac{1}{6}})^{10}$ is $10k$, then k is equal to :
- (1) 176 (2) 336
(3) 352 (4) 84
9. For a positive integer n , $\left(1 + \frac{1}{x}\right)^n$ is expanded in increasing powers of x . If three consecutive coefficients in this expansion are in the ratio, $2 : 5 : 12$, then n is equal to _____.
10. If the number of integral terms in the expansion of $(3^{1/2} + 5^{1/8})^n$ is exactly 33, then the least value of n is :
- (1) 264 (2) 256
(3) 128 (4) 248
11. If the term independent of x in the expansion of $\left(\frac{3}{2}x^2 - \frac{1}{3x}\right)^9$ is k , then $18k$ is equal to :
- (1) 9 (2) 11
(3) 5 (4) 7
12. The value of $\sum_{r=0}^{20} {}^{50-r}C_6$ is equal to :
- (1) ${}^{51}C_7 + {}^{30}C_7$ (2) ${}^{51}C_7 - {}^{30}C_7$
(3) ${}^{50}C_7 - {}^{30}C_7$ (4) ${}^{50}C_6 - {}^{30}C_6$
13. Let $(2x^2 + 3x + 4)^{10} = \sum_{r=0}^{20} a_r x^r$. Then $\frac{a_7}{a_{13}}$ is equal to _____.
14. If for some positive integer n , the coefficients of three consecutive terms in the binomial expansion of $(1+x)^{n+5}$ are in the ratio $5 : 10 : 14$, then the largest coefficient in this expansion is :-
- (1) 792 (2) 252
(3) 462 (4) 330

RELATION

1. If $R = \{(x, y) : x, y \in \mathbb{Z}, x^2 + 3y^2 \leq 8\}$ is a relation on the set of integers \mathbb{Z} , then the domain of R^{-1} is:
- (1) $\{-2, -1, 1, 2\}$ (2) $\{-1, 0, 1\}$
 (3) $\{-2, -1, 0, 1, 2\}$ (4) $\{0, 1\}$
2. Let R_1 and R_2 be two relations defined as follows:
- $R_1 = \{(a, b) \in \mathbb{R}^2 : a^2 + b^2 \in \mathbb{Q}\}$ and
 $R_2 = \{(a, b) \in \mathbb{R}^2 : a^2 + b^2 \notin \mathbb{Q}\}$,
 where \mathbb{Q} is the set of all rational numbers. Then:
- (1) R_2 is transitive but R_1 is not transitive
 (2) R_1 is transitive but R_2 is not transitive
 (3) R_1 and R_2 are both transitive
 (4) Neither R_1 nor R_2 is transitive

FUNCTION

1. If $g(x) = x^2 + x - 1$ and $(g \circ f)(x) = 4x^2 - 10x + 5$, then $f\left(\frac{5}{4}\right)$ is equal to
- (1) $\frac{3}{2}$ (2) $-\frac{1}{2}$
 (3) $-\frac{3}{2}$ (4) $\frac{1}{2}$
2. Let $f : (1, 3) \rightarrow \mathbb{R}$ be a function defined by $f(x) = \frac{x[x]}{1+x^2}$, where $[x]$ denotes the greatest integer $\leq x$. Then the range of f is
- (1) $\left(\frac{3}{5}, \frac{4}{5}\right)$ (2) $\left(\frac{2}{5}, \frac{3}{5}\right] \cup \left(\frac{3}{4}, \frac{4}{5}\right)$
 (3) $\left(\frac{2}{5}, \frac{4}{5}\right]$ (4) $\left(\frac{2}{5}, \frac{1}{2}\right) \cup \left(\frac{3}{5}, \frac{4}{5}\right]$
3. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be such that for all $x \in \mathbb{R}$ ($2^{1+x} + 2^{1-x}$), $f(x)$ and $(3^x + 3^{-x})$ are in A.P., then the minimum value of $f(x)$ is
- (1) 0 (2) 3
 (3) 2 (4) 4

4. The inverse function of

$$f(x) = \frac{8^{2x} - 8^{-2x}}{8^{2x} + 8^{-2x}}, x \in (-1, 1), \text{ is}$$

- (1) $\frac{1}{4}(\log_8 e) \log_e \left(\frac{1-x}{1+x}\right)$
 (2) $\frac{1}{4} \log_e \left(\frac{1-x}{1+x}\right)$
 (3) $\frac{1}{4}(\log_8 e) \log_e \left(\frac{1+x}{1-x}\right)$
 (4) $\frac{1}{4} \log_e \left(\frac{1+x}{1-x}\right)$

5. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a function which satisfies $f(x+y) = f(x) + f(y) \forall x, y \in \mathbb{R}$. If $f(1) = 2$ and $g(n) = \sum_{k=1}^{(n-1)} f(k)$, $n \in \mathbb{N}$ then the value of n , for which $g(n) = 20$, is :

- (1) 5 (2) 9
 (3) 20 (4) 4

6. Let $[t]$ denote the greatest integer $\leq t$. Then the equation in x , $[x]^2 + 2[x + 2] - 7 = 0$ has :

- (1) no integral solution
 (2) exactly four integral solutions
 (3) exactly two solutions
 (4) infinitely many solutions

7. Let $A = \{a, b, c\}$ and $B = \{1, 2, 3, 4\}$. Then the number of elements in the set $C = \{f : A \rightarrow B \mid 2 \in f(A) \text{ and } f \text{ is not one-one}\}$ is _____.

8. For a suitably chosen real constant a , let a function, $f : \mathbb{R} - \{-a\} \rightarrow \mathbb{R}$ be defined by

$$f(x) = \frac{a-x}{a+x}. \text{ Further suppose that for any real}$$

number $x \neq -a$ and $f(x) \neq -a$, $(f \circ f)(x) = x$. Then

$f\left(-\frac{1}{2}\right)$ is equal to :

- (1) $\frac{1}{3}$ (2) 3
 (3) -3 (4) $-\frac{1}{3}$

9. Suppose that a function $f : \mathbb{R} \rightarrow \mathbb{R}$ satisfies $f(x + y) = f(x)f(y)$ for all $x, y \in \mathbb{R}$ and $f(1) = 3$. If $\sum_{i=1}^n f(i) = 363$, then n is equal to _____.

INVERSE TRIGONOMETRY FUNCTION

1. The domain of the function

$f(x) = \sin^{-1}\left(\frac{|x|+5}{x^2+1}\right)$ is $(-\infty, -a] \cup [a, \infty)$. Then a is equal to :

- (1) $\frac{1+\sqrt{17}}{2}$ (2) $\frac{\sqrt{17}-1}{2}$
 (3) $\frac{\sqrt{17}}{2}+1$ (4) $\frac{\sqrt{17}}{2}$
2. $2\pi - \left(\sin^{-1}\frac{4}{5} + \sin^{-1}\frac{5}{13} + \sin^{-1}\frac{16}{65}\right)$ is equal to:

- (1) $\frac{7\pi}{4}$ (2) $\frac{5\pi}{4}$
 (3) $\frac{3\pi}{2}$ (4) $\frac{\pi}{2}$

3. If S is the sum of the first 10 terms of the series

$$\tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{7}\right) + \tan^{-1}\left(\frac{1}{13}\right) + \tan^{-1}\left(\frac{1}{21}\right) + \dots,$$

then $\tan(S)$ is equal to :

- (1) $\frac{5}{11}$ (2) $-\frac{6}{5}$
 (3) $\frac{10}{11}$ (4) $\frac{5}{6}$

LIMIT

1. $\lim_{x \rightarrow 2} \frac{3^x + 3^{3-x} - 12}{3^{-x/2} - 3^{1-x}}$ is equal to _____.

2. $\lim_{x \rightarrow 0} \left(\frac{3x^2 + 2}{7x^2 + 2}\right)^{\frac{1}{x^2}}$ is equal to

- (1) $\frac{1}{e}$ (2) e^2
 (3) e (4) $\frac{1}{e^2}$

3. If $\lim_{x \rightarrow 1} \frac{x + x^2 + x^3 + \dots + x^n - n}{x - 1} = 820, (n \in \mathbb{N})$ then the value of n is equal to _____.

4. $\lim_{x \rightarrow 0} \left(\tan\left(\frac{\pi}{4} + x\right)\right)^{1/x}$ is equal to :

- (1) 2 (2) e
 (3) 1 (4) e^2

5. Let $[t]$ denote the greatest integer $\leq t$. If for some

$\lambda \in \mathbb{R} - \{0, 1\}$, $\lim_{x \rightarrow 0} \left| \frac{1 - x + |x|}{\lambda - x + [x]} \right| = L$, then L is equal to :

- (1) 1 (2) 2
 (3) $\frac{1}{2}$ (4) 0

6. If $\lim_{x \rightarrow 0} \left\{ \frac{1}{x^8} \left(1 - \cos \frac{x^2}{2} - \cos \frac{x^2}{4} + \cos \frac{x^2}{2} \cos \frac{x^2}{4} \right) \right\} = 2^{-k}$, then the value of k is _____.

7. $\lim_{x \rightarrow a} \frac{(a + 2x)^{\frac{1}{3}} - (3x)^{\frac{1}{3}}}{(3a + x)^{\frac{1}{3}} - (4x)^{\frac{1}{3}}}$ ($a \neq 0$) is equal to :

- (1) $\left(\frac{2}{3}\right)\left(\frac{2}{9}\right)^{\frac{1}{3}}$ (2) $\left(\frac{2}{3}\right)^{\frac{4}{3}}$
 (3) $\left(\frac{2}{9}\right)^{\frac{4}{3}}$ (4) $\left(\frac{2}{9}\right)\left(\frac{2}{3}\right)^{\frac{1}{3}}$

8. Let $f : (0, \infty) \rightarrow (0, \infty)$ be a differentiable function such that $f(1) = e$ and

$\lim_{t \rightarrow x} \frac{t^2 f^2(x) - x^2 f^2(t)}{t - x} = 0$. If $f(x) = 1$, then x is equal to :

- (1) $2e$ (2) $\frac{1}{2e}$
 (3) e (4) $\frac{1}{e}$

9. If α is the positive root of the equation,

$$p(x) = x^2 - x - 2 = 0, \text{ then } \lim_{x \rightarrow \alpha^+} \frac{\sqrt{1 - \cos(p(x))}}{x + \alpha - 4}$$

is equal to

(1) $\frac{3}{\sqrt{2}}$ (2) $\frac{3}{2}$

(3) $\frac{1}{\sqrt{2}}$ (4) $\frac{1}{2}$

10. $\lim_{x \rightarrow 0} \frac{x(e^{(\sqrt{1+x^2+x^4}-1)/x} - 1)}{\sqrt{1+x^2+x^4} - 1}$

(1) does not exist. (2) is equal to \sqrt{e} .

(3) is equal to 0. (4) is equal to 1.

11. $\lim_{x \rightarrow 1} \left(\frac{\int_0^{(x-1)^2} t \cos(t^2) dt}{(x-1)\sin(x-1)} \right)$

(1) does not exist (2) is equal to $\frac{1}{2}$

(3) is equal to 1 (4) is equal to $-\frac{1}{2}$

CONTINUITY

1. If the function f defined on $\left(-\frac{1}{3}, \frac{1}{3}\right)$ by

$$f(x) = \begin{cases} \frac{1}{x} \log_e \left(\frac{1+3x}{1-2x} \right), & \text{when } x \neq 0 \\ k, & \text{when } x = 0 \end{cases} \text{ is}$$

continuous, then k is equal to _____

2. Let $[t]$ denote the greatest integer $\leq t$ and

$\lim_{x \rightarrow 0} x \left[\frac{4}{x} \right] = A$. Then the function, $f(x) = [x^2] \sin(\pi x)$ is discontinuous, when x is equal to :

(1) $\sqrt{A+5}$ (2) $\sqrt{A+1}$

(3) \sqrt{A} (4) $\sqrt{A+21}$

3. If $f(x) = \begin{cases} \frac{\sin(a+2)x + \sin x}{x} & ; x < 0 \\ b & ; x = 0 \\ \frac{(x+3x^2)^{\frac{1}{3}} - x^{\frac{1}{3}}}{x^{\frac{4}{3}}} & ; x > 0 \end{cases}$

is continuous at $x = 0$, then $a + 2b$ is equal to :

(1) -1 (2) 1

(3) -2 (4) 0

4. Let $f(x) = x \cdot \left[\frac{x}{2} \right]$, for $-10 < x < 10$, where $[t]$

denotes the greatest integer function. Then the number of points of discontinuity of f is equal to _____.

DIFFERENTIABILITY

1. Let S be the set of points where the function, $f(x) = |2 - |x - 3||$, $x \in \mathbb{R}$, is not differentiable.

Then $\sum_{x \in S} f(f(x))$ is equal to _____.

2. If a function $f(x)$ defined by

$$f(x) = \begin{cases} ae^x + be^{-x}, & -1 \leq x < 1 \\ cx^2, & 1 \leq x \leq 3 \\ ax^2 + 2cx, & 3 < x \leq 4 \end{cases} \text{ be continuous}$$

for some $a, b, c \in \mathbb{R}$ and $f'(0) + f'(2) = e$, then the value of a is :

(1) $\frac{e}{e^2 - 3e - 13}$ (2) $\frac{e}{e^2 + 3e + 13}$

(3) $\frac{1}{e^2 - 3e + 13}$ (4) $\frac{e}{e^2 - 3e + 13}$

3. Suppose a differentiable function $f(x)$ satisfies the identity $f(x+y) = f(x) + f(y) + xy^2 + x^2y$,

for all real x and y . If $\lim_{x \rightarrow 0} \frac{f(x)}{x} = 1$, then $f'(3)$

is equal to _____.

4. The function $f(x) = \begin{cases} \frac{\pi}{4} + \tan^{-1} x, & |x| \leq 1 \\ \frac{1}{2}(|x| - 1), & |x| > 1 \end{cases}$ is :

- (1) continuous on $\mathbb{R} - \{1\}$ and differentiable on $\mathbb{R} - \{-1, 1\}$.
- (2) both continuous and differentiable on $\mathbb{R} - \{-1\}$.
- (3) continuous on $\mathbb{R} - \{-1\}$ and differentiable on $\mathbb{R} - \{-1, 1\}$.
- (4) both continuous and differentiable on $\mathbb{R} - \{1\}$

5. If the function $f(x) = \begin{cases} k_1(x - \pi)^2 - 1, & x \leq \pi \\ k_2 \cos x, & x > \pi \end{cases}$ is

twice differentiable, then the ordered pair (k_1, k_2) is equal to :

- (1) $(\frac{1}{2}, 1)$ (2) $(1, 1)$
- (3) $(\frac{1}{2}, -1)$ (4) $(1, 0)$

6. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be defined as

$$f(x) = \begin{cases} x^5 \sin\left(\frac{1}{x}\right) + 5x^2, & x < 0 \\ 0, & x = 0 \\ x^5 \cos\left(\frac{1}{x}\right) + \lambda x^2, & x > 0 \end{cases}$$

The value

of λ for which $f''(0)$ exists, is _.

7. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a function defined by $f(x) = \max\{x, x^2\}$. Let S denote the set of all points in \mathbb{R} , where f is not differentiable. Then :

- (1) $\{0, 1\}$ (2) $\{0\}$
- (3) ϕ (an empty set) (4) $\{1\}$

METHOD OF DIFFERENTIATION

1. Let $y = y(x)$ be a function of x satisfying $y\sqrt{1-x^2} = k - x\sqrt{1-y^2}$ where k is a constant and $y(\frac{1}{2}) = -\frac{1}{4}$. Then $\frac{dy}{dx}$ at $x = \frac{1}{2}$, is equal to:

- (1) $\frac{\sqrt{5}}{2}$ (2) $-\frac{\sqrt{5}}{2}$
- (3) $\frac{2}{\sqrt{5}}$ (4) $-\frac{\sqrt{5}}{4}$

2. If $y(\alpha) = \sqrt{2\left(\frac{\tan \alpha + \cot \alpha}{1 + \tan^2 \alpha}\right) + \frac{1}{\sin^2 \alpha}}$, $\alpha \in \left(\frac{3\pi}{4}, \pi\right)$, then $\frac{dy}{d\alpha}$ at $\alpha = \frac{5\pi}{6}$ is :

- (1) 4 (2) $-\frac{1}{4}$
- (3) $\frac{4}{3}$ (4) -4

3. Let $x^k + y^k = a^k$, ($a, k > 0$) and $\frac{dy}{dx} + \left(\frac{y}{x}\right)^{\frac{1}{3}} = 0$, then k is :

- (1) $\frac{3}{2}$ (2) $\frac{1}{3}$
- (3) $\frac{2}{3}$ (4) $\frac{4}{3}$

4. Let $f(x) = (\sin(\tan^{-1}x) + \sin(\cot^{-1}x))^2 - 1$, $|x| > 1$. If $\frac{dy}{dx} = \frac{1}{2} \frac{d}{dx}(\sin^{-1}(f(x)))$ and $y(\sqrt{3}) = \frac{\pi}{6}$, then $y(-\sqrt{3})$ is equal to

- (1) $\frac{5\pi}{6}$ (2) $-\frac{\pi}{6}$
- (3) $\frac{\pi}{3}$ (4) $\frac{2\pi}{3}$

5. If $x = 2\sin\theta - \sin 2\theta$ and $y = 2\cos\theta - \cos 2\theta$, $\theta \in [0, 2\pi]$, then $\frac{d^2y}{dx^2}$ at $\theta = \pi$ is :

- (1) $\frac{3}{2}$ (2) $-\frac{3}{4}$
- (3) $\frac{3}{4}$ (4) $-\frac{3}{8}$

6. Let f and g be differentiable functions on \mathbf{R} such that $f \circ g$ is the identity function. If for some $a, b \in \mathbf{R}$, $g'(a) = 5$ and $g(a) = b$, then $f'(b)$ is equal to :

(1) $\frac{2}{5}$ (2) 1
(3) $\frac{1}{5}$ (4) 5

7. If $y = \sum_{k=1}^6 k \cos^{-1} \left\{ \frac{3}{5} \cos kx - \frac{4}{5} \sin kx \right\}$, then $\frac{dy}{dx}$ at $x = 0$ is _____.

8. If $y^2 + \log_e (\cos^2 x) = y$, $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, then :

(1) $|y''(0)| = 2$
(2) $|y'(0)| + |y''(0)| = 3$
(3) $|y'(0)| + |y''(0)| = 1$
(4) $y''(0) = 0$

9. If $(a + \sqrt{2} b \cos x)(a - \sqrt{2} b \cos y) = a^2 - b^2$,

where $a > b > 0$, then $\frac{dx}{dy}$ at $\left(\frac{\pi}{4}, \frac{\pi}{4}\right)$ is :

(1) $\frac{a-b}{a+b}$ (2) $\frac{a+b}{a-b}$
(3) $\frac{2a+b}{2a-b}$ (4) $\frac{a-2b}{a+2b}$

10. The derivative of $\tan^{-1} \left(\frac{\sqrt{1+x^2}-1}{x} \right)$ with

respect to $\tan^{-1} \left(\frac{2x\sqrt{1-x^2}}{1-2x^2} \right)$ at $x = \frac{1}{2}$ is :

(1) $\frac{\sqrt{3}}{12}$ (2) $\frac{\sqrt{3}}{10}$
(3) $\frac{2\sqrt{3}}{5}$ (4) $\frac{2\sqrt{3}}{3}$

INDEFINITE INTEGRATION

1. If $\int \frac{\cos x dx}{\sin^3 x (1 + \sin^6 x)^{2/3}} = f(x)(1 + \sin^6 x)^{1/\lambda} + c$ where c is a constant of integration, then $\lambda f\left(\frac{\pi}{3}\right)$ is equal to

(1) -2 (2) $-\frac{9}{8}$
(3) 2 (4) $\frac{9}{8}$

2. If $\int \frac{d\theta}{\cos^2 \theta (\tan 2\theta + \sec 2\theta)} = \lambda \tan \theta + 2 \log_e |f(\theta)| + C$ where C is a constant of integration, then the ordered pair $(\lambda, f(\theta))$ is equal to :

(1) $(-1, 1 + \tan \theta)$ (2) $(-1, 1 - \tan \theta)$
(3) $(1, 1 - \tan \theta)$ (4) $(1, 1 + \tan \theta)$

3. The integral $\int \frac{dx}{(x+4)^{\frac{8}{7}}(x-3)^{\frac{6}{7}}}$ is equal to :

(where C is a constant of integration)

(1) $\left(\frac{x-3}{x+4}\right)^{\frac{1}{7}} + C$ (2) $-\left(\frac{x-3}{x+4}\right)^{\frac{1}{7}} + C$
(3) $\frac{1}{2}\left(\frac{x-3}{x+4}\right)^{\frac{3}{7}} + C$ (4) $-\frac{1}{13}\left(\frac{x-3}{x+4}\right)^{\frac{13}{7}} + C$

4. If $\int \sin^{-1} \left(\sqrt{\frac{x}{1+x}} \right) dx = A(x) \tan^{-1}(\sqrt{x}) + B(x) + C$, where C is a constant of integration, then the ordered pair $(A(x), B(x))$ can be :

(1) $(x-1, \sqrt{x})$ (2) $(x+1, \sqrt{x})$
(3) $(x+1, -\sqrt{x})$ (4) $(x-1, -\sqrt{x})$

5. The integral $\int \left(\frac{x}{x \sin x + \cos x} \right)^2 dx$ is equal to:

(where C is a constant of integration)

(1) $\sec x + \frac{x \tan x}{x \sin x + \cos x} + C$

(2) $\sec x - \frac{x \tan x}{x \sin x + \cos x} + C$

(3) $\tan x + \frac{x \sec x}{x \sin x + \cos x} + C$

(4) $\tan x - \frac{x \sec x}{x \sin x + \cos x} + C$

6. If

$$\int (e^{2x} + 2e^x - e^{-x} - 1)e^{(e^x + e^{-x})} dx = g(x)e^{(e^x + e^{-x})} + c,$$

where c is a constant of integration, then g(0) is equal to:

(1) 2 (2) e^2

(3) e (4) 1

7. If $\int \frac{\cos \theta}{5 + 7 \sin \theta - 2 \cos^2 \theta} d\theta = A \log_e |B(\theta)| + C,$

where C is a constant of integration, then $\frac{B(\theta)}{A}$

can be:

(1) $\frac{2 \sin \theta + 1}{5(\sin \theta + 3)}$ (2) $\frac{2 \sin \theta + 1}{\sin \theta + 3}$

(3) $\frac{5(\sin \theta + 3)}{2 \sin \theta + 1}$ (4) $\frac{5(2 \sin \theta + 1)}{\sin \theta + 3}$

DEFINITE INTEGRATION

1. If θ_1 and θ_2 be respectively the smallest and the largest values of θ in $(0, 2\pi) - \{\pi\}$ which satisfy

the equation, $2 \cot^2 \theta - \frac{5}{\sin \theta} + 4 = 0$, then

$\int_{\theta_1}^{\theta_2} \cos^2 3\theta d\theta$ is equal to:

(1) $\frac{2\pi}{3}$ (2) $\frac{\pi}{3} + \frac{1}{6}$

(3) $\frac{\pi}{9}$ (4) $\frac{\pi}{3}$

2. The value of α for which $4\alpha \int_{-1}^2 e^{-\alpha|x|} dx = 5$, is:

(1) $\log_e \left(\frac{3}{2} \right)$ (2) $\log_e \left(\frac{4}{3} \right)$

(3) $\log_e 2$ (4) $\log_e \sqrt{2}$

3. If $f(a + b + 1 - x) = f(x)$, for all x, where a and b are fixed positive real numbers, then

$\frac{1}{a+b} \int_a^b x(f(x) + f(x+1)) dx$ is equal to:

(1) $\int_{a+1}^{b+1} f(x) dx$ (2) $\int_{a+1}^{b+1} f(x+1) dx$

(3) $\int_{a-1}^{b-1} f(x+1) dx$ (4) $\int_{a-1}^{b-1} f(x) dx$

4. If $I = \int_1^2 \frac{dx}{\sqrt{2x^3 - 9x^2 + 12x + 4}}$, then:

(1) $\frac{1}{9} < I^2 < \frac{1}{8}$ (2) $\frac{1}{16} < I^2 < \frac{1}{9}$

(3) $\frac{1}{6} < I^2 < \frac{1}{2}$ (4) $\frac{1}{8} < I^2 < \frac{1}{4}$

5. $\lim_{x \rightarrow 0} \frac{\int_0^x t \sin(10t) dt}{x}$ is equal to

(1) 0 (2) $-\frac{1}{5}$

(3) $-\frac{1}{10}$ (4) $\frac{1}{10}$

6. The value of $\int_0^{2\pi} \frac{x \sin^8 x}{\sin^8 x + \cos^8 x} dx$ is equal to:

(1) 2π (2) 4π

(3) $2\pi^2$ (4) π^2

7. If for all real triplets (a, b, c), $f(x) = a + bx + cx^2$;

then $\int_0^1 f(x) dx$ is equal to:

(1) $\frac{1}{2} \left\{ f(1) + 3f\left(\frac{1}{2}\right) \right\}$

(2) $2 \left\{ 3f(1) + 2f\left(\frac{1}{2}\right) \right\}$

(3) $\frac{1}{6} \left\{ f(0) + f(1) + 4f\left(\frac{1}{2}\right) \right\}$

(4) $\frac{1}{3} \left\{ f(0) + f\left(\frac{1}{2}\right) \right\}$

8. The integral $\int_0^2 |x-1| - x \, dx$ is equal to _____.

9. Let $[t]$ denote the greatest integer less than or equal to t . Then the value of $\int_1^2 |2x - [3x]| \, dx$ is _____.

10. $\int_{-\pi}^{\pi} |\pi - |x|| \, dx$ is equal to :

(1) π^2 (2) $2\pi^2$

(3) $\sqrt{2}\pi^2$ (4) $\frac{\pi^2}{2}$

11. If the value of the integral $\int_0^{1/2} \frac{x^2}{(1-x^2)^{3/2}} \, dx$ is

$\frac{k}{6}$, then k is equal to :

(1) $2\sqrt{3} - \pi$ (2) $3\sqrt{2} + \pi$

(3) $3\sqrt{2} - \pi$ (4) $2\sqrt{3} + \pi$

12. Let $f(x) = |x-2|$ and $g(x) = f(f(x))$, $x \in [0, 4]$.

Then $\int_0^3 (g(x) - f(x)) \, dx$ is equal to :

(1) $\frac{3}{2}$ (2) 0

(3) $\frac{1}{2}$ (4) 1

13. Let $f(x) = \int \frac{\sqrt{x}}{(1+x)^2} \, dx$ ($x \geq 0$). Then $f(3) - f(1)$

is equal to :

(1) $-\frac{\pi}{6} + \frac{1}{2} + \frac{\sqrt{3}}{4}$ (2) $\frac{\pi}{6} + \frac{1}{2} - \frac{\sqrt{3}}{4}$

(3) $-\frac{\pi}{12} + \frac{1}{2} + \frac{\sqrt{3}}{4}$ (4) $\frac{\pi}{12} + \frac{1}{2} - \frac{\sqrt{3}}{4}$

14. $\int_{\pi/6}^{\pi/3} \tan^3 x \cdot \sin^2 3x (2\sec^2 x \cdot \sin^2 3x + 3\tan x \cdot \sin 6x) \, dx$ is equal to :

(1) $\frac{9}{2}$ (2) $-\frac{1}{9}$

(3) $-\frac{1}{18}$ (4) $\frac{7}{18}$

15. Let $\{x\}$ and $[x]$ denote the fractional part of x and the greatest integer $\leq x$ respectively of a real number x . If $\int_0^n \{x\} \, dx$, $\int_0^n [x] \, dx$ and $10(n^2 - n)$, ($n \in \mathbb{N}$, $n > 1$) are three consecutive terms of a G.P., then n is equal to _____

16. The value of $\int_{-\pi/2}^{\pi/2} \frac{1}{1+e^{\sin x}} \, dx$ is

(1) π (2) $\frac{3\pi}{2}$

(3) $\frac{\pi}{4}$ (4) $\frac{\pi}{2}$

17. If $I_1 = \int_0^1 (1-x^{50})^{100} \, dx$ and $I_2 = \int_0^1 (1-x^{50})^{101} \, dx$ such that $I_2 = \alpha I_1$ then α equals to

(1) $\frac{5050}{5051}$ (2) $\frac{5050}{5049}$

(3) $\frac{5049}{5050}$ (4) $\frac{5051}{5050}$

18. The integral $\int_1^2 e^x \cdot x^x (2 + \log_e x) \, dx$ equal :

(1) $e(4e + 1)$ (2) $e(2e - 1)$

(3) $4e^2 - 1$ (4) $e(4e - 1)$

TANGENT & NORMAL

1. The length of the perpendicular from the origin, on the normal to the curve, $x^2 + 2xy - 3y^2 = 0$ at the point (2,2) is

(1) $4\sqrt{2}$ (2) $2\sqrt{2}$

(3) 2 (4) $\sqrt{2}$

2. Let the normal at a point P on the curve $y^2 - 3x^2 + y + 10 = 0$ intersect the y-axis at $(0, \frac{3}{2})$. If m is the slope of the tangent at P to the curve, then $|m|$ is equal to

3. If the tangent to the curve $y = x + \sin y$ at a point (a, b) is parallel to the line joining $\left(0, \frac{3}{2}\right)$ and $\left(\frac{1}{2}, 2\right)$, then :
- (1) $b = a$ (2) $b = \frac{\pi}{2} + a$
 (3) $|b - a| = 1$ (4) $|a + b| = 1$
4. The equation of the normal to the curve $y = (1+x)^{2y} + \cos^2(\sin^{-1}x)$ at $x = 0$ is :
- (1) $y = 4x + 2$ (2) $x + 4y = 8$
 (3) $y + 4x = 2$ (4) $2y + x = 4$
5. If the surface area of a cube is increasing at a rate of $3.6 \text{ cm}^2/\text{sec}$, retaining its shape; then the rate of change of its volume (in cm^3/sec), when the length of a side of the cube is 10 cm, is :
- (1) 9 (2) 18
 (3) 10 (4) 20
6. If the tangent of the curve, $y = e^x$ at a point (c, e^c) and the normal to the parabola, $y^2 = 4x$ at the point $(1, 2)$ intersect at the same point on the x-axis, then the value of c is _____.
7. If the lines $x + y = a$ and $x - y = b$ touch the curve $y = x^2 - 3x + 2$ at the points where the curve intersects the x-axis, then $\frac{a}{b}$ is equal to _____.
8. The position of a moving car at time t is given by $f(t) = at^2 + bt + c$, $t > 0$, where a , b and c are real numbers greater than 1. Then the average speed of the car over the time interval $[t_1, t_2]$ is attained at the point :
- (1) $a(t_2 - t_1) + b$ (2) $(t_2 - t_1)/2$
 (3) $2a(t_1 + t_2) + b$ (4) $(t_1 + t_2)/2$

MONOTONICITY

1. The value of c in the Lagrange's mean value theorem for the function $f(x) = x^3 - 4x^2 + 8x + 11$, when $x \in [0, 1]$ is :
- (1) $\frac{2}{3}$ (2) $\frac{\sqrt{7}-2}{3}$
 (3) $\frac{4-\sqrt{5}}{3}$ (4) $\frac{4-\sqrt{7}}{3}$

2. Let the function, $f: [-7, 0] \rightarrow \mathbb{R}$ be continuous on $[-7, 0]$ and differentiable on $(-7, 0)$. If $f(-7) = -3$ and $f'(x) \leq 2$, for all $x \in (-7, 0)$, then for all such functions f , $f(-1) + f(0)$ lies in the interval:
- (1) $[-6, 20]$ (2) $(-\infty, 20]$
 (3) $(-\infty, 11]$ (4) $[-3, 11]$
3. Let S be the set of all functions $f: [0, 1] \rightarrow \mathbb{R}$, which are continuous on $[0, 1]$ and differentiable on $(0, 1)$. Then for every f in S , there exists a $c \in (0, 1)$, depending on f , such that
- (1) $|f(c) - f(1)| < (1 - c)|f'(c)|$
 (2) $|f(c) - f(1)| < |f'(c)|$
 (3) $|f(c) + f(1)| < (1 + c)|f'(c)|$
 (4) $\frac{f(1) - f(c)}{1 - c} = f'(c)$
4. If c is a point at which Rolle's theorem holds for the function, $f(x) = \log_e \left(\frac{x^2 + \alpha}{7x} \right)$ in the interval $[3, 4]$, where $\alpha \in \mathbb{R}$, then $f''(c)$ is equal to
- (1) $\frac{\sqrt{3}}{7}$ (2) $\frac{1}{12}$
 (3) $-\frac{1}{24}$ (4) $-\frac{1}{12}$
5. Let $f(x) = x \cos^{-1}(-\sin|x|)$, $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, then which of the following is true ?
- (1) f' is decreasing in $\left(-\frac{\pi}{2}, 0\right)$ and increasing in $\left(0, \frac{\pi}{2}\right)$
 (2) f is not differentiable at $x = 0$
 (3) $f'(0) = -\frac{\pi}{2}$
 (4) f' is increasing in $\left(-\frac{\pi}{2}, 0\right)$ and decreasing in $\left(0, \frac{\pi}{2}\right)$

6. Let f be any function continuous on $[a, b]$ and twice differentiable on (a, b) . If for all $x \in (a, b)$, $f'(x) > 0$ and $f''(x) < 0$, then for any $c \in (a, b)$,

$\frac{f(c) - f(a)}{f(b) - f(c)}$ is greater than :

(1) $\frac{b+a}{b-a}$ (2) $\frac{b-c}{c-a}$

(3) $\frac{c-a}{b-c}$ (4) 1

7. Let $f : (-1, \infty) \rightarrow \mathbb{R}$ be defined by $f(0) = 1$ and

$f(x) = \frac{1}{x} \log_e(1+x), x \neq 0$. Then the function f :

- (1) decreases in $(-1, \infty)$
 (2) decreases in $(-1, 0)$ and increases in $(0, \infty)$
 (3) increases in $(-1, \infty)$
 (4) increases in $(-1, 0)$ and decreases in $(0, \infty)$

8. The function, $f(x) = (3x - 7)x^{2/3}$, $x \in \mathbb{R}$, is increasing for all x lying in :

(1) $(-\infty, 0) \cup \left(\frac{3}{7}, \infty\right)$

(2) $(-\infty, 0) \cup \left(\frac{14}{15}, \infty\right)$

(3) $\left(-\infty, \frac{14}{15}\right)$

(4) $\left(-\infty, -\frac{14}{15}\right) \cup (0, \infty)$

9. Let f be a twice differentiable function on $(1, 6)$. If $f(2) = 8$, $f'(2) = 5$, $f'(x) \geq 1$ and $f''(x) \geq 4$, for all $x \in (1, 6)$, then :

- (1) $f(5) \leq 10$ (2) $f'(5) + f''(5) \leq 20$
 (3) $f(5) + f'(5) \geq 28$ (4) $f(5) + f'(5) \leq 26$

10. For all twice differentiable functions $f : \mathbb{R} \rightarrow \mathbb{R}$, with $f(0) = f(1) = f'(0) = 0$

- (1) $f''(x) = 0$, for some $x \in (0, 1)$
 (2) $f''(0) = 0$
 (3) $f''(x) \neq 0$ at every point $x \in (0, 1)$
 (4) $f''(x) = 0$ at every point $x \in (0, 1)$

11. If the tangent to the curve, $y = f(x) = x \log_e x$, ($x > 0$) at a point $(c, f(c))$ is parallel to the line - segment joining the points $(1, 0)$ and (e, e) , then c is equal to :

(1) $\frac{1}{e-1}$ (2) $e^{\left(\frac{1}{1-e}\right)}$

(3) $e^{\left(\frac{1}{e-1}\right)}$ (4) $\frac{e-1}{e}$

MAXIMA & MINIMA

1. Let $f(x)$ be a polynomial of degree 5 such that $x = \pm 1$ are its critical points. If $\lim_{x \rightarrow 0} \left(2 + \frac{f(x)}{x^3}\right) = 4$,

then which one of the following is not true?

- (1) f is an odd function
 (2) $x = 1$ is a point of minima and $x = -1$ is a point of maxima of f .
 (3) $x = 1$ is a point of maxima and $x = -1$ is a point of minimum of f .
 (4) $f(1) - 4f(-1) = 4$

2. Let $f(x)$ be a polynomial of degree 3 such that $f(-1) = 10$, $f(1) = -6$, $f(x)$ has a critical point at $x = -1$ and $f'(x)$ has a critical point at $x = 1$. Then $f(x)$ has a local minima at $x = \underline{\hspace{2cm}}$.

3. Let a function $f : [0, 5] \rightarrow \mathbb{R}$ be continuous, $f(1) = 3$ and F be defined as : $F(x) = \int_1^x t^2 g(t) dt$,

where $g(t) = \int_1^t f(u) du$. Then for the function F ,

the point $x = 1$ is :

- (1) a point of local minima.
 (2) not a critical point.
 (3) a point of inflection.
 (4) a point of local maxima.

4. A spherical iron ball of 10 cm radius is coated with a layer of ice of uniform thickness the melts at a rate of $50 \text{ cm}^3/\text{min}$. When the thickness of ice is 5 cm, then the rate (in cm/min .) at which of the thickness of ice decreases, is :

- (1) $\frac{1}{36\pi}$ (2) $\frac{5}{6\pi}$
 (3) $\frac{1}{18\pi}$ (4) $\frac{1}{54\pi}$

5. Let $P(h, k)$ be a point on the curve $y = x^2 + 7x + 2$, nearest to the line, $y = 3x - 3$. Then the equation of the normal to the curve at P is :

- (1) $x + 3y - 62 = 0$ (2) $x - 3y - 11 = 0$
 (3) $x - 3y + 22 = 0$ (4) $x + 3y + 26 = 0$

6. If $p(x)$ be a polynomial of degree three that has a local maximum value 8 at $x = 1$ and a local minimum value 4 at $x = 2$; then $p(0)$ is equal to:

- (1) 12 (2) -24
 (3) 6 (4) -12

7. Suppose $f(x)$ is a polynomial of degree four, having critical points at $-1, 0, 1$. If $T = \{x \in \mathbb{R} | f(x) = f(0)\}$, then the sum of squares of all the elements of T is:

- (1) 6 (2) 8
 (3) 4 (4) 2

8. If $x = 1$ is a critical point of the function $f(x) = (3x^2 + ax - 2 - a) e^x$, then :

- (1) $x = 1$ is a local minima and $x = -\frac{2}{3}$ is a local maxima of f .
 (2) $x = 1$ is a local maxima and $x = -\frac{2}{3}$ is a local minima of f .
 (3) $x = 1$ and $x = -\frac{2}{3}$ are local minima of f .
 (4) $x = 1$ and $x = -\frac{2}{3}$ are local maxima of f .

9. Let AD and BC be two vertical poles at A and B respectively on a horizontal ground. If $AD = 8 \text{ m}$, $BC = 11 \text{ m}$ and $AB = 10 \text{ m}$; then the distance (in meters) of a point M on AB from the point A such that $MD^2 + MC^2$ is minimum is_.

10. The set of all real values of λ for which the function $f(x) = (1 - \cos^2 x) \cdot (\lambda + \sin x)$, $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, has exactly one maxima and

exactly one minima, is :

- (1) $\left(-\frac{1}{2}, \frac{1}{2}\right) - \{0\}$ (2) $\left(-\frac{1}{2}, \frac{1}{2}\right)$
 (3) $\left(-\frac{3}{2}, \frac{3}{2}\right)$ (4) $\left(-\frac{3}{2}, \frac{3}{2}\right) - \{0\}$

DIFFERENTIAL EQUATION

1. Let $y = y(x)$ be the solution curve of the differential equation, $(y^2 - x) \frac{dy}{dx} = 1$, satisfying $y(0) = 1$. This curve intersects the x -axis at a point whose abscissa is :

- (1) $2 + e$ (2) 2
 (3) $2 - e$ (4) $-e$

2. If $y = y(x)$ is the solution of the differential equation, $e^y \left(\frac{dy}{dx} - 1\right) = e^x$ such that $y(0) = 0$, then $y(1)$ is equal to :

- (1) $2 + \log_e 2$ (2) $2e$
 (3) $\log_e 2$ (4) $1 + \log_e 2$

3. The differential equation of the family of curves, $x^2 = 4b(y + b)$, $b \in \mathbb{R}$, is

- (1) $x(y')^2 = x + 2yy'$
 (2) $x(y')^2 = 2yy' - x$
 (3) $xy'' = y'$
 (4) $x(y')^2 = x - 2yy'$

4. Let $y = y(x)$ be a solution of the differential equation, $\sqrt{1-x^2} \frac{dy}{dx} + \sqrt{1-y^2} = 0, |x| < 1$. If $y\left(\frac{1}{2}\right) = \frac{\sqrt{3}}{2}$, then $y\left(\frac{-1}{\sqrt{2}}\right)$ is equal to
- (1) $-\frac{\sqrt{3}}{2}$ (2) $\frac{1}{\sqrt{2}}$
 (3) $\frac{\sqrt{3}}{2}$ (4) $-\frac{1}{\sqrt{2}}$
5. If $\frac{dy}{dx} = \frac{xy}{x^2 + y^2}$; $y(1) = 1$; then a value of x satisfying $y(x) = e$ is :
- (1) $\sqrt{2}e$ (2) $\frac{e}{\sqrt{2}}$
 (3) $\frac{1}{2}\sqrt{3}e$ (4) $\sqrt{3}e$
6. If $f'(x) = \tan^{-1}(\sec x + \tan x)$, $-\frac{\pi}{2} < x < \frac{\pi}{2}$, and $f(0) = 0$, then $f(1)$ is equal to :
- (1) $\frac{\pi-1}{4}$ (2) $\frac{\pi+2}{4}$
 (3) $\frac{\pi+1}{4}$ (4) $\frac{1}{4}$
7. If for $x \geq 0$, $y = y(x)$ is the solution of the differential equation $(x+1)dy = ((x+1)^2 + y-3)dx$, $y(2) = 0$, then $y(3)$ is equal to —
8. Let $y = y(x)$ be the solution of the differential equation, $\frac{2+\sin x}{y+1} \cdot \frac{dy}{dx} = -\cos x, y > 0, y(0) = 1$. If $y(\pi) = a$ and $\frac{dy}{dx}$ at $x = \pi$ is b , then the ordered pair (a, b) is equal to :
- (1) $(2, 1)$ (2) $\left(2, \frac{3}{2}\right)$
 (3) $(1, -1)$ (4) $(1, 1)$
9. If a curve $y = f(x)$, passing through the point $(1, 2)$, is the solution of the differential equation, $2x^2 dy = (2xy + y^2)dx$, then $f\left(\frac{1}{2}\right)$ is equal to:
- (1) $\frac{1}{1-\log_e 2}$ (2) $\frac{1}{1+\log_e 2}$
 (3) $\frac{-1}{1+\log_e 2}$ (4) $1+\log_e 2$
10. The solution curve of the differential equation, $(1 + e^{-x})(1 + y^2) \frac{dy}{dx} = y^2$, which passes through the point $(0, 1)$, is :
- (1) $y^2 = 1 + y \log_e \left(\frac{1+e^x}{2}\right)$
 (2) $y^2 + 1 = y \left(\log_e \left(\frac{1+e^x}{2}\right) + 2\right)$
 (3) $y^2 = 1 + y \log_e \left(\frac{1+e^{-x}}{2}\right)$
 (4) $y^2 + 1 = y \left(\log_e \left(\frac{1+e^{-x}}{2}\right) + 2\right)$
11. If $x^3 dy + xy dx = x^2 dy + 2y dx$; $y(2) = e$ and $x > 1$, then $y(4)$ is equal to :
- (1) $\frac{3}{2} + \sqrt{e}$ (2) $\frac{3}{2}\sqrt{e}$
 (3) $\frac{1}{2} + \sqrt{e}$ (4) $\frac{\sqrt{e}}{2}$
12. Let $y = y(x)$ be the solution of the differential equation, $xy' - y = x^2(x \cos x + \sin x)$, $x > 0$. If $y(\pi) = \pi$, then $y''\left(\frac{\pi}{2}\right) + y\left(\frac{\pi}{2}\right)$ is equal to :
- (1) $2 + \frac{\pi}{2}$ (2) $1 + \frac{\pi}{2}$
 (3) $1 + \frac{\pi}{2} + \frac{\pi^2}{4}$ (4) $2 + \frac{\pi}{2} + \frac{\pi^2}{4}$

13. The solution of the differential equation

$$\frac{dy}{dx} - \frac{y+3x}{\log_e(y+3x)} + 3 = 0 \text{ is :-}$$

(where C is a constant of integration.)

(1) $x - 2 \log_e(y+3x) = C$

(2) $x - \log_e(y+3x) = C$

(3) $x - \frac{1}{2} (\log_e(y+3x))^2 = C$

(4) $y + 3x - \frac{1}{2} (\log_e x)^2 = C$

14. If $y = y(x)$ is the solution of the differential

equation $\frac{5+e^x}{2+y} \cdot \frac{dy}{dx} + e^x = 0$ satisfying

$y(0) = 1$, then a value of $y(\log_e 13)$ is :

(1) 1 (2) -1

(3) 2 (4) 0

15. Let $y = y(x)$ be the solution of the differential

equation $\cos x \frac{dy}{dx} + 2y \sin x = \sin 2x$,

$x \in \left(0, \frac{\pi}{2}\right)$. If $y(\pi/3) = 0$, then $y(\pi/4)$ is equal

to :

(1) $\sqrt{2} - 2$ (2) $\frac{1}{\sqrt{2}} - 1$

(3) $2 - \sqrt{2}$ (4) $2 + \sqrt{2}$

16. Which of the following points lies on the

tangent to the curve $x^4 e^y + 2\sqrt{y+1} = 3$ at the point (1, 0)?

(1) (2, 2) (2) (-2, 6)

(3) (-2, 4) (4) (2, 6)

17. The general solution of the differential equation

$$\sqrt{1+x^2+y^2+x^2y^2} + xy \frac{dy}{dx} = 0 \text{ is :}$$

(where C is a constant of integration)

(1) $\sqrt{1+y^2} + \sqrt{1+x^2} = \frac{1}{2} \log_e \left(\frac{\sqrt{1+x^2+1}}{\sqrt{1+x^2-1}} \right) + C$

(2) $\sqrt{1+y^2} - \sqrt{1+x^2} = \frac{1}{2} \log_e \left(\frac{\sqrt{1+x^2+1}}{\sqrt{1+x^2-1}} \right) + C$

(3) $\sqrt{1+y^2} + \sqrt{1+x^2} = \frac{1}{2} \log_e \left(\frac{\sqrt{1+x^2-1}}{\sqrt{1+x^2+1}} \right) + C$

(4) $\sqrt{1+y^2} - \sqrt{1+x^2} = \frac{1}{2} \log_e \left(\frac{\sqrt{1+x^2-1}}{\sqrt{1+x^2+1}} \right) + C$

18. If $y = \left(\frac{2}{\pi}x - 1\right) \operatorname{cosec} x$ is the solution of the

differential equation, $\frac{dy}{dx} + p(x)y = \frac{2}{\pi} \operatorname{cosec} x$,

$0 < x < \frac{\pi}{2}$, then the function $p(x)$ is equal to

(1) $\cot x$ (2) $\tan x$

(3) $\operatorname{cosec} x$ (4) $\sec x$

AREA UNDER THE CURVE

1. The area (in sq. units) of the region $\{(x, y) \in \mathbb{R}^2 | 4x^2 \leq y \leq 8x + 12\}$ is :

(1) $\frac{127}{3}$ (2) $\frac{125}{3}$

(3) $\frac{124}{3}$ (4) $\frac{128}{3}$

2. The area of the region, enclosed by the circle $x^2 + y^2 = 2$ which is not common to the region bounded by the parabola $y^2 = x$ and the straight line $y = x$, is :

(1) $\frac{1}{3}(12\pi - 1)$ (2) $\frac{1}{6}(12\pi - 1)$

(3) $\frac{1}{6}(24\pi - 1)$ (4) $\frac{1}{3}(6\pi - 1)$

3. The area (in sq. units) of the region $\{(x, y) \in \mathbb{R}^2 : x^2 \leq y \leq 3 - 2x\}$, is
- (1) $\frac{29}{3}$ (2) $\frac{31}{3}$
 (3) $\frac{34}{3}$ (4) $\frac{32}{3}$
4. For $a > 0$, let the curves $C_1 : y^2 = ax$ and $C_2 : x^2 = ay$ intersect at origin O and a point P. Let the line $x = b$ ($0 < b < a$) intersect the chord OP and the x-axis at points Q and R, respectively. If the line $x = b$ bisects the area bounded by the curves, C_1 and C_2 , and the area of $\Delta OQR = \frac{1}{2}$, then 'a' satisfies the equation
- (1) $x^6 - 12x^3 + 4 = 0$
 (2) $x^6 - 12x^3 - 4 = 0$
 (3) $x^6 + 6x^3 - 4 = 0$
 (4) $x^6 - 6x^3 + 4 = 0$
5. Given : $f(x) = \begin{cases} x & , 0 \leq x < \frac{1}{2} \\ \frac{1}{2} & , x = \frac{1}{2} \\ 1-x & , \frac{1}{2} < x \leq 1 \end{cases}$ and $g(x) = \left(x - \frac{1}{2}\right)^2, x \in \mathbb{R}$. Then the area (in sq. units) of the region bounded by the curves, $y = f(x)$ and $y = g(x)$ between the lines, $2x = 1$ and $2x = \sqrt{3}$, is :
- (1) $\frac{1}{3} + \frac{\sqrt{3}}{4}$ (2) $\frac{\sqrt{3}}{4} - \frac{1}{3}$
 (3) $\frac{1}{2} + \frac{\sqrt{3}}{4}$ (4) $\frac{1}{2} - \frac{\sqrt{3}}{4}$
6. Area (in sq. units) of the region outside $\frac{|x|}{2} + \frac{|y|}{3} = 1$ and inside the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$ is :
- (1) $3(4 - \pi)$ (2) $6(\pi - 2)$
 (3) $3(\pi - 2)$ (4) $6(4 - \pi)$
7. Consider a region $R = \{(x, y) \in \mathbb{R}^2 : x^2 \leq y \leq 2x\}$. If a line $y = \alpha$ divides the area of region R into two equal parts, then which of the following is true?
- (1) $\alpha^3 - 6\alpha^2 + 16 = 0$
 (2) $3\alpha^2 - 8\alpha + 8 = 0$
 (3) $\alpha^3 - 6\alpha^{3/2} - 16 = 0$
 (4) $3\alpha^2 - 8\alpha^{3/2} + 8 = 0$
8. The area (in sq. units) of the region $\{(x, y) : 0 \leq y \leq x^2 + 1, 0 \leq y \leq x + 1, \frac{1}{2} \leq x \leq 2\}$ is:
- (1) $\frac{79}{16}$ (2) $\frac{23}{6}$
 (3) $\frac{79}{24}$ (4) $\frac{23}{16}$
9. The area (in sq. units) of the region $A = \{(x, y) : (x - 1)[x] \leq y \leq 2\sqrt{x}, 0 \leq x \leq 2\}$, where $[t]$ denotes the greatest integer function, is :
- (1) $\frac{8}{3}\sqrt{2} - \frac{1}{2}$ (2) $\frac{8}{3}\sqrt{2} - 1$
 (3) $\frac{4}{3}\sqrt{2} - \frac{1}{2}$ (4) $\frac{4}{3}\sqrt{2} + 1$
10. The area (in sq. units) of the region $A = \{(x, y) : |x| + |y| \leq 1, 2y^2 \geq |x|\}$ is :
- (1) $\frac{1}{6}$ (2) $\frac{1}{3}$
 (3) $\frac{7}{6}$ (4) $\frac{5}{6}$
11. The area (in sq. units) of the region enclosed by the curves $y = x^2 - 1$ and $y = 1 - x^2$ is equal to :
- (1) $\frac{4}{3}$ (2) $\frac{8}{3}$
 (3) $\frac{16}{3}$ (4) $\frac{7}{2}$

MATRICES

1. Let $A = [a_{ij}]$ and $B = [b_{ij}]$ be two 3×3 real matrices such that $b_{ij} = (3)^{(i+j-2)}a_{ij}$, where $i, j = 1, 2, 3$. If the determinant of B is 81, then the determinant of A is :

- (1) 3
- (2) 1/3
- (3) 1/81
- (4) 1/9

2. Let α be a root of the equation $x^2 + x + 1 = 0$

and the matrix $A = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha^4 \end{bmatrix}$, then the

matrix A^{31} is equal to:

- (1) A^3
- (2) A
- (3) A^2
- (4) I_3

3. If $A = \begin{pmatrix} 2 & 2 \\ 9 & 4 \end{pmatrix}$ and $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, then $10A^{-1}$ is

equal to

- (1) $4I - A$
- (2) $A - 6I$
- (3) $6I - A$
- (4) $A - 4I$

4. The number of all 3×3 matrices A , with entries from the set $\{-1, 0, 1\}$ such that the sum of the diagonal elements of AA^T is 3, is

5. If the matrices $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 3 & 4 \\ 1 & -1 & 3 \end{bmatrix}$, $B = \text{adj}A$ and

$C = 3A$, then $\frac{|\text{adj}B|}{|C|}$ is equal to :

- (1) 72
- (2) 2
- (3) 8
- (4) 16

6. Let A be a 2×2 real matrix with entries from $\{0, 1\}$ and $|A| \neq 0$. Consider the following two statements:

(P) If $A \neq I_2$, then $|A| = -1$

(Q) If $|A| = 1$, then $\text{tr}(A) = 2$,

where I_2 denotes 2×2 identity matrix and $\text{tr}(A)$ denotes the sum of the diagonal entries of A .

Then:

- (1) (P) is true and (Q) is false
- (2) Both (P) and (Q) are false
- (3) Both (P) and (Q) are true
- (4) (P) is false and (Q) is true

7. Let $a, b, c \in \mathbb{R}$ be all non-zero and satisfy

$$a^3 + b^3 + c^3 = 2. \text{ If the matrix } A = \begin{pmatrix} a & b & c \\ b & c & a \\ c & a & b \end{pmatrix}$$

satisfies $A^T A = I$, then a value of abc can be:

- (1) $\frac{2}{3}$
- (2) $-\frac{1}{3}$
- (3) 3
- (4) $\frac{1}{3}$

8. Let $A = \{X = (x, y, z)^T : PX = 0 \text{ and}$

$$x^2 + y^2 + z^2 = 1\}$$
 where $P = \begin{bmatrix} 1 & 2 & 1 \\ -2 & 3 & -4 \\ 1 & 9 & -1 \end{bmatrix}$,

then the set A :

- (1) is a singleton
- (2) contains exactly two elements
- (3) contains more than two elements
- (4) is an empty set

9. Let $A = \begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}$, $x \in \mathbb{R}$ and $A^4 = [a_{ij}]$.

If $a_{11} = 109$, then a_{22} is equal to _____ .

10. Let A be a 3×3 matrix such that adj

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 0 & 2 \\ 1 & -2 & -1 \end{bmatrix} \text{ and } B = \text{adj}(\text{adj} A).$$

If $|A| = \lambda$ and $|(B^{-1})^T| = \mu$, then the ordered pair, $(|\lambda|, \mu)$ is equal to :

- (1) $(9, \frac{1}{9})$
- (2) $(9, \frac{1}{81})$
- (3) $(3, \frac{1}{81})$
- (4) (3, 81)

11. Suppose the vectors x_1 , x_2 and x_3 are the solutions of the system of linear equations, $Ax = b$ when the vector b on the right side is equal to b_1 , b_2 and b_3 respectively. If

$$x = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, x_2 = \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}, x_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, b_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix},$$

$$b_2 = \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix} \text{ and } b_3 = \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}, \text{ then the determinant of}$$

A is equal to :-

- (1) $\frac{1}{2}$ (2) 4
 (3) $\frac{3}{2}$ (4) 2
12. Let $\theta = \frac{\pi}{5}$ and $A = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$.

If $B = A + A^4$, then $\det(B)$:

- (1) is one (2) lies in (1, 2)
 (3) is zero (4) lies in (2, 3)

VECTORS

1. Let \vec{a} , \vec{b} and \vec{c} be three units vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$. If $\lambda = \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$ and $\vec{d} = \vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}$, then the ordered pair, (λ, \vec{d}) is equal to :

- (1) $\left(-\frac{3}{2}, 3\vec{a} \times \vec{b}\right)$ (2) $\left(-\frac{3}{2}, 3\vec{c} \times \vec{b}\right)$
 (3) $\left(\frac{3}{2}, 3\vec{b} \times \vec{c}\right)$ (4) $\left(\frac{3}{2}, 3\vec{a} \times \vec{c}\right)$

2. A vector $\vec{a} = \alpha\hat{i} + 2\hat{j} + \beta\hat{k}$ ($\alpha, \beta \in \mathbb{R}$) lies in the plane of the vectors $\vec{b} = \hat{i} + \hat{j}$ and $\vec{c} = \hat{i} - \hat{j} + 4\hat{k}$. If \vec{a} bisects the angle between \vec{b} and \vec{c} , then:

- (1) $\vec{a} \cdot \hat{i} + 1 = 0$ (2) $\vec{a} \cdot \hat{i} + 3 = 0$
 (3) $\vec{a} \cdot \hat{k} + 4 = 0$ (4) $\vec{a} \cdot \hat{k} + 2 = 0$

3. Let $\vec{a} = \hat{i} - 2\hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$ be two vectors. If \vec{c} is a vector such that $\vec{b} \times \vec{c} = \vec{b} \times \vec{a}$ and $\vec{c} \cdot \vec{a} = 0$, then $\vec{c} \cdot \vec{b}$ is equal to

(1) $\frac{1}{2}$ (2) -1

(3) $-\frac{1}{2}$ (4) $-\frac{3}{2}$

4. Let the volume of a parallelepiped whose coterminous edges are given by $\vec{u} = \hat{i} + \hat{j} + \lambda\hat{k}$, $\vec{v} = \hat{i} + \hat{j} + 3\hat{k}$ and $\vec{w} = 2\hat{i} + \hat{j} + \hat{k}$ be 1 cu. unit. If θ be the angle between the edges \vec{u} and \vec{w} , then $\cos\theta$ can be

(1) $\frac{7}{6\sqrt{3}}$ (2) $\frac{5}{7}$

(3) $\frac{7}{6\sqrt{6}}$ (4) $\frac{5}{3\sqrt{3}}$

5. Let \vec{a} , \vec{b} and \vec{c} be three vectors such that $|\vec{a}| = \sqrt{3}$, $|\vec{b}| = 5$, $\vec{b} \cdot \vec{c} = 10$ and the angle between \vec{b} and \vec{c} is $\frac{\pi}{3}$. If \vec{a} is perpendicular to the vector $\vec{b} \times \vec{c}$,

then $|\vec{a} \times (\vec{b} \times \vec{c})|$ is equal to _____.

6. If the vectors, $\vec{p} = (a+1)\hat{i} + a\hat{j} + a\hat{k}$, $\vec{q} = a\hat{i} + (a+1)\hat{j} + a\hat{k}$ and $\vec{r} = a\hat{i} + a\hat{j} + (a+1)\hat{k}$ ($a \in \mathbb{R}$) are coplanar and $3(\vec{p} \cdot \vec{q})^2 - \lambda|\vec{r} \times \vec{q}|^2 = 0$, then the value of λ is _____.

7. The projection of the line segment joining the points $(1, -1, 3)$ and $(2, -4, 11)$ on the line joining the points $(-1, 2, 3)$ and $(3, -2, 10)$ is _____.

8. Let \vec{a} , \vec{b} and \vec{c} be three unit vectors such that $|\vec{a} - \vec{b}|^2 + |\vec{a} - \vec{c}|^2 = 8$. Then $|\vec{a} + 2\vec{b}|^2 + |\vec{a} + 2\vec{c}|^2$ is equal to _____.

9. Let the position vectors of points 'A' and 'B' be $\hat{i} + \hat{j} + \hat{k}$ and $2\hat{i} + \hat{j} + 3\hat{k}$, respectively. A point 'P' divides the line segment AB internally in the ratio $\lambda : 1$ ($\lambda > 0$). If O is the origin and $\overline{OB} \cdot \overline{OP} - 3|\overline{OA} \times \overline{OP}|^2 = 6$, then λ is equal to _____.

10. The lines $\vec{r} = (\hat{i} - \hat{j}) + \ell(2\hat{i} + \hat{k})$ and $\vec{r} = (2\hat{i} - \hat{j}) + m(\hat{i} + \hat{j} - \hat{k})$

(1) Intersect when $\ell = 1$ and $m = 2$

(2) Intersect when $\ell = 2$ and $m = \frac{1}{2}$

(3) Do not intersect for any values of ℓ and m

(4) Intersect for all values of ℓ and m

11. Let a plane P contain two lines $\vec{r} = \hat{i} + \lambda(\hat{i} + \hat{j})$, $\lambda \in \mathbb{R}$ and $\vec{r} = -\hat{j} + \mu(\hat{j} - \hat{k})$, $\mu \in \mathbb{R}$. If $Q(\alpha, \beta, \gamma)$ is the foot of the perpendicular drawn from the point $M(1, 0, 1)$ to P, then $3(\alpha + \beta + \gamma)$ equals _____.

12. Let x_0 be the point of local maxima of $f(x) = \vec{a} \cdot (\vec{b} \times \vec{c})$, where $\vec{a} = x\hat{i} - 2\hat{j} + 3\hat{k}$, $\vec{b} = -2\hat{i} + x\hat{j} - \hat{k}$ and $\vec{c} = 7\hat{i} - 2\hat{j} + x\hat{k}$. Then the value of $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$ at $x = x_0$ is :

- (1) -30 (2) 14
(3) -4 (4) -22

13. If $\vec{a} = 2\hat{i} + \hat{j} + 2\hat{k}$, then the value of $|\hat{i} \times (\vec{a} \times \hat{i})|^2 + |\hat{j} \times (\vec{a} \times \hat{j})|^2 + |\hat{k} \times (\vec{a} \times \hat{k})|^2$ is equal to _____.

14. If the volume of a parallelepiped, whose coterminal edges are given by the vectors $\vec{a} = \hat{i} + \hat{j} + n\hat{k}$, $\vec{b} = 2\hat{i} + 4\hat{j} - n\hat{k}$ and $\vec{c} = \hat{i} + n\hat{j} + 3\hat{k}$ ($n \geq 0$), is 158 cu. units, then :

- (1) $\vec{a} \cdot \vec{c} = 17$ (2) $\vec{b} \cdot \vec{c} = 10$
(3) $n = 7$ (4) $n = 9$

15. Let the vectors $\vec{a}, \vec{b}, \vec{c}$ be such that $|\vec{a}| = 2, |\vec{b}| = 4$ and $|\vec{c}| = 4$. If the projection of \vec{b} on \vec{a} is equal to the projection of \vec{c} on \vec{a} and \vec{b} is perpendicular to \vec{c} , then the value of $|\vec{a} + \vec{b} - \vec{c}|$ is _____.

16. If \vec{a} and \vec{b} are unit vectors, then the greatest value of $\sqrt{3}|\vec{a} + \vec{b}| + |\vec{a} - \vec{b}|$ is _____.

17. If \vec{x} and \vec{y} be two non-zero vectors such that $|\vec{x} + \vec{y}| = |\vec{x}|$ and $2\vec{x} + \lambda\vec{y}$ is perpendicular to \vec{y} , then the value of λ is _____.

3D

1. If the foot of the perpendicular drawn from the point $(1, 0, 3)$ on a line passing through $(\alpha, 7, 1)$ is $(\frac{5}{3}, \frac{7}{3}, \frac{17}{3})$, then α is equal to _____.

2. Let P be a plane passing through the points $(2, 1, 0), (4, 1, 1)$ and $(5, 0, 1)$ and R be any point $(2, 1, 6)$. Then the image of R in the plane P is :

(1) $(6, 5, -2)$ (2) $(4, 3, 2)$
(3) $(3, 4, -2)$ (4) $(6, 5, 2)$

3. The mirror image of the point $(1, 2, 3)$ in a plane is $(-\frac{7}{3}, -\frac{4}{3}, -\frac{1}{3})$. Which of the following points lies on this plane ?

- (1) $(-1, -1, -1)$ (2) $(-1, -1, 1)$
(3) $(1, 1, 1)$ (4) $(1, -1, 1)$

4. The shortest distance between the lines $\frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1}$ and $\frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}$ is

- (1) $\frac{7}{2}\sqrt{30}$ (2) $3\sqrt{30}$
(3) 3 (4) $2\sqrt{30}$

5. If the distance between the plane, $23x - 10y - 2z + 48 = 0$ and the plane containing the lines $\frac{x+1}{2} = \frac{y-3}{4} = \frac{z+1}{3}$ and $\frac{x+3}{2} = \frac{y+2}{6} = \frac{z-1}{\lambda}$ ($\lambda \in \mathbb{R}$) is equal to $\frac{k}{\sqrt{633}}$, then k is equal to _____.
6. If for some α and β in \mathbb{R} , the intersection of the following three planes
 $x + 4y - 2z = 1$
 $x + 7y - 5z = \beta$
 $x + 5y + \alpha z = 5$
 is a line in \mathbb{R}^3 , then $\alpha + \beta$ is equal to :
 (1) 10 (2) -10
 (3) 2 (4) 0
7. The plane passing through the points $(1, 2, 1)$, $(2, 1, 2)$ and parallel to the line, $2x = 3y$, $z = 1$ also passes through the point :
 (1) $(0, 6, -2)$ (2) $(-2, 0, 1)$
 (3) $(0, -6, 2)$ (4) $(2, 0, -1)$
8. A plane passing through the point $(3, 1, 1)$ contains two lines whose direction ratios are $1, -2, 2$ and $2, 3, -1$ respectively. If this plane also passes through the point $(\alpha, -3, 5)$, then α is equal to:
 (1) -10 (2) 5
 (3) 10 (4) -5
9. The foot of the perpendicular drawn from the point $(4, 2, 3)$ to the line joining the points $(1, -2, 3)$ and $(1, 1, 0)$ lies on the plane :
 (1) $x + 2y - z = 1$ (2) $x - 2y + z = 1$
 (3) $x - y - 2z = 1$ (4) $2x + y - z = 1$
10. The plane which bisects the line joining the points $(4, -2, 3)$ and $(2, 4, -1)$ at right angles also passes through the point :
 (1) $(4, 0, -1)$ (2) $(4, 0, 1)$
 (3) $(0, 1, -1)$ (4) $(0, -1, 1)$
11. If the equation of a plane P, passing through the intersection of the planes, $x + 4y - z + 7 = 0$ and $3x + y + 5z = 8$ is $ax + by + 6z = 15$ for some $a, b \in \mathbb{R}$, then the distance of the point $(3, 2, -1)$ from the plane P is _____.
12. The distance of the point $(1, -2, 3)$ from the plane $x - y + z = 5$ measured parallel to the line $\frac{x}{2} = \frac{y}{3} = \frac{z}{-6}$ is:
 (1) 7 (2) 1
 (3) $\frac{1}{7}$ (4) $\frac{7}{5}$
13. If (a, b, c) is the image of the point $(1, 2, -3)$ in the line, $\frac{x+1}{2} = \frac{y-3}{-2} = \frac{z}{-1}$, then $a + b + c$ is equal to
 (1) -1 (2) 2
 (3) 3 (4) 1
14. If for some $\alpha \in \mathbb{R}$, the lines
 $L_1 : \frac{x+1}{2} = \frac{y-2}{-1} = \frac{z-1}{1}$ and
 $L_2 : \frac{x+2}{\alpha} = \frac{y+1}{5-\alpha} = \frac{z+1}{1}$ are coplanar, then the line L_2 passes through the point :
 (1) $(-2, 10, 2)$ (2) $(10, 2, 2)$
 (3) $(10, -2, -2)$ (4) $(2, -10, -2)$
15. The shortest distance between the lines $\frac{x-1}{0} = \frac{y+1}{-1} = \frac{z}{1}$ and $x + y + z + 1 = 0$, $2x - y + z + 3 = 0$ is :
 (1) $\frac{1}{2}$ (2) 1
 (3) $\frac{1}{\sqrt{2}}$ (4) $\frac{1}{\sqrt{3}}$
16. A plane P meets the coordinate axes at A, B and C respectively. The centroid of ΔABC is given to be $(1, 1, 2)$. Then the equation of the line through this centroid and perpendicular to the plane P is:
 (1) $\frac{x-1}{1} = \frac{y-1}{2} = \frac{z-2}{2}$
 (2) $\frac{x-1}{2} = \frac{y-1}{2} = \frac{z-2}{1}$
 (3) $\frac{x-1}{2} = \frac{y-1}{1} = \frac{z-2}{1}$
 (4) $\frac{x-1}{1} = \frac{y-1}{1} = \frac{z-2}{2}$

PARABOLA

1. If $y = mx + 4$ is a tangent to both the parabolas, $y^2 = 4x$ and $x^2 = 2by$, then b is equal to :
 - (1) 128
 - (2) -64
 - (3) -128
 - (4) -32
2. Let a line $y = mx$ ($m > 0$) intersect the parabola, $y^2 = x$ at a point P , other than the origin. Let the tangent to it at P meet the x -axis at the point Q . If area (ΔOPQ) = 4 sq. units, then m is equal to _____.
3. The locus of a point which divides the line segment joining the point $(0, -1)$ and a point on the parabola, $x^2 = 4y$, internally in the ratio $1 : 2$ is-
 - (1) $9x^2 - 3y = 2$
 - (2) $9x^2 - 12y = 8$
 - (3) $x^2 - 3y = 2$
 - (4) $4x^2 - 3y = 2$
4. If one end of a focal chord AB of the parabola $y^2 = 8x$ is at $A\left(\frac{1}{2}, -2\right)$, then the equation of the tangent to it at B is :
 - (1) $2x + y - 24 = 0$
 - (2) $x - 2y + 8 = 0$
 - (3) $2x - y - 24 = 0$
 - (4) $x + 2y + 8 = 0$
5. The area (in sq. units) of an equilateral triangle inscribed in the parabola $y^2 = 8x$, with one of its vertices on the vertex of this parabola, is :
 - (1) $64\sqrt{3}$
 - (2) $256\sqrt{3}$
 - (3) $192\sqrt{3}$
 - (4) $128\sqrt{3}$
6. Let P be a point on the parabola, $y^2 = 12x$ and N be the foot of the perpendicular drawn from P on the axis of the parabola. A line is now drawn through the mid-point M of PN , parallel to its axis which meets the parabola at Q . If the y -intercept of the line NQ is $\frac{4}{3}$, then :
 - (1) $MQ = \frac{1}{3}$
 - (2) $PN = 3$
 - (3) $MQ = \frac{1}{4}$
 - (4) $PN = 4$
7. Let the latus rectum of the parabola $y^2 = 4x$ be the common chord to the circles C_1 and C_2 each of them having radius $2\sqrt{5}$. Then, the distance between the centres of the circles C_1 and C_2 is :
 - (1) 8
 - (2) $4\sqrt{5}$
 - (3) 12
 - (4) $8\sqrt{5}$
8. The area (in sq. units) of the largest rectangle $ABCD$ whose vertices A and B lie on the x -axis and vertices C and D lie on the parabola, $y = x^2 - 1$ below the x -axis, is :
 - (1) $\frac{4}{3\sqrt{3}}$
 - (2) $\frac{1}{3\sqrt{3}}$
 - (3) $\frac{4}{3}$
 - (4) $\frac{2}{3\sqrt{3}}$
9. If the common tangent to the parabolas, $y^2 = 4x$ and $x^2 = 4y$ also touches the circle, $x^2 + y^2 = c^2$, then c is equal to :
 - (1) $\frac{1}{2}$
 - (2) $\frac{1}{2\sqrt{2}}$
 - (3) $\frac{1}{\sqrt{2}}$
 - (4) $\frac{1}{4}$
10. Let L_1 be a tangent to the parabola $y^2 = 4(x + 1)$ and L_2 be a tangent to the parabola $y^2 = 8(x + 2)$ such that L_1 and L_2 intersect at right angles. Then L_1 and L_2 meet on the straight line :
 - (1) $x + 3 = 0$
 - (2) $x + 2y = 0$
 - (3) $2x + 1 = 0$
 - (4) $x + 2 = 0$
11. The centre of the circle passing through the point $(0, 1)$ and touching the parabola $y = x^2$ at the point $(2, 4)$ is :
 - (1) $\left(\frac{3}{10}, \frac{16}{5}\right)$
 - (2) $\left(\frac{-16}{5}, \frac{53}{10}\right)$
 - (3) $\left(\frac{6}{5}, \frac{53}{10}\right)$
 - (4) $\left(\frac{-53}{10}, \frac{16}{5}\right)$

ELLIPSE

1. If $3x + 4y = 12\sqrt{2}$ is a tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{9} = 1$ for some $a \in \mathbb{R}$, then the distance between the foci of the ellipse is :
- (1) 4 (2) $2\sqrt{7}$
(3) $2\sqrt{5}$ (4) $2\sqrt{2}$
2. If the distance between the foci of an ellipse is 6 and the distance between its directrices is 12, then the length of its latus rectum is :
- (1) $\sqrt{3}$ (2) $2\sqrt{3}$
(3) $3\sqrt{2}$ (4) $\frac{3}{\sqrt{2}}$
3. Let the line $y = mx$ and the ellipse $2x^2 + y^2 = 1$ intersect at a point P in the first quadrant. If the normal to this ellipse at P meets the co-ordinate axes at $(-\frac{1}{3\sqrt{2}}, 0)$ and $(0, \beta)$, then β is equal to
- (1) $\frac{2}{\sqrt{3}}$ (2) $\frac{2\sqrt{2}}{3}$
(3) $\frac{2}{3}$ (4) $\frac{\sqrt{2}}{3}$
4. The length of the minor axis (along y-axis) of an ellipse in the standard form is $\frac{4}{\sqrt{3}}$. If this ellipse touches the line, $x + 6y = 8$; then its eccentricity is :
- (1) $\sqrt{\frac{5}{6}}$ (2) $\frac{1}{2}\sqrt{\frac{11}{3}}$
(3) $\frac{1}{3}\sqrt{\frac{11}{3}}$ (4) $\frac{1}{2}\sqrt{\frac{5}{3}}$
5. Let e_1 and e_2 be the eccentricities of the ellipse, $\frac{x^2}{25} + \frac{y^2}{b^2} = 1 (b < 5)$ and the hyperbola, $\frac{x^2}{16} - \frac{y^2}{b^2} = 1$ respectively satisfying $e_1 e_2 = 1$. If α and β are the distances between the foci of the ellipse and the foci of the hyperbola respectively, then the ordered pair (α, β) is equal to :
- (1) (8, 10) (2) (8, 12)
(3) $(\frac{20}{3}, 12)$ (4) $(\frac{24}{5}, 10)$
6. Let $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 (a > b)$ be a given ellipse, length of whose latus rectum is 10. If its eccentricity is the maximum value of the function, $\phi(t) = \frac{5}{12} + t - t^2$, then $a^2 + b^2$ is equal to :
- (1) 126 (2) 135
(3) 145 (4) 116
7. Let $x = 4$ be a directrix to an ellipse whose centre is at the origin and its eccentricity is $\frac{1}{2}$. If $P(1, \beta)$, $\beta > 0$ is a point on this ellipse, then the equation of the normal to it at P is :-
- (1) $7x - 4y = 1$ (2) $4x - 2y = 1$
(3) $4x - 3y = 2$ (4) $8x - 2y = 5$
8. If the point P on the curve, $4x^2 + 5y^2 = 20$ is farthest from the point $Q(0, -4)$, then PQ^2 is equal to:
- (1) 21 (2) 36
(3) 48 (4) 29
9. If the co-ordinates of two points A and B are $(\sqrt{7}, 0)$ and $(-\sqrt{7}, 0)$ respectively and P is any point on the conic, $9x^2 + 16y^2 = 144$, then $PA + PB$ is equal to :
- (1) 8 (2) 6
(3) 16 (4) 9

10. Which of the following points lies on the locus of the foot of perpendicular drawn upon any tangent to the ellipse, $\frac{x^2}{4} + \frac{y^2}{2} = 1$ from any of its foci ?
- (1) $(-1, \sqrt{3})$ (2) $(-1, \sqrt{2})$
 (3) $(-2, \sqrt{3})$ (4) $(1, 2)$
11. If the normal at an end of a latus rectum of an ellipse passes through an extremity of the minor axis, then the eccentricity e of the ellipse satisfies :
- (1) $e^2 + 2e - 1 = 0$ (2) $e^2 + e - 1 = 0$
 (3) $e^4 + 2e^2 - 1 = 0$ (4) $e^4 + e^2 - 1 = 0$

HYPERBOLA

1. If a hyperbola passes through the point $P(10, 16)$ and it has vertices at $(\pm 6, 0)$, then the equation of the normal to it at P is
- (1) $x + 2y = 42$ (2) $3x + 4y = 94$
 (3) $2x + 5y = 100$ (4) $x + 3y = 58$
2. If e_1 and e_2 are the eccentricities of the ellipse, $\frac{x^2}{18} + \frac{y^2}{4} = 1$ and the hyperbola, $\frac{x^2}{9} - \frac{y^2}{4} = 1$ respectively and (e_1, e_2) is a point on the ellipse, $15x^2 + 3y^2 = k$, then k is equal to :
- (1) 15 (2) 14
 (3) 17 (4) 16
3. A line parallel to the straight line $2x - y = 0$ is tangent to the hyperbola $\frac{x^2}{4} - \frac{y^2}{2} = 1$ at the point (x_1, y_1) . Then $x_1^2 + 5y_1^2$ is equal to :
- (1) 5 (2) 6
 (3) 8 (4) 10

4. For some $\theta \in \left(0, \frac{\pi}{2}\right)$, if the eccentricity of the hyperbola, $x^2 - y^2 \sec^2 \theta = 10$ is $\sqrt{5}$ times the eccentricity of the ellipse, $x^2 \sec^2 \theta + y^2 = 5$, then the length of the latus rectum of the ellipse, is :
- (1) $\sqrt{30}$ (2) $\frac{4\sqrt{5}}{3}$
 (3) $2\sqrt{6}$ (4) $\frac{2\sqrt{5}}{3}$
5. A hyperbola having the transverse axis of length $\sqrt{2}$ has the same foci as that of the ellipse $3x^2 + 4y^2 = 12$, then this hyperbola does not pass through which of the following points ?
- (1) $\left(1, -\frac{1}{\sqrt{2}}\right)$ (2) $\left(\frac{\sqrt{3}}{2}, \frac{1}{\sqrt{2}}\right)$
 (3) $\left(\frac{1}{\sqrt{2}}, 0\right)$ (4) $\left(-\sqrt{\frac{3}{2}}, 1\right)$
6. Let $P(3, 3)$ be a point on the hyperbola, $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. If the normal to it at P intersects the x -axis at $(9, 0)$ and e is its eccentricity, then the ordered pair (a^2, e^2) is equal to :
- (1) $\left(\frac{9}{2}, 3\right)$ (2) $\left(\frac{9}{2}, 2\right)$
 (3) $\left(\frac{3}{2}, 2\right)$ (4) $(9, 3)$
7. If the line $y = mx + c$ is a common tangent to the hyperbola $\frac{x^2}{100} - \frac{y^2}{64} = 1$ and the circle $x^2 + y^2 = 36$, then which one of the following is true?
- (1) $5m = 4$ (2) $4c^2 = 369$
 (3) $c^2 = 369$ (4) $8m + 5 = 0$

COMPLEX NUMBER

1. If $\frac{3+i\sin\theta}{4-i\cos\theta}$, $\theta \in [0, 2\pi]$, is a real number, then an argument of $\sin\theta + i\cos\theta$ is :
- (1) $-\tan^{-1}\left(\frac{3}{4}\right)$ (2) $\tan^{-1}\left(\frac{4}{3}\right)$
 (3) $\pi - \tan^{-1}\left(\frac{4}{3}\right)$ (4) $\pi - \tan^{-1}\left(\frac{3}{4}\right)$
2. If $\operatorname{Re}\left(\frac{z-1}{2z+i}\right) = 1$, where $z = x + iy$, then the point (x, y) lies on a :
- (1) circle whose centre is at $\left(-\frac{1}{2}, -\frac{3}{2}\right)$
 (2) circle whose diameter is $\frac{\sqrt{5}}{2}$
 (3) straight line whose slope is $\frac{3}{2}$
 (4) straight line whose slope is $-\frac{2}{3}$
3. Let $\alpha = \frac{-1+i\sqrt{3}}{2}$. If $a = (1+\alpha) \sum_{k=0}^{100} \alpha^{2k}$ and $b = \sum_{k=0}^{100} \alpha^{3k}$, then a and b are the roots of the quadratic equation :
- (1) $x^2 - 102x + 101 = 0$
 (2) $x^2 + 101x + 100 = 0$
 (3) $x^2 - 101x + 100 = 0$
 (4) $x^2 + 102x + 101 = 0$
4. If the equation, $x^2 + bx + 45 = 0$ ($b \in \mathbb{R}$) has conjugate complex roots and they satisfy $|z+1| = 2\sqrt{10}$, then
- (1) $b^2 - b = 42$ (2) $b^2 + b = 12$
 (3) $b^2 + b = 72$ (4) $b^2 - b = 30$
5. If z be a complex number satisfying $|\operatorname{Re}(z)| + |\operatorname{Im}(z)| = 4$, then $|z|$ cannot be
- (1) $\sqrt{\frac{17}{2}}$ (2) $\sqrt{10}$
 (3) $\sqrt{8}$ (4) $\sqrt{7}$
6. Let z be complex number such that $\left|\frac{z-i}{z+2i}\right| = 1$ and $|z| = \frac{5}{2}$. Then the value of $|z+3i|$ is :
- (1) $\sqrt{10}$ (2) $2\sqrt{3}$
 (3) $\frac{7}{2}$ (4) $\frac{15}{4}$
7. The value of $\left(\frac{1+\sin\frac{2\pi}{9}+i\cos\frac{2\pi}{9}}{1+\sin\frac{2\pi}{9}-i\cos\frac{2\pi}{9}}\right)^3$ is :
- (1) $\frac{1}{2}(\sqrt{3}-i)$ (2) $-\frac{1}{2}(\sqrt{3}-i)$
 (3) $-\frac{1}{2}(1-i\sqrt{3})$ (4) $\frac{1}{2}(1-i\sqrt{3})$
8. The imaginary part of $(3+2\sqrt{-54})^{1/2} z - (3-2\sqrt{-54})^{1/2}$ can be :
- (1) $-2\sqrt{6}$ (2) 6
 (3) $\sqrt{6}$ (4) $-\sqrt{6}$
9. If $\left(\frac{1+i}{1-i}\right)^m = \left(\frac{1+i}{i-1}\right)^n = 1$, ($m, n \in \mathbb{N}$) then the greatest common divisor of the least values of m and n is _____ .
10. If z_1, z_2 are complex numbers such that $\operatorname{Re}(z_1) = |z_1 - 1|$, $\operatorname{Re}(z_2) = |z_2 - 1|$ and $\arg(z_1 - z_2) = \frac{\pi}{6}$, then $\operatorname{Im}(z_1 + z_2)$ is equal to :
- (1) $\frac{\sqrt{3}}{2}$ (2) $\frac{2}{\sqrt{3}}$
 (3) $\frac{1}{\sqrt{3}}$ (4) $2\sqrt{3}$

11. If $A = \begin{bmatrix} \cos \theta & i \sin \theta \\ i \sin \theta & \cos \theta \end{bmatrix}$, $\left(\theta = \frac{\pi}{24}\right)$ and $A^5 = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, where $i = \sqrt{-1}$, then which one of the following is not true?
 (1) $0 \leq a^2 + b^2 \leq 1$ (2) $a^2 - d^2 = 0$
 (3) $a^2 - b^2 = \frac{1}{2}$ (4) $a^2 - c^2 = 1$

12. Let $u = \frac{2z+i}{z-ki}$, $z = x + iy$ and $k > 0$. If the curve represented by $\text{Re}(u) + \text{Im}(u) = 1$ intersects the y-axis at the points P and Q where $PQ = 5$, then the value of k is :

- (1) $3/2$ (2) 4
 (3) 2 (4) $1/2$

13. If a and b are real numbers such that $(2 + \alpha)^4 = a + b\alpha$, where $\alpha = \frac{-1+i\sqrt{3}}{2}$, then

- a + b is equal to:
 (1) 57 (2) 33
 (3) 24 (4) 9

14. If the four complex numbers $z, \bar{z}, \bar{z} - 2\text{Re}(\bar{z})$ and $z - 2\text{Re}(z)$ represent the vertices of a square of side 4 units in the Argand plane, then $|z|$ is equal to :

- (1) 4 (2) 2
 (3) $4\sqrt{2}$ (4) $2\sqrt{2}$

15. The value of $\left(\frac{-1+i\sqrt{3}}{1-i}\right)^{30}$ is :

- (1) $2^{15}i$ (2) -2^{15}
 (3) $-2^{15}i$ (4) 6^5

16. The region represented by $\{z = x + iy \in \mathbb{C} : |z| - \text{Re}(z) \leq 1\}$ is also given by the inequality:

- (1) $y^2 \geq x + 1$ (2) $y^2 \geq 2(x + 1)$
 (3) $y^2 \leq x + \frac{1}{2}$ (4) $y^2 \leq 2\left(x + \frac{1}{2}\right)$

17. Let $z = x + iy$ be a non-zero complex number such that $z^2 = i|z|^2$, where $i = \sqrt{-1}$, then z lies on the:
 (1) imaginary axis
 (2) real axis
 (3) line, $y = x$
 (4) line, $y = -x$

PROBABILITY

1. In a workshop, there are five machines and the probability of any one of them to be out of service on a day is $\frac{1}{4}$. If the probability that at most two machines will be out of service on the same day is $\left(\frac{3}{4}\right)^3 k$, then k is equal to :

- (1) $\frac{17}{2}$ (2) 4

- (3) $\frac{17}{8}$ (4) $\frac{17}{4}$

2. An unbiased coin is tossed 5 times. Suppose that a variable X is assigned the value k when k consecutive heads are obtained for $k = 3, 4, 5$ otherwise X takes the value -1 . Then the expected value of X, is :

- (1) $\frac{3}{16}$ (2) $-\frac{3}{16}$

- (3) $\frac{1}{8}$ (4) $-\frac{1}{8}$

3. Let A and B be two events such that the probability that exactly one of them occurs is $\frac{2}{5}$ and the probability that A or B occurs is $\frac{1}{2}$, then the probability of both of them occur together is

- (1) 0.02 (2) 0.01
 (3) 0.20 (4) 0.10

4. Let A and B be two independent events such that $P(A) = \frac{1}{3}$ and $P(B) = \frac{1}{6}$. Then, which of the following is TRUE ?
- (1) $P(A/B) = \frac{2}{3}$
- (2) $P(A/(A \cup B)) = \frac{1}{4}$
- (3) $P(A/B') = \frac{1}{3}$
- (4) $P(A'/B') = \frac{1}{3}$
5. A random variable X has the following probability distribution :
- | | | | | | |
|--------|-------|------|-----|------|--------|
| X : | 1 | 2 | 3 | 4 | 5 |
| P(X) : | K^2 | $2K$ | K | $2K$ | $5K^2$ |
- Then $P(X > 2)$ is equal to :
- (1) $\frac{7}{12}$ (2) $\frac{23}{36}$
- (3) $\frac{1}{36}$ (4) $\frac{1}{6}$
6. If 10 different balls are to be placed in 4 distinct boxes at random, then the probability that two of these boxes contain exactly 2 and 3 balls is :
- (1) $\frac{945}{2^{11}}$ (2) $\frac{965}{2^{11}}$
- (3) $\frac{945}{2^{10}}$ (4) $\frac{965}{2^{10}}$
7. In a box, there are 20 cards, out of which 10 are labelled as A and the remaining 10 are labelled as B. Cards are drawn at random, one after the other and with replacement, till a second A-card is obtained. The probability that the second A-card appears before the third B-card is :
- (1) $\frac{11}{16}$ (2) $\frac{13}{16}$
- (3) $\frac{9}{16}$ (4) $\frac{15}{16}$
8. Box I contains 30 cards numbered 1 to 30 and Box II contains 20 cards numbered 31 to 50. A box is selected at random and a card is drawn from it. The number on the card is found to be a non-prime number. The probability that the card was drawn from Box I is :
- (1) $\frac{8}{17}$ (2) $\frac{2}{3}$
- (3) $\frac{4}{17}$ (4) $\frac{2}{5}$
9. Let E^c denote the complement of an event E. Let E_1, E_2 and E_3 be any pairwise independent events with $P(E_1) > 0$ and $P(E_1 \cap E_2 \cap E_3) = 0$. Then $P(E_2^c \cap E_3^c / E_1)$ is equal to :
- (1) $P(E_3^c) - P(E_2)$ (2) $P(E_2^c) + P(E_3)$
- (3) $P(E_3^c) - P(E_2^c)$ (4) $P(E_3) - P(E_2^c)$
10. A die is thrown two times and the sum of the scores appearing on the die is observed to be a multiple of 4. Then the conditional probability that the score 4 has appeared atleast once is :
- (1) $\frac{1}{8}$ (2) $\frac{1}{9}$
- (3) $\frac{1}{3}$ (4) $\frac{1}{4}$
11. The probability that a randomly chosen 5-digit number is made from exactly two digits is :
- (1) $\frac{121}{10^4}$ (2) $\frac{150}{10^4}$
- (3) $\frac{135}{10^4}$ (4) $\frac{134}{10^4}$
12. The probability of a man hitting a target is $\frac{1}{10}$. The least number of shots required, so that the probability of his hitting the target at least once is greater than $\frac{1}{4}$, is _____.

13. In a game two players A and B take turns in throwing a pair of fair dice starting with player A and total of scores on the two dice, in each throw is noted. A wins the game if he throws a total of 6 before B throws a total of 7 and B wins the game if he throws a total of 7 before A throws a total of six. The game stops as soon as either of the players wins. The probability of A winning the game is :

- (1) $\frac{31}{61}$ (2) $\frac{5}{6}$
 (3) $\frac{5}{31}$ (4) $\frac{30}{61}$

14. Four fair dice are thrown independently 27 times. Then the expected number of times, at least two dice show up a three or a five, is ____.

15. In a bombing attack, there is 50% chance that a bomb will hit the target. At least two independent hits are required to destroy the target completely. Then the minimum number of bombs, that must be dropped to ensure that there is at least 99% chance of completely destroying the target, is _____.

16. Two families with three members each and one family with four members are to be seated in a row. In how many ways can they be seated so that the same family members are not separated ?

- (1) $2!3!4!$ (2) $(3!)^3 \cdot (4!)$
 (3) $(3!)^2 \cdot (4!)$ (4) $3!(4!)^3$

17. Out of 11 consecutive natural numbers if three numbers are selected at random (without repetition), then the probability that they are in A.P. with positive common difference, is :

- (1) $\frac{15}{101}$ (2) $\frac{5}{101}$
 (3) $\frac{5}{33}$ (4) $\frac{10}{99}$

18. The probabilities of three events A, B and C are given by $P(A) = 0.6$, $P(B) = 0.4$ and $P(C) = 0.5$. If $P(A \cup B) = 0.8$, $P(A \cap C) = 0.3$, $P(A \cap B \cap C) = 0.2$, $P(B \cap C) = \beta$ and $P(A \cup B \cup C) = \alpha$, where $0.85 \leq \alpha \leq 0.95$, then β lies in the interval:

- (1) [0.36, 0.40] (2) [0.35, 0.36]
 (3) [0.25, 0.35] (4) [0.20, 0.25]

STATISTICS

1. If the mean and variance of eight numbers 3, 7, 9, 12, 13, 20, x and y be 10 and 25 respectively, then $x \cdot y$ is equal to _____

2. If the variance of the first n natural numbers is 10 and the variance of the first m even natural numbers is 16, then $m + n$ is equal to _____.

3. The mean and variance of 20 observations are found to be 10 and 4, respectively. On rechecking, it was found that an observation 9 was incorrect and the correct observation was 11. Then the correct variance is

- (1) 3.99 (2) 3.98
 (3) 4.02 (4) 4.01

4. The mean and the standard deviation (s.d.) of 10 observations are 20 and 2 respectively. Each of these 10 observations is multiplied by p and then reduced by q, where $p \neq 0$ and $q \neq 0$. If the new mean and new s.d. become half of their original values, then q is equal to

- (1) -20 (2) 10
 (3) -10 (4) -5

5. Let the observations $x_i (1 \leq i \leq 10)$ satisfy the equations, $\sum_{i=1}^{10} (x_i - 5) = 10$ and $\sum_{i=1}^{10} (x_i - 5)^2 = 40$. If μ and λ are the mean and the variance of the observations, $x_1 - 3, x_2 - 3, \dots, x_{10} - 3$, then the ordered pair (μ, λ) is equal to :

- (1) (6, 6) (2) (3, 6)
 (3) (6, 3) (4) (3, 3)

6. Let $X = \{x \in \mathbb{N} : 1 \leq x \leq 17\}$ and $Y = \{ax + b : x \in X \text{ and } a, b \in \mathbb{R}, a > 0\}$. If mean and variance of elements of Y are 17 and 216 respectively then $a + b$ is equal to :
- (1) -7 (2) 7
(3) 9 (4) -27
7. If the variance of the terms in an increasing A.P., $b_1, b_2, b_3, \dots, b_{11}$ is 90, then the common difference of this A.P. is _____.
8. For the frequency distribution :
- | | | | |
|-----------------|-------|-------|--------------------|
| Variate (x) : | x_1 | x_2 | $x_3 \dots x_{15}$ |
| Frequency (f) : | f_1 | f_2 | $f_3 \dots f_{15}$ |
- where $0 < x_1 < x_2 < x_3 < \dots < x_{15} = 10$ and $\sum_{i=1}^{15} f_i > 0$, the standard deviation cannot be :
- (1) 2 (2) 1
(3) 4 (4) 6
9. Let x_i ($1 \leq i \leq 10$) be ten observations of a random variable X . If $\sum_{i=1}^{10} (x_i - p) = 3$ and $\sum_{i=1}^{10} (x_i - p)^2 = 9$ where $0 \neq p \in \mathbb{R}$, then the standard deviation of these observations is :
- (1) $\sqrt{\frac{3}{5}}$ (2) $\frac{7}{10}$
(3) $\frac{9}{10}$ (4) $\frac{4}{5}$
10. The mean and variance of 8 observations are 10 and 13.5, respectively. If 6 of these observations are 5, 7, 10, 12, 14, 15, then the absolute difference of the remaining two observations is :
- (1) 7 (2) 3
(3) 5 (4) 9
11. If the variance of the following frequency distribution:
- | | | | |
|-------------|-------|-------|-------|
| Class : | 10-20 | 20-30 | 30-40 |
| Frequency : | 2 | x | 2 |
- is 50, then x is equal to _____
12. The mean and variance of 7 observations are 8 and 16, respectively. If five observations are 2, 4, 10, 12, 14, then the absolute difference of the remaining two observations is :
- (1) 2 (2) 4
(3) 3 (4) 1

13. If the mean and the standard deviation of the data 3, 5, 7, a , b are 5 and 2 respectively, then a and b are the roots of the equation :
- (1) $2x^2 - 20x + 19 = 0$
(2) $x^2 - 10x + 19 = 0$
(3) $x^2 - 10x + 18 = 0$
(4) $x^2 - 20x + 18 = 0$
14. If $\sum_{i=1}^n (x_i - a) = n$ and $\sum_{i=1}^n (x_i - a)^2 = na$, ($n, a > 1$) then the standard deviation of n observations x_1, x_2, \dots, x_n is
- (1) $n\sqrt{a-1}$
(2) $\sqrt{a-1}$
(3) $a-1$
(4) $\sqrt{n(a-1)}$
15. Consider the data on x taking the values 0, 2, 4, 8, ..., 2^n with frequencies ${}^n C_0, {}^n C_1, {}^n C_2, \dots, {}^n C_n$ respectively. If the mean of this data is $\frac{728}{2^n}$, then n is equal to _____.

MATHEMATICAL REASONING

1. Let A, B, C and D be four non-empty sets. The contrapositive statement of "If $A \subseteq B$ and $B \subseteq D$, then $A \subseteq C$ " is :
- (1) If $A \subseteq C$, then $B \subset A$ or $D \subset B$
(2) If $A \not\subseteq C$, then $A \not\subseteq B$ or $B \not\subseteq D$
(3) If $A \not\subseteq C$, then $A \subseteq B$ and $B \subseteq D$
(4) If $A \not\subseteq C$, then $A \not\subseteq B$ and $B \subseteq D$
2. The logical statement $(p \Rightarrow q) \wedge (q \Rightarrow \sim p)$ is equivalent to :
- (1) p
(2) q
(3) $\sim p$
(4) $\sim q$

3. Which of the following statements is a tautology?
- (1) $\sim(p \vee \sim q) \rightarrow p \vee q$
 (2) $\sim(p \wedge \sim q) \rightarrow p \vee q$
 (3) $\sim(p \vee \sim q) \rightarrow p \wedge q$
 (4) $p \vee (\sim q) \rightarrow p \wedge q$
4. Which one of the following is a tautology ?
- (1) $P \wedge (P \vee Q)$ (2) $P \vee (P \wedge Q)$
 (3) $Q \rightarrow (P \wedge (P \rightarrow Q))$ (4) $(P \wedge (P \rightarrow Q)) \rightarrow Q$
5. If $p \rightarrow (p \wedge \sim q)$ is false, then the truth values of p and q are respectively :
- (1) F, T (2) T, T
 (3) F, F (4) T, F
6. Negation of the statement :
 $\sqrt{5}$ is an integer or 5 is irrational is :
- (1) $\sqrt{5}$ is irrational or 5 is an integer.
 (2) $\sqrt{5}$ is not an integer and 5 is not irrational.
 (3) $\sqrt{5}$ is an integer and 5 is irrational.
 (4) $\sqrt{5}$ is not an integer or 5 is not irrational.
7. The contrapositive of the statement "If I reach the station in time, then I will catch the train" is :
- (1) If I will catch the train, then I reach the station in time.
 (2) If I do not reach the station in time, then I will not catch the train.
 (3) If I will not catch the train, then I do not reach the station in time.
 (4) If I do not reach the station in time, then I will catch the train.
8. Which of the following is a tautology ?
- (1) $(\sim p) \wedge (p \vee q) \rightarrow q$
 (2) $(q \rightarrow p) \vee \sim(p \rightarrow q)$
 (3) $(p \rightarrow q) \wedge (q \rightarrow p)$
 (4) $(\sim q) \vee (p \wedge q) \rightarrow q$
9. The proposition $p \rightarrow \sim(p \wedge \sim q)$ is equivalent to:
- (1) $(\sim p) \vee q$
 (2) q
 (3) $(\sim p) \wedge q$
 (4) $(\sim p) \vee (\sim q)$
10. Let p, q, r be three statements such that the truth value of $(p \wedge q) \rightarrow (\sim q \vee r)$ is F. Then the truth values of p, q, r are respectively :
- (1) T, F, T
 (2) F, T, F
 (3) T, T, F
 (4) T, T, T
11. Given the following two statements :
- (S₁) : $(q \vee p) \rightarrow (p \leftrightarrow \sim q)$ is a tautology.
 (S₂) : $\sim q \wedge (\sim p \leftrightarrow q)$ is a fallacy.
 Then :
- (1) only (S₁) is correct.
 (2) both (S₁) and (S₂) are correct.
 (3) both (S₁) and (S₂) are not correct.
 (4) only (S₂) is correct.
12. Contrapositive of the statement:
 'If a function f is differentiable at a , then it is also continuous at a ', is :-
- (1) If a function f is continuous at a , then it is not differentiable at a .
 (2) If a function f is not continuous at a , then it is differentiable at a .
 (3) If a function f is not continuous at a , then it is not differentiable at a .
 (4) If a function f is continuous at a , then it is differentiable at a .
13. The negation of the Boolean expression $x \leftrightarrow \sim y$ is equivalent to :
- (1) $(\sim x \wedge y) \vee (\sim x \wedge \sim y)$
 (2) $(x \wedge \sim y) \vee (\sim x \wedge y)$
 (3) $(x \wedge y) \vee (\sim x \wedge \sim y)$
 (4) $(x \wedge y) \wedge (\sim x \vee \sim y)$

Sequence & Progression										
Que.	1	2	3	4	5	6	7	8	9	10
Ans.	1	1	3	3	2	504	1540.00	3	4	14
Que.	11	12	13	14	15	16	17	18	19	20
Ans.	1	1	4	2	3	4	3	39	2	3
Que.	21	22	23	24	25	26	27	28		
Ans.	1	1	3	4	2	2	3	2		

Trigonometric Equation				
Que.	1	2	3	
Ans.	8.00	4	1	

Solution of Triangle	
Que.	1
Ans.	3

Height & Distance				
Que.	1	2	3	4
Ans.	4	1	80.00	1

Determinant										
Que.	1	2	3	4	5	6	7	8	9	10
Ans.	13.00	4	4	4	3	1	3	8	3	5
Que.	11	12	13	14	15	16	17	18		
Ans.	3	4	1	2	2	4	1	3.00		

Straight Line										
Que.	1	2	3	4	5	6	7	8	9	10
Ans.	3	5	3	2	4	3	4	30	2	3

Circle									
Que.	1	2	3	4	5	6	7	8	
Ans.	4	2	36	9.00	3	4	7	2	

Permutation & Combination										
Que.	1	2	3	4	5	6	7	8	9	10
Ans.	1	2454	4	490.00	1	309.00	3	3	54	135
Que.	11	12	13							
Ans.	240	4	120.00							

Binomial Theorem

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	3	2	30	3	4	51	615.00	2	118	2
Que.	11	12	13	14	15	16	17	18		
Ans.	4	2	8	3	13	120.00	1	3		

Logarithm

Que.	1	2	3	4	5	6	7			
Ans.	29.00	1	8	4	4	4	28.00			

Relation

Que.	1	2								
Ans.	2	4								

Function

Que.	1	2	3	4	5	6	7	8	9	
Ans.	2	4	2	3	1	4	19.00	2	5	

Inverse trigonometry function

Que.	1	2	3							
Ans.	1	3	4							

Limit

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	36	4	40.00	4	2	8	1	4	1	4
Que.	11									
Ans.	1									

Continuity

Que.	1	2	3	4						
Ans.	5.00	2	4	8						

Differentiability

Que.	1	2	3	4	5	6	7			
Ans.	3	4	10	1	1	5.00	1			

Method of differentiation

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	2	1	3	1	Bonus	3	91	1	2	2

Indefinite Integration

Que.	1	2	3	4	5	6	7	
Ans.	1	1	1	3	4	1	4	

Definite Integration

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	4	3	1	1	1	4	3	1.50	1.0	1
Que.	11	12	13	14	15	16	17	18		
Ans.	1	4	4	3	21	4	1	4		

Tangent & Normal

Que.	1	2	3	4	5	6	7	8	
Ans.	2	4.00	3	2	1	4	0.50	4	

Monotonicity

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	4	2	2	2	1	3	1	2	3	1
Que.	11									
Ans.	3									

Maxima & Minima

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	2	3	1	3	4	4	3	1	5.00	4

Differential Equation

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	3	4	1	2	4	3	3.00	4	2	1
Que.	11	12	13	14	15	16	17	18		
Ans.	2	1	3	2	1	2	1	1		

Logarithm

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	4	2	4	1	2	2	4	3	1	4
Que.	11									
Ans.	2									

Matrices										
Que.	1	2	3	4	5	6	7	8	9	10
Ans.	4	1	2	672.00	3	4	4	2	10	3
Que.	11	12								
Ans.	4	2								

Vectors										
Que.	1	2	3	4	5	6	7	8	9	10
Ans.	1	4	3	1	30	1.00	8.00	2.00	0.8	3
Que.	11	12	13	14	15	16	17			
Ans.	5	4	18	2	6.00	4.00	1.00			

3D										
Que.	1	2	3	4	5	6	7	8	9	10
Ans.	4.00	1	4	2	3	1	2	2	4	1
Que.	11	12	13	14	15	16				
Ans.	3	2	2	4	4	2				

Parabola										
Que.	1	2	3	4	5	6	7	8	9	10
Ans.	3	0.50	2	2	3	3	1	1	3	1
Que.	11									
Ans.	2									

Ellipse										
Que.	1	2	3	4	5	6	7	8	9	10
Ans.	2	3	4	2	1	1	2	2	1	1
Que.	11									
Ans.	4									

Hyperbola										
Que.	1	2	3	4	5	6	7			
Ans.	3	4	2	2	2	1	2			

Complex Number										
Que.	1	2	3	4	5	6	7	8	9	10
Ans.	3	2	1	4	4	3	2	1	4	4
Que.	11	12	13	14	15	16	17			
Ans.	3	3	4	4	3	4	3			

Probability										
Que.	1	2	3	4	5	6	7	8	9	10
Ans.	3	3	4	3	2	3	1	1	1	2
Que.	11	12	13	14	15	16	17	18		
Ans.	3	3	4	11	11.00	2	3	3		

Statistics										
Que.	1	2	3	4	5	6	7	8	9	10
Ans.	54.00	18	1	1	4	1	3.00	4	3	1
Que.	11	12	13	14	15					
Ans.	4	1	2	2	6.00					

Mathematical Reasoning										
Que.	1	2	3	4	5	6	7	8	9	10
Ans.	2.00	3	1	4	2	2	3	1	1	3
Que.	11	12	13	14	15	16				
Ans.	3	3	3	3	3	2				

