UNIT & DIMENSION

1. Expression for time in terms of G (universal gravitational constant), h (Planck constant) and c (speed of light) is proportional to :

(1) $\sqrt{\frac{Gh}{c^3}}$ (2) $\sqrt{\frac{hc^5}{G}}$

(3) $\sqrt{\frac{c^3}{Gh}}$ (4) $\sqrt{\frac{Gh}{c^5}}$

2. The density of a material in SI units is 128 kg m^{-3}. In certain units in which the unit of length is 25 cm and the unit of mass is 50 g, the numerical value of density of the material is :

(1) 410 (2) 640 (3) 16 (4) 40

3. If speed (V), acceleration (A) and force (F) are considered as fundamental units, the dimension of Young's modulus will be :-

(1) $V^{-2}\ A^2F^2$ (2) $V^{-4}A^2F$

(3) $V^{-4}A^{-2}F$ (4) $V^{-2}A^2F^{-2}$

4. The force of interaction between two atoms is given by $F = \alpha\beta\exp\left(-\frac{x^2}{\alpha kt}\right)$; where x is the distance, k is the Boltzmann constant and T is temperature and α and β are two constants. The dimension of β is :

(1) $M^2L^2T^{-2}$ (2) M^2LT^{-4}

(3) $M^0L^2T^{-4}$ (4) MLT^{-2}

5. Let ℓ, r, c and ν represent inductance, resistance, capacitance and voltage, respectively. The dimension of $\frac{\ell}{rcv}$ in SI units will be:

(1) [LTA] (2) $[LA^{-2}]$

(3) $[A^{-1}]$ (4) $[LT^2]$

6. If surface tension (S), Moment of inertia (I) and Planck's constant (h), were to be taken as the fundamental units, the dimensional formula for linear momentum would be :-

(1) $S^{3/2}I^{1/2}h^0$ (2) $S^{1/2}I^{1/2}h^0$

(3) $S^{1/2}I^{1/2}h^{-1}$ (4) $S^{1/2}I^{3/2}h^{-1}$

7. In SI units, the dimesions of $\sqrt{\frac{\epsilon_0}{\mu_0}}$ is :

(1) $A^{-1}\ TML^3$ (2) $A^2T^3M^{-1}L^{-2}$

(3) $AT^2M^{-1}L^{-1}$ (4) $AT^{-3}ML^{3/2}$

8. Which of the following combinations has the dimension of electrical resistance (ε_0 is the permittivity of vacuum and μ_0 is the permeability of vacuum) ?

(1) $\sqrt{\frac{\varepsilon_0}{\mu_0}}$ (2) $\frac{\mu_0}{\varepsilon_0}$

(3) $\sqrt{\frac{\mu_0}{\varepsilon_0}}$ (4) $\frac{\varepsilon_0}{\mu_0}$

SOLUTION

1. Ans. (4)

$F = \frac{GM^2}{R^2} \Rightarrow G = [M^{-1}L^3T^{-2}]$

$E = h\nu \Rightarrow h = [ML^2T^{-1}]$

$C = [LT^{-1}]$

$t \propto G^x h^y C^z$

$[T] = [M^{-1}L^3T^{-2}]^x[ML^2T^{-1}]^y[LT^{-1}]^z$

$[M^0L^0T^1] = [M^{-x+y}L^{3x+2y+z}T^{-2x-y-z}]$

on comparing the powers of M, L, T

$-x + y = 0 \Rightarrow x = y$

$3x + 2y + z = 0 \Rightarrow 5x + z = 0$ ....(i)

$-2x - y - z = 1 \Rightarrow 3x + z = -1$ ...(ii)

on solving (i) & (ii) $x = y = \frac{1}{2}, z = -\frac{5}{2}$

$t \propto \sqrt{\frac{Gh}{C^5}}$

2. Ans. (4)

$\frac{128\text{kg}}{\text{m}^3} = \frac{125(50\text{g})(20)}{(25\text{cm})^3(4)^3}$

$= \frac{128}{64}(20)\text{units}$

$= 40$ units

3. Ans. (2)

$\frac{F}{A} = y.\frac{\Delta\ell}{\ell}$

$[Y] = \frac{F}{A}$

Now from dimension

$F = \frac{ML}{T^2}$

$L = \frac{F}{M}.T^2$

$L^2 = \frac{F^2}{M^2}\left(\frac{V}{A}\right)^4 \quad \because T = \frac{V}{A}$

$L^2 = \frac{F^2}{M^2A^2}\frac{v^4}{A^2} \qquad F = MA$

$L^2 = \frac{V^4}{A^2}$

$[Y] = \frac{[F]}{[A]} = F^1\ V^{-4}\ A^2$

4. Ans. (2)

$F = \alpha\beta e^{\left(\frac{-x^2}{\alpha KT}\right)}$

$\left[\frac{x^2}{\alpha KT}\right] = M^\circ L^\circ T^\circ$

$\frac{L^2}{[\alpha]ML^2T^{-2}} = M^\circ L^\circ T^\circ$

$\Rightarrow [\alpha] = M^{-1}T^2$

$[F] = [\alpha]\ [\beta]$

$MLT^{-2} = M^{-1}T^2[\beta]$

$\Rightarrow [\beta] = M^2LT^{-4}$

5. Ans. (3)

$\left[\frac{\ell}{r}\right] = T$

$[CV] = AT$

So, $\left[\frac{\ell}{rCV}\right] = \frac{T}{AT} = A^{-1}$

6. Ans. (2)

Sol. $p = k\ s^a I^b h^c$

where k is dimensionless constant

$MLT^{-1} = (MT^{-2})^a(ML^2)^b(ML^2T^{-1})^c$

$a + b + c = 1$

$2b + 2c = 1$

$-2a - c = -1$

$a = \frac{1}{2} \qquad b = \frac{1}{2} \qquad c = 0$

$s^{1/2}I^{1/2}h^0$

7. Ans. (2)

Sol. dimension of $\sqrt{\frac{\varepsilon_0}{\mu_0}}$

$[\varepsilon_0] = [M^{-1}L^{-3}T^4A^2]$

$[\mu_0] = [MLT^{-2}A^{-2}]$

dimensions of $\sqrt{\frac{\varepsilon_0}{\mu_0}} = \left[\frac{M^{-1}L^{-3}T^4A^2}{MLT^{-2}A^{-2}}\right]^{\frac{1}{2}}$

$= [M^{-2}L^{-4}T^6A^4]^{1/2}$

$= [M^{-1}L^{-2}T^3A^2]$

8. Ans. (3)

Sol. $[\varepsilon_0] = M^{-1}\ L^{-3}\ T^4\ A^2$

$[\mu_0] = M\ L\ T^{-2}\ A^{-2}$

$[R] = M\ L^2\ T^{-3}\ A^{-2}$

$[R] = \left[\sqrt{\frac{\mu_0}{\varepsilon_0}}\right]$