SHM

1. A rod of mass 'M' and length '2L' is suspended at its middle by a wire. It exhibits torsional oscillations; If two masses each of 'm' are attached at distance 'L/2' from its centre on both sides, it reduces the oscillation frequency by 20%. The value of ratio m/M is close to :

(1) 0.17 (2) 0.37

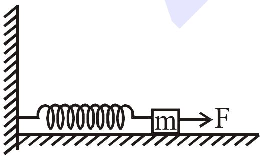
(3) 0.57 (4) 0.77

2. A particle is executing simple harmonic motion (SHM) of amplitude A, along the x-axis, about x = 0. When its potential Energy (PE) equals kinetic energy (KE), the position of the particle will be :

(1) $\frac{A}{2}$ (2) $\frac{A}{2\sqrt{2}}$

(3) $\frac{A}{\sqrt{2}}$ (4) A

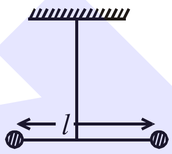
3. A block of mass m, lying on a smooth horizontal surface, is attached to a spring (of negligible mass) of spring constant k. The other end of the spring is fixed, as shown in the figure. The block is initally at rest in its equilibrium position. If now the block is pulled with a constant force F, the maximum speed of the block is :



(1) $\frac{\pi F}{\sqrt{mk}}$ (2) $\frac{2F}{\sqrt{mk}}$

(3) $\frac{F}{\sqrt{mk}}$ (4) $\frac{F}{\pi\sqrt{mk}}$

4. Two masses m and $\frac{m}{2}$ are connected at the two ends of a massless rigid rod of length l. The rod is suspended by a thin wire of torsional constant k at the centre of mass of the rod-mass system(see figure). Because of torsional constant k, the restoring torque is $\tau = k\theta$ for angular displacement 0. If the rod is rota ted by θ_0 and released, the tension in it when it passes through its mean position will be:



(1) $\frac{3k\theta_0^2}{l}$ (2) $\frac{k\theta_0^2}{2l}$

(3) $\frac{2k\theta_0^2}{l}$ (4) $\frac{k\theta_0^2}{l}$

5. A particle executes simple harmonic motion with an amplitude of 5 cm. When the particle is at 4 cm from the mean position, the magnitude of its velocity in SI units is equal to that of its acceleration. Then, its periodic time in seconds is :

(1) $\frac{7}{3}\pi$ (2) $\frac{3}{8}\pi$

(3) $\frac{4\pi}{3}$ (4) $\frac{8\pi}{3}$

6. A simple pendulum of length 1 m is oscillating with an angular frequency 10 rad/s. The support of the pendulum starts oscillating up and down with a small angular frequency of 1 rad/s and an amplitude of 10^{-2} m. The relative change in the angular frequency of the pendulum is best given by :-

(1) 10^{-3} rad/s (2) 10^{-1} rad/s

(3) 1 rad/s (4) 10^{-5} rad/s

7. A pendulum is executing simple harmonic motion and its maximum kinetic energy is K_1. If the length of the pendulum is doubled and it performs simple harmonic motion with the same amplitude as in the first case, its maximum kinetic energy is K_2. Then :-

(1) $K_2 = \frac{K_1}{4}$

(2) $K_2 = \frac{K_1}{2}$

(3) $K_2 = 2K_1$

(4) $K_2 = K_1$

8. A particle undergoing simple harmonic motion has time dependent displacement given by $x(t) = A\sin\frac{\pi t}{90}$. The ratio of kinetic to potential energy of this particle at t = 210 s will be :

(1) 2 (2) $\frac{1}{9}$

(3) 3 (4) 1

9. A simple harmonic motion is represented by:

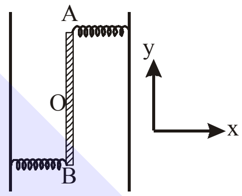
$y = 5(\sin 3\pi t + \sqrt{3}\cos 3\pi t)$ cm

The amplitude and time period of the motion are:

(1) 5cm, $\frac{3}{2}$s (2) 5cm, $\frac{2}{3}$s

(3) 10cm, $\frac{3}{2}$s (4) 10cm, $\frac{2}{3}$s

10. Two light identical springs of spring constant k are attached horizontally at the two ends of a uniform horizontal rod AB of length ℓ and mass m. The rod is pivoted at its centre 'O' and can rotate freely in horizontal plane. The other ends of the two springs are fixed to rigid supports as shown in figure. The rod is gently pushed through a small angle and released. The frequency of resulting oscillation is:



(1) $\frac{1}{2\pi}\sqrt{\frac{6k}{m}}$

(2) $\frac{1}{2\pi}\sqrt{\frac{2k}{m}}$

(3) $\frac{1}{2\pi}\sqrt{\frac{k}{m}}$

(4) $\frac{1}{2\pi}\sqrt{\frac{3k}{m}}$

11. A simple pendulum oscillating in air has period T. The bob of the pendulum is completely immersed in a non-viscous liquid. The density of the liquid is $\frac{1}{16}$th of the material of the bob. If the bob is inside liquid all the time, its period of oscillation in this liquid is :

(1) $4T\sqrt{\frac{1}{15}}$ (2) $2T\sqrt{\frac{1}{10}}$

(3) $4T\sqrt{\frac{1}{14}}$ (4) $2T\sqrt{\frac{1}{14}}$

SOLUTION

1. Ans. (2)

Frequency of torsonal oscillations is given by

$$f = \frac{k}{\sqrt{I}}$$

$$f_1 = \frac{k}{\sqrt{\frac{M(2L)^2}{12}}}$$

$$f_2 = \frac{k}{\sqrt{\frac{M(2L)^2}{12} + 2m\left(\frac{L}{2}\right)^2}}$$

$f_2 = 0.8\ f_1$

$$\frac{m}{M} = 0.375$$

2. Ans. (3)

Potential energy (U) = $\frac{1}{2}kx^2$

Kinetic energy (K) = $\frac{1}{2}kA^2 - \frac{1}{2}kx^2$

According to the question, U = k

$$\therefore\ \frac{1}{2}kx^2 = \frac{1}{2}kA^2 - \frac{1}{2}kx^2$$

$$\mathbf{x} = \pm\frac{A}{\sqrt{2}}$$

$\therefore$ Correct answer is (3)

3. Ans. (3)

Maximum speed is at mean position (equilibrium). F = kx

$$x = \frac{F}{k}$$

$W_F + W_{sp} = \Delta KE$

$$F(x) - \frac{1}{2}kx^2 = \frac{1}{2}mv^2 - 0$$

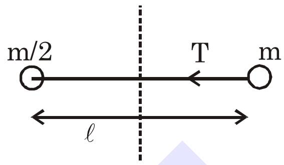
$$F\left(\frac{F}{k}\right) - \frac{1}{2}k\left(\frac{F}{k}\right)^2 = \frac{1}{2}mv^2$$

$$\Rightarrow v_{max} = \frac{F}{\sqrt{mk}}$$

4. Ans. (4)

$$\omega = \sqrt{\frac{k}{I}}$$

$$\omega = \sqrt{\frac{3k}{m\ell^2}}$$



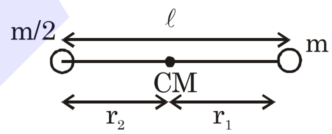
$\Omega = \omega\theta_0$ = average velocity

$T = m\Omega^2 r_1$

$$T = m\Omega^2\frac{\ell}{3} = m\omega^2\theta_0^2\frac{\ell}{3} = m\frac{3k}{m\ell^2}\theta_0^2\frac{\ell}{3} = \frac{k\theta_0^2}{\ell}$$

$$I = \mu\ell^2 = \frac{\frac{m^2}{2}}{\frac{3m}{2}}\ell^2$$

$$= \frac{m\ell^2}{3}$$



$$\frac{r_1}{r_2} = \frac{1}{2} \Rightarrow r_1 = \frac{\ell}{3}$$

5. Ans. (4)

$v = \omega\sqrt{A^2 - x^2}$ ____(1)

$a = -\omega^2 x$ ____(2)

$|v| = |a|$ ____(3)

$$\omega\sqrt{A^2 - x^2} = \omega^2 x$$

$A^2 - x^2 = \omega^2 x^2$

$5^2 - 4^2 = \omega^2(4^2)$

$\Rightarrow 3 = \omega \times 4$

$T = 2\pi/\omega$

6. Ans. (1)

Angular frequency of pendulum

$$\omega = \sqrt{\frac{g_{eff}}{\ell}}$$

$$\therefore \quad \frac{\Delta\omega}{\omega} = \frac{1}{2}\frac{\Delta g_{eff}}{g_{eff}}$$

$$\Delta\omega = \frac{1}{2}\frac{\Delta g}{g} \times \omega$$

[ω_s = angular frequency of support]

$$\Delta\omega = \frac{1}{2} \times \frac{2A\omega_s^2}{100} \times 100$$

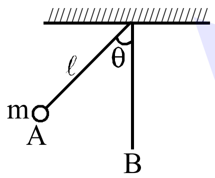
$\Delta\omega = 10^{-3}$ rad/sec.

7. Ans. (2)

Maximum kinetic energy at lowest point B is given by

$K = mgl\,(1 - \cos\theta)$

where θ = angular amp.



$K_1 = mg\ell\,(1 - \cos\theta)$

$K_2 = mg(2\ell)\,(1 - \cos\theta)$

$K_2 = 2K_1$.

8. Ans. (3)

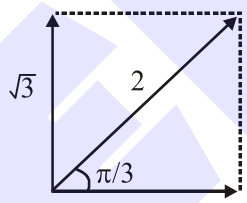
$$k = \frac{1}{2}m\omega^2A^2\cos^2\omega t$$

$$U = \frac{1}{2}m\omega^2A^2\sin^2\omega t$$

$$\frac{k}{U} = \cot^2\omega t = \cot^2\frac{\pi}{90}(210) = \frac{1}{3}$$

No answer is matching as correct answer is 1/3. Hence ratio is 3 (most appropriate)

9. Ans. (4)



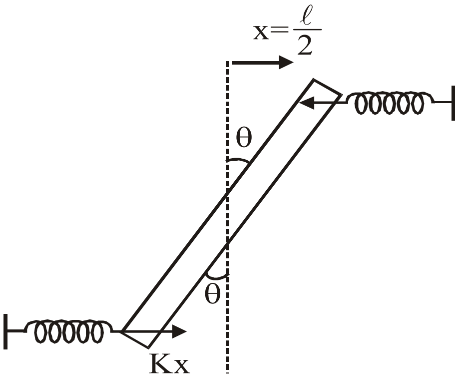
$$y = 5\left[\sin(3\pi t) + \sqrt{3}\cos(3\pi t)\right]$$

$$= 10\sin\left(3\pi t + \frac{\pi}{3}\right)$$

Amplitude = 10 cm

$$T = \frac{2\pi}{w} = \frac{2\pi}{3\pi} = \frac{2}{3}\text{sec}$$

10. Ans. (1)



$$\tau = -2Kx\frac{\ell}{2}\cos\theta$$

$$\Rightarrow \tau = \left(\frac{K\ell^2}{2}\right)\theta = -C\theta$$

$$\Rightarrow f = \frac{1}{2\pi}\sqrt{\frac{C}{I}} = \frac{1}{2\pi}\sqrt{\frac{\frac{K\ell^2}{2}}{\frac{M\ell^2}{12}}}$$

$$\Rightarrow f = \frac{1}{2\pi}\sqrt{\frac{6K}{M}}$$

11. Ans. (1)

Sol. For a simple pendulum $T = 2\pi\sqrt{\frac{L}{g_{eff}}}$

situation 1 : when pendulum is in air $\rightarrow g_{eff} = g$

situation 2 : when pendulum is in liquid

$$\rightarrow g_{eff} = g\left(1-\frac{\rho_{liquid}}{\rho_{body}}\right) = g\left(1-\frac{1}{16}\right) = \frac{15g}{16}$$

So, $$\frac{T'}{T} = \frac{2\pi\sqrt{\frac{L}{15g/16}}}{2\pi\sqrt{\frac{L}{g}}} \Rightarrow T' = \frac{4T}{\sqrt{15}}$$

Option (1)