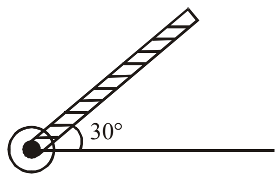
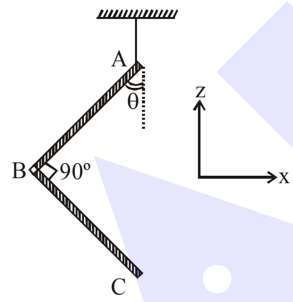
ROTATIONAL MECHANICS

1. A rod of length 50cm is pivoted at one end. It is raised such that if makes an angle of 30° from the horizontal as shown and released from rest. Its angular speed when it passes through the horizontal (in rad s^{-1}) will be
($g = 10ms^{-2}$)



(1) $\sqrt{30}$ (2) $\sqrt{\frac{30}{2}}$ (3) $\frac{\sqrt{30}}{2}$ (4) $\frac{\sqrt{20}}{3}$

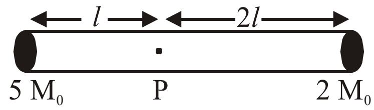
2. An L-shaped object, made of thin rods of uniform mass density, is suspended with a string as shown in figure. If AB = BC, and the angle made by AB with downward vertical is θ, then :



(1) $\tan\theta = \frac{2}{\sqrt{3}}$ (2) $\tan\theta = \frac{1}{3}$

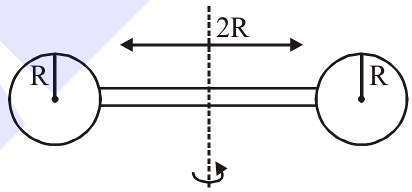
(3) $\tan\theta = \frac{1}{2}$ (4) $\tan\theta = \frac{1}{2\sqrt{3}}$

3. A rigid massless rod of length $3l$ has two masses attached at each end as shown in the figure. The rod is pivoted at point P on the horizontal axis (see figure). When released from initial horizontal position, its instantaneous angular acceleration will be :



(1) $\frac{g}{2l}$ (2) $\frac{7g}{3l}$ (3) $\frac{g}{13l}$ (4) $\frac{g}{3l}$

4. Two identical spherical balls of mass M and radius R each are stuck on two ends of a rod of length 2R and mass M (see figure). The moment of inertia of the system about the axis passing perpendicularly through the centre of the rod is :



(1) $\frac{152}{15}MR^2$ (2) $\frac{17}{15}MR^2$

(3) $\frac{137}{15}MR^2$ (4) $\frac{209}{15}MR^2$

5. To mop-clean a floor, a cleaning machine presses a circular mop of radius R vertically down with a total force F and rotates it with a constant angular speed about its axis. If the force F is distributed uniformly over the mop and if coefficient of friction between the mop and the floor is μ, the torque, applied by the machine on the mop is :

(1) $\frac{2}{3}\mu FR$ (2) $\mu FR/3$

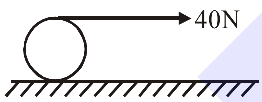
(3) $\mu FR/2$ (4) $\mu FR/6$

6. A homogeneous solid cylindrical roller of radius R and mass M is pulled on a cricket pitch by a horizontal force. Assuming rolling without slipping, angular acceleration of the cylinder is :

(1) $\frac{3F}{2m R}$ (2) $\frac{F}{3m R}$

(3) $\frac{2F}{3m R}$ (4) $\frac{F}{2m R}$

7. A string is wound around a hollow cylinder of mass 5 kg and radius 0.5 m. If the string is now pulled with a horizontal force of 40 N, and the cylinder is rolling without slipping on a horizontal surface (see figure), then the angular acceleration of the cylinder will be (Neglect the mass and thickness of the string):-



(1) 12 rad/s^2 (2) 16 rad/s^2

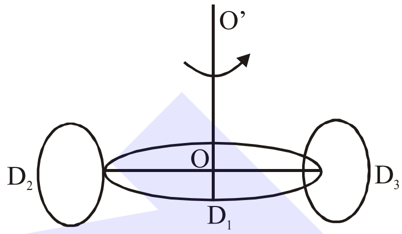
(3) 10 rad/s^2 (4) 20 rad/s^2

8. The magnitude of torque on a particle of mass 1kg is 2.5 Nm about the origin. If the force acting on it is 1 N, and the distance of the particle from the origin is 5m, the angle between the force and the position vector is (in radians) :-

(1) $\frac{\pi}{8}$ (2) $\frac{\pi}{6}$

(3) $\frac{\pi}{4}$ (4) $\frac{\pi}{3}$

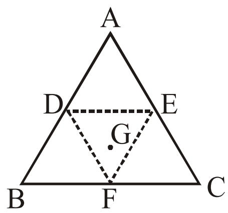
9. A circular disc D_1 of mass M and radius R has two identical discs D_2 and D_3 of the same mass M and radius R attached rigidly at its opposite ends (see figure). The moment of inertia of the system about the axis OO', passing through the centre of D_1, as shown in the figure, will be:-



(1) 3MR^2 (2) $\frac{2}{3}$MR^2

(3) MR^2 (4) $\frac{4}{5}$ MR^2

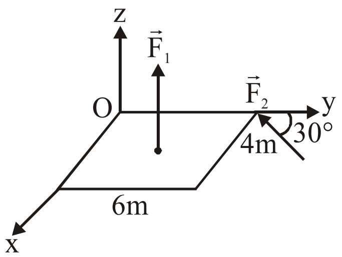
10. An equilateral triangle ABC is cut from a thin solid sheet of wood. (see figure) D, E and F are the mid-points of its sides as shown and G is the centre of the triangle. The moment of inertia of the triangle about an axis passing through G and perpendicular to the plane of the triangle is I_0. It the smaller triangle DEF is removed from ABC, the moment of inertia of the remaining figure about the same axis is I. Then:



(1) $I = \frac{9}{16}I_0$ (2) $I = \frac{3}{4}I_0$

(3) $I = \frac{I_0}{4}$ (4) $I = \frac{15}{16}I_0$

11. A slob is subjected to two forces $\vec{F}_1$ and $\vec{F}_2$ of same magnitude F as shown in the figure. Force $\vec{F}_2$ is in XY-plane while force F_1 acts along z-axis at the point $(2\vec{i}+3\vec{j})$. The moment of these forces about point O will be :



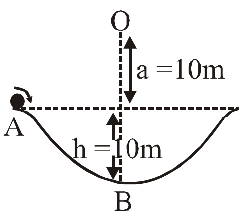
(1) $(3\hat{i}-2\hat{j}-3\hat{k})F$

(2) $(3\hat{i}+2\hat{j}+3\hat{k})F$

(3) $(3\hat{i}+2\hat{j}-3\hat{k})F$

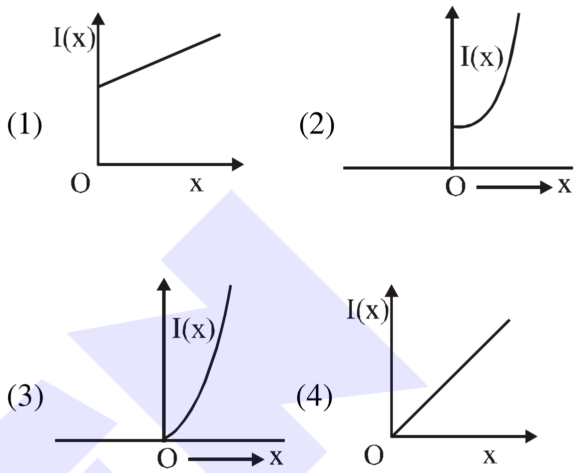
(4) $(3\hat{i}-2\hat{j}+3\hat{k})F$

12. A particle of mass 20 g is released with an initial velocity 5 m/s along the curve from the point A, as shown in the figure. The point A is at height h from point B. The particle slides along the frictionless surface. When the particle reaches point B, its angular momentum about O will be : (Take g= 10 m/s^2)



(1) 8kg-m^2/s (2) 6kg-m^2/s
(3) 3kg-m^2/s (4) 2kg-m^2/s

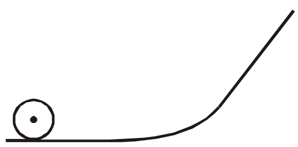
13. The moment of inertia of a solid sphere, about an axis parallel to its diameter and at a distance of x from it, is I(x)'. Which one of the graphs represents the variation of I(x) with x correctlv?



14. Let the moment of inertia of a hollow cylinder of length 30 cm (inner radius 10 cm and outer radius 20 cm), about its axis be I. The radius of a thin cylinder of the same mass such that its moment of inertia about its axis is also I, is:

(1) 12 cm (2) 18 cm
(3) 16 cm (4) 14 cm

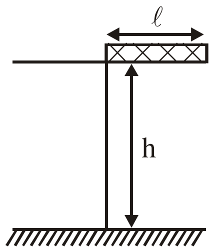
15. A solid sphere and solid cylinder of identical radii approach an incline with the same linear velocity (see figure). Both roll without slipping all throughout. The two climb maximum heights h_{sph} and h_{cyl} on the incline. The ratio $\frac{h_{sph}}{h_{cyl}}$ is given by :-



(1) $\frac{14}{15}$ (2) $\frac{4}{5}$

(3) 1 (4) $\frac{2}{\sqrt{5}}$

16. A rectangular solid box of length 0.3 m is held horizontally, with one of its sides on the edge of a platform of height 5m. When released, it slips off the table in a very short time $\tau = 0.01$s, remaining essentially horizontal. The angle by which it would rotate when it hits the ground will be (in radians) close to :-



(1) 0.02 (2) 0.28 (3) 0.5 (4) 0.3

17. A thin circular plate of mass M and radius R has its density varying as $\rho(r) = \rho_0 r$ with ρ_0 as constant and r is the distance from its centre. The moment of Inertia of the circular plate about an axis perpendicular to the plate and passing through its edge is $I = aMR^2$. The value of the coefficient a is :

(1) $\frac{3}{2}$ (2) $\frac{1}{2}$ (3) $\frac{3}{5}$ (4) $\frac{8}{5}$

18. Moment of inertia of a body about a given axis is 1.5 kg m^2. Initially the body is at rest. In order to produce a rotational kinetic energy of 1200 J, the angular accleration of 20 rad/s^2 must be applied about the axis for a duration of :-

(1) 2 s (2) 5s (3) 2.5 s (4) 3 s

19. A thin smooth rod of length L and mass M is rotating freely with angular speed ω_0 about an axis perpendicular to the rod and passing through its center. Two beads of mass m and negligible size are at the center of the rod initially. The beads are free to slide along the rod. The angular speed of the system, when the beads reach the opposite ends of the rod, will be :-

(1) $\frac{M\omega_0}{M+3m}$ (2) $\frac{M\omega_0}{M+m}$

(3) $\frac{M\omega_0}{M+2m}$ (4) $\frac{M\omega_0}{M+6m}$

20. The following bodies are made to roll up (without slipping) the same inclined plane from a horizontal plane. : (i) a ring of radius R, (ii) a solid cylinder of radius $\frac{R}{2}$ and (iii) a solid sphere of radius $\frac{R}{4}$. If in each case, the speed of the centre of mass at the bottom of the incline is same, the ratio of the maximum heights they climb is :

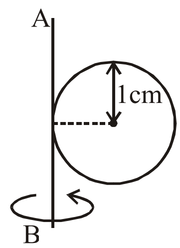
(1) 4 : 3 : 2 (2) 14 : 15 : 20

(3) 10 : 15 : 7 (4) 2 : 3 : 4

21. A stationary horizontal disc is free to rotate about its axis. When a torque is applied on it, its kinetic energy as a function of θ, where θ is the angle by which it has rotated, is given as $k\theta^2$. If its moment of inertia is I then the angular acceleration of the disc is :

(1) $\frac{k}{2I}\theta$ (2) $\frac{k}{I}\theta$ (3) $\frac{k}{4I}\theta$ (4) $\frac{2k}{I}\theta$

22. A metal coin of mass 5 g and radius 1 cm is fixed to a thin stick AB of negligible mass as shown in the figure. The system is initially at rest. The constant torque, that will make the system rotate about AB at 25 rotations per second in 5 s, is close to :



(1) 4.0×10^{-6} Nm

(2) 2.0×10^{-5} Nm

(3) 1.6×10^{-5} Nm

(4) 7.9×10^{-6} Nm

23. The time dependence of the position of a particle of mass m = 2 is given by $\vec{r}(t) = 2t\hat{i} - 3t^2\hat{j}$. Its angular momentum, with respect to the origin, at time t = 2 is :

(1) $36\,\hat{k}$ (2) $-34(\hat{k}-\hat{i})$

(3) $48(\hat{i}+\hat{j})$ (4) $-48\hat{k}$

24. A solid sphere of mass M and radius R is divided into two unequal parts. The first part has a mass of $\frac{7M}{8}$ and is converted into a uniform disc of radius 2R. The second part is converted into a uniform solid sphere. Let I_1 be the moment of inertia of the disc about its axis and I_2 be the moment of inertia of the new sphere about its axis. The ratio I_1/I_2 is given by :

(1) 185 (2) 65 (3) 285 (4) 140

25. A thin disc of mass M and radius R has mass per unit area $\sigma(r) = kr^2$ where r is the distance from its centre. Its moment of inertia about an axis going through its centre of mass and perpendicular to its plane is :

(1) $\frac{MR^2}{6}$ (2) $\frac{MR^2}{3}$

(3) $\frac{2MR^2}{3}$ (4) $\frac{MR^2}{2}$

26. Two coaxial discs, having moments of inertia I_1 and $\frac{I_1}{2}$, are rotating with respective angular velocities ω_1 and $\frac{\omega_1}{2}$, about their common axis. They are brought in contact with each other and thereafter they rotate with a common angular velocity. If E_f and E_i are the final and initial total energies, then $(E_f - E_i)$ is :

(1) $\frac{I_1\omega_1^2}{12}$ (2) $\frac{3}{8}I_1\omega_1^2$

(3) $\frac{I_1\omega_1^2}{6}$ (4) $\frac{I_1\omega_1^2}{24}$

27. A particle of mass m is moving along a trajectory given by

$x = x_0 + a\cos\omega_1 t$

$y = y_0 + b\sin\omega_2 t$

The torque, acting on the particle about the origin, at t = 0 is :

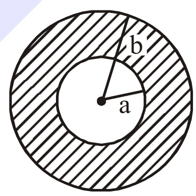
(1) $m(-x_0b + y_0a)\,\omega_1^2\hat{k}$

(2) $+my_0a\,\omega_1^2\hat{k}$

(3) $-m(x_0b\omega_2^2 - y_0a\omega_1^2)\hat{k}$

(4) Zero

28. A circular disc of radius b has a hole of radius a at its centre (see figure). If the mass per unit area of the disc varies as $\left(\frac{\sigma_0}{r}\right)$, then the radius of gyration of the disc about its axis passing through the centre is :



(1) $\frac{a+b}{2}$ (2) $\frac{a+b}{3}$

(3) $\sqrt{\frac{a^2+b^2+ab}{2}}$ (4) $\sqrt{\frac{a^2+b^2+ab}{3}}$

29. A person of mass M is, sitting on a swing of length L and swinging with an angular amplitude θ_0. If the person stands up when the swing passes through its lowest point, the work done by him, assuming that his centre of mass moves by a distance ℓ ($\ell << L$), is close to :

(1) $Mg\ell$

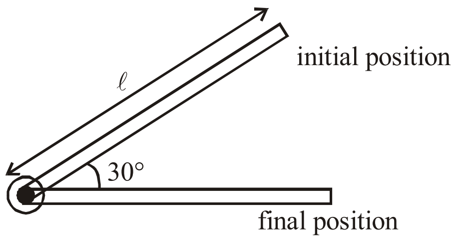
(2) $Mg\ell(1 + \theta_0^2)$

(3) $Mg\ell(1 - \theta_0^2)$

(4) $Mg\ell\left(1+\frac{\theta_0^2}{2}\right)$

SOLUTION

1. Ans. (1)



Work done by gravity from initial to final position is,

$$W = mg\frac{\ell}{2}\sin 30°$$

$$= \frac{mg\ell}{4}$$

According to work energy theorem

$$W = \frac{1}{2}I\omega^2$$

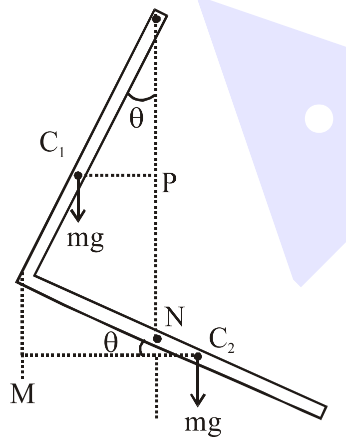
$$\Rightarrow \frac{1}{2}\frac{m\ell^2}{3}\omega^2 = \frac{mg\ell}{4}$$

$$\omega = \sqrt{\frac{3g}{2\ell}} = \sqrt{\frac{3\times 10}{2\times 0.5}}$$

$$\omega = \sqrt{30}\ \text{rad/sec}$$

$\therefore$ correct answer is (1)

2. Ans. (2)



Let mass of one rod is m.

Balancing torque about hinge point.

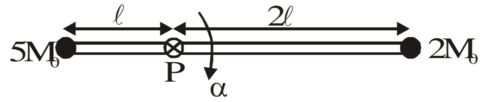
mg (C_1P) = mg (C_2N)

$$mg\left(\frac{L}{2}\sin\theta\right) = mg\left(\frac{L}{2}\cos\theta - L\sin\theta\right)$$

$$\Rightarrow \frac{3}{2}mgL\sin\theta = \frac{mgL}{2}\cos\theta$$

$$\Rightarrow \tan\theta = \frac{1}{3}$$

3. Ans. (3)



Applying torque equation about point P.

$2M_0\,(2l) - 5\,M_0\,gl = I\alpha$

$I = 2M_0\,(2l)^2 + 5M_0\,l^2 = 13\,M_0l^2d$

$$\therefore\ \alpha = -\frac{M_0 g\ell}{13M_0\ell^2} \Rightarrow \alpha = -\frac{g}{13\ell}$$

$$\therefore\ \alpha = \frac{g}{13\ell}\ \text{anticlockwise}$$

4. Ans. (3)

For Ball

using parallel axis theorem.

$$I_{ball} = \frac{2}{5}MR^2 + M(2R)^2$$

$$= \frac{22}{5}MR^2$$

2 Balls so $\frac{44}{5}MR^2$

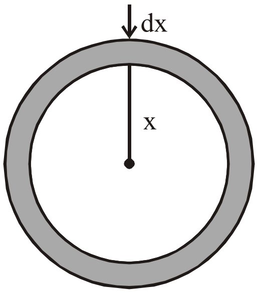
Irod = for rod $\frac{M(2R)^2}{R} = \frac{MR^2}{3}$

$$I_{system} = I_{Ball} + I_{rod}$$

$$= \frac{44}{5}MR^2 + \frac{MR^2}{3}$$

$$= \frac{137}{15}MR^2$$

5. Ans. (1)



Consider a strip of radius x & thickness dx,

Torque due to friction on this strip.

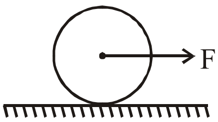
$$\int d\tau = \int_0^R \frac{x\mu F \cdot 2\pi x dx}{\pi R^2}$$

$$\tau = \frac{2\mu F}{R^2} \cdot \frac{R^3}{3}$$

$$\tau = \frac{2\mu FR}{3}$$

$\therefore$ correct answer is (1)

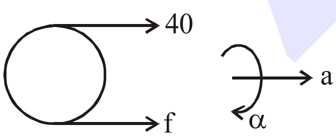
6. Ans. (3)



$$FR = \frac{3}{2} MR^2 \alpha$$

$$\alpha = \frac{2F}{3MR}$$

7. Ans. (2)



$40 + f = m(R\alpha)$ .....(i)

$40 \times R - f \times R = mR^2\alpha$

$40 - f = mR\alpha$ ...... (ii)

From (i) and (ii)

$$\alpha = \frac{40}{mR} = 16$$

8. Ans. (2)

$2.5 = 1 \times 5 \sin\theta$

$\sin\theta = 0.5 = \frac{1}{2}$

$\theta = \frac{\pi}{6}$

9. Ans. (1)

$$I = \frac{MR^2}{2} + 2\left(\frac{MR^2}{4} + MR^2\right)$$

$$= \frac{MR^2}{2} + \frac{MR^2}{2} + 2MR^2$$

$$= 3\ MR^2$$

10. Ans. (4)

Suppose M is mass and a is side of larger triangle, then $\frac{M}{4}$ and $\frac{a}{2}$ will be mass and side length of smaller triangle.

$$\frac{I_{removed}}{I_{original}} = \frac{\frac{M}{4}}{M} \cdot \frac{\left(\frac{a}{2}\right)^2}{(a)^2}$$

$$I_{removed} = \frac{I_0}{16}$$

$$\text{So, } I = I_0 - \frac{I_0}{16} = \frac{15 I_0}{16}$$

11. Ans. (4)

Torque for F_1 force

$$\vec{F}_1 = \frac{F}{2}(-\hat{i}) + \frac{F\sqrt{3}}{2}(-\hat{j})$$

$$\vec{r}_1 = 0\hat{i} + 6\hat{j}$$

$$\vec{\tau}_{F_1} = \vec{r}_1 \times \vec{F}_1 = 3F\hat{k}$$

Torque for F_2 force

$$\vec{F}_2 = F\hat{k}$$

$$\vec{r}_2 = 2\hat{i} + 3\hat{j}$$

$$\vec{\tau}_{F_2} = \vec{r}_2 \times \vec{F}_2 = 3F\hat{i} + 2F(-\hat{j})$$

$$\vec{\tau}_{net} = \vec{\tau}_{F_1} + \vec{\tau}_{F_2}$$

$$= 3F\hat{i} + 2F(-\hat{j}) + 3F(\hat{k})$$

12. Ans. (2)

13. Ans. (2)

$$I = \frac{2}{5}mR^2 + mx^2$$

14. Ans. (3)

15. Ans. (1)

Sol. for solid sphere

$$\frac{1}{2}mv^2 + \frac{1}{2}.\frac{2}{5}mR^2.\frac{v^2}{R^2} = mgh_{sph.}$$

for solid cylinder

$$\frac{1}{2}mv^2 + \frac{1}{2}.\frac{1}{2}mR^2.\frac{v^2}{R^2} = mgh_{cyl.}$$

$$\Rightarrow \frac{h_{sph.}}{h_{cyl.}} = \frac{7/5}{3/2} = \frac{14}{15}$$

16. Ans. (3)

Sol. Angular impulse = change in angular momentum

$\tau \Delta t = \Delta L$

$$mg\frac{\ell}{2}\times .01 = \frac{m\ell^2}{3}\omega$$

$$\omega = \frac{3g\times 0.01}{2\ell}$$

$$= \frac{3\times 10\times .01}{2\times 0.3}$$

$$= \frac{1}{2} = 0.5\text{ rad/s}$$

time taken by rod to hit the ground

$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2\times 5}{10}} = 1\text{ sec.}$$

in this time angle rotate by rod

$\theta = \omega t = 0.5 \times 1 = 0.5$ radian

17. Ans. (4)

Sol. $$M = \int_0^R \rho_0 r(2\pi r dr) = \frac{\rho_0\times 2\pi\times R^3}{3}$$

$$\underset{(\text{MOI about COM})}{I_0} = \int_0^R \rho_0 r(2\pi r dr)\times r^2 = \frac{\rho_0\times 2\pi R^5}{5}$$

by parallel axis theorem

$I = I_0 + MR^2$

$$= \frac{\rho_0\times 2\pi R^5}{5} + \frac{\rho_0\times 2\pi R^3}{3}\times R^2 = \rho_0 2\pi R^5\times\frac{8}{15}$$

$$= MR^2\times\frac{8}{5}$$

18. Ans. (1)

Sol. Given moment of inertia 'I' = 1.5 kgm^2

Angular Acc. "α" = 20 Rad/s^2

$$KE = \frac{1}{2}I\omega^2$$

$$1200 = \frac{1}{2}1.5\times\omega^2$$

$$\omega^2 = \frac{1200\times 2}{1.5} = 1600$$

$\omega = 40\ rad/s^2$

$\omega = \omega_0 + \alpha t$

$40 = 0 + 20\ t$

t = 2sec.

19. Ans. (4)

Sol. Applying angular momentum conservation, about axis of rotation

$L_i = L_f$

$$\frac{ML^2}{12}\omega_0 = \left(\frac{ML^2}{12} + m\left(\frac{L}{2}\right)^2\times 2\right)\omega$$

$$\Rightarrow \omega = \frac{M\omega_0}{M+6m}.$$

20. Ans. (2)

Sol. $\frac{1}{2}\left(m+\frac{I}{R^2}\right)v^2 = mgh$

if radius of gyration is k, then

$h = \frac{\left(1+\frac{k^2}{R^2}\right)v^2}{2g}$, $\frac{k_{ring}}{R_{ring}} = 1$, $\frac{k_{solid\ cylinder}}{R_{solid\ cylinder}} = \frac{1}{\sqrt{2}}$

$\frac{k_{solid\ sphere}}{R_{solid\ sphere}} = \sqrt{\frac{2}{5}}$

$h_1 : h_2 : h_3 :: (1+1) : \left(1+\frac{1}{2}\right) : \left(1+\frac{2}{5}\right) :: 20 : 15 : 14$

Therefor **most appropriate option is (2)** athough which in not in correct sequence

21. Ans. (4)

Sol. Kinetic energy KE = $\frac{1}{2}I\omega^2 = k\theta^2$

$\Rightarrow \omega^2 = \frac{2k\theta^2}{I} \Rightarrow \omega = \sqrt{\frac{2k}{I}}\,\theta$ ....(1)

Differentiate (1) wrt time →

$\frac{d\omega}{dt} = \alpha = \sqrt{\frac{2k}{I}}\left(\frac{d\theta}{dt}\right)$

$\Rightarrow \alpha = \sqrt{\frac{2k}{I}} \cdot \sqrt{\frac{2k}{I}}\,\theta$ {by (1)}

$\Rightarrow \alpha = \frac{2k}{I}\theta$

Option (4)

22. Ans. (2)

Sol. $\alpha = \frac{\Delta\omega}{\Delta t} = \frac{25\times 2\pi}{5} = 10\pi \text{ rad/sec}^2$

$\tau = \left(\frac{5}{4}MR^2\right)\alpha$

$= \frac{5}{4}\times 5\times 10^{-3}\times(10^{-2})^2\times 10\pi$

$= 1.9625 \times 10^{-5}$ Nm

$\simeq 2.0 \times 10^{-5}$ Nm

23. Ans. (4)

Sol. $\vec{L} = m[\vec{r}\times\vec{v}]$

m = 2 kg

$\vec{r} = 2t\,\hat{i} - 3t^2\hat{j}$

$= 4\,\hat{i} - 12\,\hat{j}$ (At t = 2 sec)

$\vec{v} = \frac{d\vec{r}}{dt} = 2\,\hat{i} - 6t\,\hat{j} = 2\hat{i} - 12\hat{j}$

$\vec{r}\times\vec{v} = (4\hat{i}-12\hat{j})\times(2\hat{i}-12\hat{j})$

$= -24\,\hat{k}$

$\vec{L} = m(\vec{r}\times\vec{v})$

$= -48\hat{k}$

24. Ans. (4)

Sol. $I_1 = \frac{\left(\frac{7M}{8}\right)(2R)^2}{2} = \left(\frac{7}{16}\times 4\right)MR^2 = \frac{7}{4}MR^2$

$I_2 = \frac{2}{5}\left(\frac{M}{8}\right)R_1^2 = \frac{2}{5}\left(\frac{M}{8}\right)\frac{R^2}{4} = \frac{MR^2}{80}$

$\frac{4}{3}\pi R^3 = 8\left(\frac{4}{3}\pi R_1^3\right)$

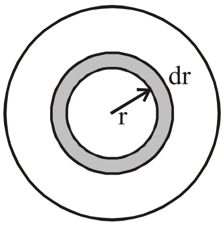
$R^3 = 8R_1^3$

$R = 2R_1$

$\therefore \frac{I_1}{I_2} = \frac{7/4\,MR^2}{\frac{MR^2}{80}} = \frac{7}{4}\times 80 = 140$

25. Ans. (3)

Sol.



$$I_{Disc} = \int_0^R (dm)r^2 \Rightarrow I_{Disc} = \int_0^R (\sigma 2\pi r dr)r^2$$

$$I_{Disc} = \int_0^R (kr^2 2\pi r dr)r^2 \qquad \text{Mass of disc}$$

$$I_{Disc} = 2\pi k\int_0^R r^5 dr \qquad M = \int_0^R 2\pi r dr\ kr^2$$

$$I_{Disc} = 2\pi k\left(\frac{r^6}{6}\right)_0^R \qquad M = 2\pi k\int_0^R r^3 dr$$

$$I_{Disc} = 2\pi k\frac{R^6}{6} \qquad M = 2\pi k\left.\frac{r^4}{4}\right|_0^R$$

$$I_{Disc} = \frac{\pi k R^6}{3} = \left(\frac{\pi k R^4}{2}\right)\frac{R^2 2}{3} \qquad M = 2\pi k\frac{R^4}{4}$$

$$I_{Disc} = \frac{M2R^2}{3}$$

$$I_{Disc} = \frac{2}{3}MR^2$$

26. Ans. (4)

Sol. $E_i = \frac{1}{2}I_1 \times \omega_1^2 + \frac{1}{2}\frac{I_1}{2} \times \frac{\omega_1^2}{4}$

$$= \frac{I_1\omega_1^2}{2}\left(\frac{9}{8}\right) = \frac{9}{16}I_1\omega_1^2$$

$$I_1\omega_1 + \frac{I_1\omega_1}{4} = \frac{3I_1}{2}\omega$$

$$\frac{5}{4}I_1\omega_1 = \frac{3I_1}{2}\omega$$

$$\omega = \frac{5}{6}\omega_1$$

$$E_f = \frac{1}{2}\times\frac{3I_1}{2}\times\frac{25}{36}\omega_1^2$$

$$= \frac{25}{48}I_1\omega_1^2$$

$$\Rightarrow E_f - E_i = I_1\omega_1^2\left(\frac{25}{48} - \frac{9}{16}\right) = \frac{-2}{48}I_1\omega_1^2$$

$$= \frac{-I_1\omega_1^2}{24}$$

27. Ans. (2)

Sol. $F = -m\ (a\omega_1^2 \cos\omega_1 t\hat{i} + b\omega_2^2 \sin\omega_2 t\hat{j})$

$$\vec{r} = (x_0 + a\cos\omega_1 t)\hat{i} + (y_0 + b\sin\omega_2 t)\hat{j}$$

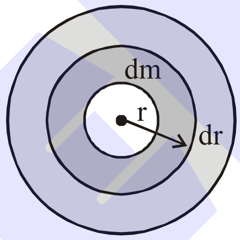
$$\vec{T} = \vec{r}\times\vec{F} = -m(x_0 + a\cos\omega_1 t)b\omega_2^2 \sin\omega_2 t\,\hat{k}$$

$$+m(y_0 + b\sin\omega_2 t)a\omega_1^2\cos\omega_1 t\,\hat{k}$$

$$= ma\omega_1^2 y_0\hat{k}$$

28. Ans. (4)

Sol.



$$dI = (dm)r^2$$

$$= (\sigma\ dA)r^2$$

$$= \left(\frac{\sigma_0}{r}2\pi r dr\right)r^2$$

$$= (\sigma_0\ 2\pi)r^2 dr$$

$$I = \int dI = \int_a^b \sigma_0 2\pi r^2 dr$$

$$= \sigma_0 2\pi\left(\frac{b^3 - a^3}{3}\right)$$

$$m = \int dm = \int \sigma dA$$

$$= \sigma_0 2\pi\int_a^b dr$$

$$m = \sigma_0\ 2\pi\ (b\text{-}a)$$

Radius of gyration

$$k = \sqrt{\frac{I}{m}} = \sqrt{\frac{(b^3 - a^3)}{3(b-a)}}$$

$$= \sqrt{\left(\frac{a^3 + b^3 + ab}{3}\right)}$$

29. Ans. (2)

Sol. Angular momentum conservation.

$$\cancel{M}V_0L = \cancel{M}V_1(L-\ell)$$

$$V_1 = V_0\left(\frac{L}{L-\ell}\right)$$

$$w_g + w_p = \Delta KE$$

$$-mg\ell + w_p = \frac{1}{2}m\left(V_1^2 - V_0^2\right)$$

$$w_p = mg\ell + \frac{1}{2}mV_0^2\left(\left(\frac{L}{L-\ell}\right)^2 - 1\right)$$

$$= mg\ell + \frac{1}{2}mV_0^2\left(\left(1-\frac{\ell}{L}\right)^{-2} - 1\right)$$

Now, $\ell << L$

By, Binomial approximation

$$= mg\ell + \frac{1}{2}mV_0^2\left(\left(1+\frac{2\ell}{L}\right) - 1\right)$$

$$= mg\ell + \frac{1}{2}mV_0^2\left(\frac{2\ell}{L}\right)$$

$$W_p = mg\ell + mv_0^2\frac{\ell}{L}$$

here, V_0 = maximum velocity

$= \omega \times A$

$$= \left(\sqrt{\frac{g}{L}}\right)(\theta_0 L)$$

$$V_0 = \theta_0\sqrt{gL}$$

so, $w_p = mg\ell + m\left(\theta_0\sqrt{gL}\right)^2\frac{\ell}{L}$

$$= mg\ell\left(1+\theta_0^2\right)$$