# KINEMATICS

**1.** The position co-ordinates of a particle moving in a 3-D coordinate system is given by

$x = a\cos\omega t$

$y = a\sin\omega t$

and $z = a\omega t$

The speed of the particle is :

(1) $a\omega$ (2) $\sqrt{3}\,a\omega$

(3) $\sqrt{2}\,a\omega$ (4) $2a\omega$

**2.** In a car race on straight road, car A takes a time t less than car B at the finish and passes finishing point with a speed 'v' more than that of car B. Both the cars start from rest and travel with constant acceleration $a_1$ and $a_2$ respectively. Then 'v' is equal to :

(1) $\frac{a_1 + a_2}{2}t$ (2) $\sqrt{2a_1a_2}\,t$

(3) $\frac{2a_1a_2}{a_1 + a_2}t$ (4) $\sqrt{a_1a_2}\,t$

**3.** A particle is moving with a velocity $\bar{v} = K\left(y\hat{i} + x\hat{j}\right)$, where K is a constant. The general equation for its path is:

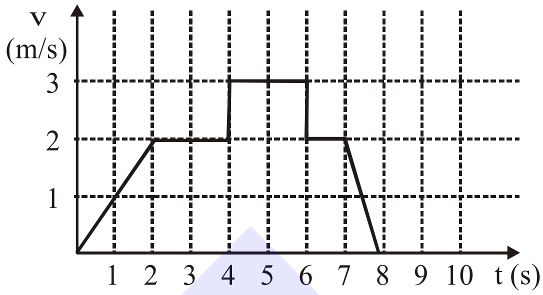
(1) xy = constant

(2) $y^2 = x^2$ + constant

(3) $y = x^2$ + constant

(4) $y^2 = x$ + constant

**4.** A particle starts from the origin at time t = 0 and moves along the positive x-axis. The graph of velocity with respect to time is shown in figure. What is the position of the particle at time t = 5s ?




(1) 6 m (2) 9 m (3) 3 m (4) 10 m

**5.** Two guns A and B can fire bullets at speeds 1 km/s and 2 km/s respectively. From a point on a horizontal ground, they are fired in all possible directions. The ratio of maximum areas covered by the bullets fired by the two guns, on the ground is :

(1) 1 : 2 (2) 1 : 4 (3) 1 : 8 (4) 1 : 16

**6.** A particle moves from the point $\left(2.0\hat{i} + 4.0\hat{j}\right)$ m, at t = 0, with an initial velocity $\left(5.0\hat{i} + 4.0\hat{j}\right)$ $ms^{-1}$. It is acted upon by a constant force which produces a constant acceleration $\left(4.0\hat{i} + 4.0\hat{j}\right)$ $ms^{-2}$. What is the distance of the particle from the origin at time 2 s ?

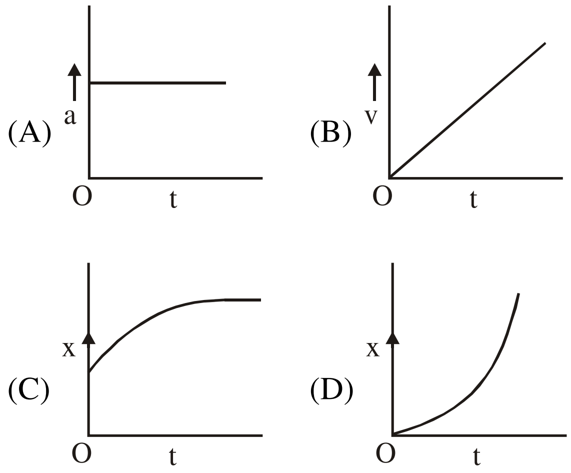
(1) $20\sqrt{2}$ m (2) $10\sqrt{2}$ m

(3) 5 m (4) 15 m

**7.** A passenger train of length 60m travels at a speed of 80 km/hr. Another freight train of length 120 m travels at a speed of 30 km/hr. The ratio of times taken by the passenger train to completely cross the freight train when : (i) they are moving in the same direction, and (ii) in the opposite directions is :

(1) $\frac{5}{2}$ (2) $\frac{25}{11}$ (3) $\frac{3}{2}$ (4) $\frac{11}{5}$

**8.** A particle starts from origin O from rest and moves with a uniform acceleration along the positive x-axis. Identify all figures that correctly represent the motion qualitatively. (a = acceleration, v = velocity, x = displacement, t = time)




(1) (A), (B), (C) (2) (A)

(3) (A), (B), (D) (4) (B), (C)

**9.** Ship A is sailing towards north-east with velocity $\vec{v} = 30\hat{i} + 50\hat{j}$ km/hr where $\hat{i}$ points east and $\hat{j}$, north. Ship B is at a distance of 80 km east and 150 km north of Ship A and is sailing towards west at 10 km/hr. A will be at minimum distance from B in :

(1) 4.2 hrs. (2) 2.2 hrs.

(3) 3.2 hrs. (4) 2.6 hrs.

**10.** The position of a particle as a function of time t, is given by

$x(t) = at + bt^2 - ct^3$

where a, b and c are constants. When the particle attains zero acceleration, then its velocity will be :

(1) $a + \frac{b^2}{4c}$ (2) $a + \frac{b^2}{c}$

(3) $a + \frac{b^2}{2c}$ (4) $a + \frac{b^2}{3c}$

**11.** The position vector of a particle changes with time according to the relation

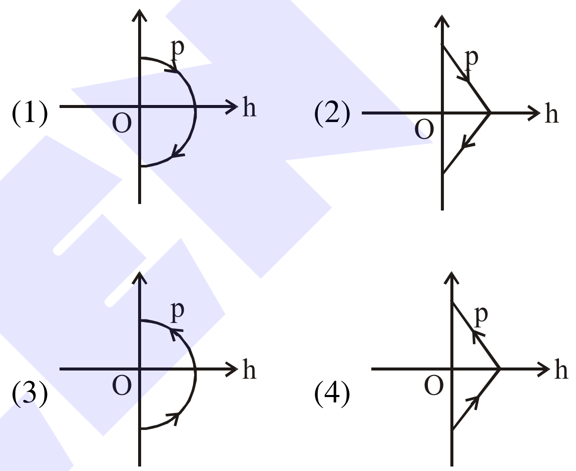
$\vec{r}(t) = 15t^2\hat{i} + (4 - 20t^2)\hat{j}$.

What is the magnitude of the acceleration at t = 1 ?

(1) 40 (2) 100

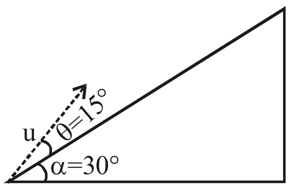
(3) 25 (4) 50

**12.** A ball is thrown vertically up (taken as +z-axis) from the ground. The correct momentum-height (p-h) diagram is :




**13.** A plane is inclined at an angle α = 30° with a respect to the horizontal. A particle is projected with a speed u = 2 $ms^{-1}$ from the base of the plane, making an angle θ = 15° with respect to the plane as shown in the figure. The distance from the base, at which the particle hits the plane is close to :

(Take g = 10 $ms^{-2}$)




(1) 14 cm (2) 20 cm

(3) 18 cm (4) 26 cm

**14.** A particle is moving with speed $v = b\sqrt{x}$ along positive x-axis. Calculate the speed of the particle at time $t = \tau$ (assume that the particle is at origin at t = 0).

(1) $\frac{b^2\tau}{4}$ (2) $\frac{b^2\tau}{2}$

(3) $b^2\tau$ (4) $\frac{b^2\tau}{\sqrt{2}}$

**15.** Two particles are projected from the same point with the same speed u such that they have the same range R, but different maximum heights, $h_1$ and $h_2$. Which of the following is correct ?

(1) $R^2 = 2h_1h_2$ (2) $R^2 = 16h_1h_2$

(3) $R^2 = 4h_1h_2$ (4) $R^2 = h_1h_2$

**16.** The trajectory of a projectile near the surface of the earth is given as $y = 2x - 9x^2$. If it were launched at an angle $\theta_0$ with speed $v_0$ then ($g = 10\ ms^{-2}$) :

(1) $\theta_0 = \cos^{-1}\left(\frac{1}{\sqrt{5}}\right)$ and $v_0 = \frac{5}{3}ms^{-1}$

(2) $\theta_0 = \sin^{-1}\left(\frac{1}{\sqrt{5}}\right)$ and $v_0 = \frac{5}{3}ms^{-1}$

(3) $\theta_0 = \sin^{-1}\left(\frac{2}{\sqrt{5}}\right)$ and $v_0 = \frac{3}{5}ms^{-1}$

(4) $\theta_0 = \cos^{-1}\left(\frac{2}{\sqrt{5}}\right)$ and $v_0 = \frac{3}{5}ms^{-1}$

**17.** A shell is fired from a fixed artillery gun with an initial speed u such that it hits the target on the ground at a distance R from it. If $t_1$ and $t_2$ are the values of the time taken by it to hit the target in two possible ways, the product $t_1t_2$ is :

(1) R/g (2) R/4g

(3) 2R/g (4) R/2g

## SOLUTION

**1. Ans. (3)**

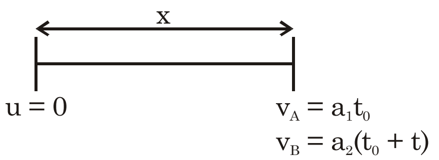
$v_x = -a\omega\sin\omega t \Rightarrow v_y = a\omega\cos\omega t$

$v_z = a\omega \Rightarrow v = \sqrt{v_x^2 + v_y^2 + v_z^2}$

$v = \sqrt{2}a\omega$

**2. Ans. (4)**

For A & B let time taken by A is $t_0$




from ques.

$v_A - v_B = v = (a_1 - a_2)t_0 - a_2t$ ....(i)

$x_B = x_A = \frac{1}{2}a_1t_0^2 = \frac{1}{2}a_2(t_0 + t)^2$

$\Rightarrow \sqrt{a_1}t_0 = \sqrt{a_2}(t_0 + t)$

$\Rightarrow \left(\sqrt{a_2} - \sqrt{a_2}\right)t_0 = \sqrt{a_2}t$ ....(ii)

putting $t_0$ in equation

$v = (a_1 - a_2)\frac{\sqrt{a_2}t}{\sqrt{a_1} - \sqrt{a_2}} - a_2t$

$= \left(\sqrt{a_1} + \sqrt{a_2}\right)\sqrt{a_2}t - a_2t \Rightarrow v = \sqrt{a_1a_2}t$

$\Rightarrow \sqrt{a_1a_2}t + a_2t - a_2t$

**3. Ans. (2)**

$\frac{dx}{dt} = ky, \frac{dy}{dt} = kx$

Now, $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{x}{y}$

$\Rightarrow ydy = xdx$

Integrating both side

$y^2 = x^2 + c$

**4. Ans. (2)**

S = Area under graph

$\frac{1}{2} \times 2 \times 2 + 2 \times 2 + 3 \times 1 = 9$ m

**5. Ans. (4)**

$R = \frac{u^2 \sin 2\theta}{g}$

$A = \pi R^2$

$A \propto R^2$

$A \propto u^4$

$\frac{A_1}{A_2} = \frac{u_1^4}{u_2^4} = \left[\frac{1}{2}\right]^4 = \frac{1}{16}$

**6. Ans. (1)**

$\vec{S} = (5\hat{i} + 4\hat{j})2 + \frac{1}{2}(4\hat{i} + 4\hat{j})4$

$= 10\hat{i} + 8\hat{j} + 8\hat{i} + 8\hat{j}$

$\vec{r}_f - \vec{r}_i = 18\hat{i} + 16\hat{j}$

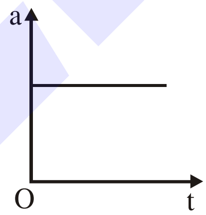
$\vec{r}_f = 20\hat{i} + 20\hat{j}$

$|\vec{r}_f| = 20\sqrt{2}$

**7. Ans. (4)**

**8. Ans. (3)**

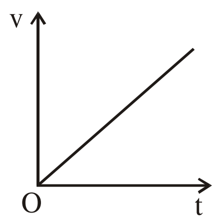
**Sol.** Given initial velocity u = 0 and acceleration is constant

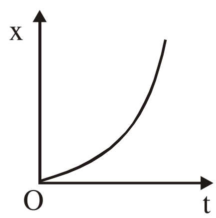



At time t

$v = 0 + at \Rightarrow v = at$

also $x = 0(t) + \frac{1}{2}at^2 \Rightarrow x = \frac{1}{2}at^2$

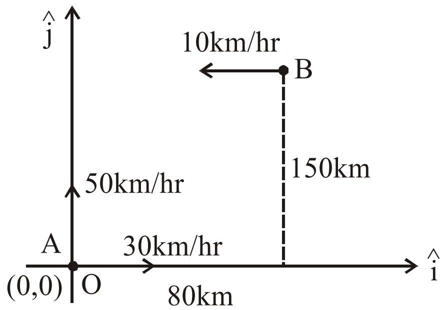



Graph (A) ; (B) and (D) are correct.

**9. Ans. (4)**

**Sol.** If we take the position of ship 'A' as origin then positions and velocities of both ships can be given as :




$$\vec{v}_A = \left(30\hat{i} + 50\hat{j}\right) \text{km/hr}$$

$$\vec{v}_B = -10\hat{i} \text{ km/hr}$$

$$\vec{r}_A = 0\hat{i} + 0\hat{j}$$

$$\vec{r}_B = \left(80\hat{i} + 150\hat{j}\right) \text{km}$$

Time after which distance between them will be minimum

$$t = -\frac{\vec{r}_{BA} \cdot \vec{v}_{BA}}{\left|\vec{v}_{BA}\right|^2};$$

where $\vec{r}_{BA} = \left(80\hat{i} + 150\hat{j}\right) \text{km}$

$$\vec{v}_{BA} = -10\hat{i} - \left(30\hat{i} + 50\hat{j}\right)$$

$$\left(-40\hat{i} - 50\hat{j}\right) \text{km/hr}$$

$$\therefore t = -\frac{\left(80\hat{i} + 150\hat{j}\right) \cdot \left(-40\hat{i} - 50\hat{j}\right)}{\left|-40\hat{i} - 50\hat{j}\right|^2}$$

$$= \frac{3200 + 7500}{4100} \text{hr} = \frac{10700}{4100} \text{hr} = 2.6 \text{hrs}$$

**10. Ans. (4)**

**Sol.** $x = at + bt^2 - ct^3$

$$v = \frac{dx}{dt} = a + 2bt - 3ct^2$$

$$a = \frac{dv}{dt} = 2b - 6ct = 0 \Rightarrow t = \frac{b}{3c}$$

$$v_{\left(\text{at } t = \frac{b}{3c}\right)} = a + 2b\left(\frac{b}{3c}\right) - 3c\left(\frac{b}{3c}\right)$$

$$= a + \frac{b^2}{3c}.$$

**11. Ans. (4)**

**Sol.** $\vec{r} = 15t^2\hat{i} + (4 - 20t^2)\hat{j}$

$$\vec{v} = \frac{d\vec{r}}{dt} = 30t\hat{i} + (-40t)\hat{j}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = 30\hat{i} - 40\hat{j}$$

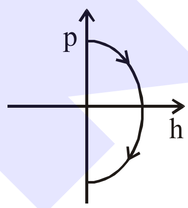
$$|\vec{a}| = 50 \text{m/s}^2.$$

**12. Ans. (1)**

**Sol.** Momentum $p = mv$ .....(1)

and for motion under gravity $h = \dfrac{u^2 - v^2}{2g}$ ...(2)

$$h = \frac{u^2 - p^2/m^2}{2g}$$




Option (1)

**13. Ans. (2)**

**Sol.** $t = \dfrac{2 \times 2 \times \sin 15°}{g \cos 30°}$

$$S = 2\cos 15° \times t - \frac{1}{2} g \sin 30° t^2$$

Put values and solve

$S \simeq 20\text{cm}$

**14. Ans. (2)**

**Sol.** $v = b\sqrt{x}$

$$\frac{dv}{dt} = \frac{b}{2\sqrt{x}} \frac{dx}{dt}$$

$$a = \frac{bv}{2\sqrt{x}}$$

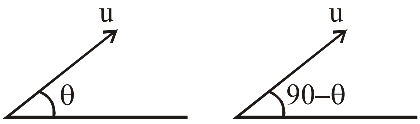
$$a = \frac{b\left(b\sqrt{x}\right)}{2\sqrt{x}}$$

$$\frac{dv}{dt} = a = \frac{b^2}{2}$$

$$v = \frac{b^2}{2}\tau$$

**15. Ans. (2)**

**Sol.**




For same range angle of projection will be θ & 90 – θ

$$R = \frac{u^2 2\sin\theta\cos\theta}{g}$$

$$h_1 = \frac{u^2\sin^2\theta}{g}$$

$$h_2 = \frac{u^2\sin^2(90-\theta)}{g}$$

$$\frac{R^2}{h_1h_2} = 16$$

**16. Ans. (1)**

**Sol.** Equation of trajectory is given as

$y = 2x - 9x^2$ ........ (1)

Comparing with equation :

$y = x\tan\theta - \frac{g}{2u^2\cos^2\theta}.x^2$ ........ (2)

We get;

$\tan\theta = 2$

$$\therefore \boxed{\cos\theta = \frac{1}{\sqrt{5}}}$$

Also, $\frac{g}{2u^2\cos^2\theta} = 9$

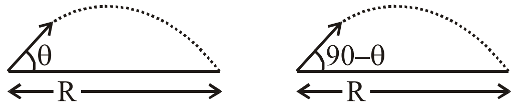
$$\Rightarrow \frac{10}{2\times 9\times\left(\frac{1}{\sqrt{5}}\right)^2} = u^2$$

$$\Rightarrow u^2 = \frac{25}{9}$$

$$\Rightarrow \boxed{u = \frac{5}{3}\,m/s}$$

**17. Ans. (3)**

**Sol.** Range will be same for time $t_1$ & $t_2$, so angles of projection will be 'θ' & '90° – θ'




$t_1 = \frac{2u\sin\theta}{g}$ $t_2 = \frac{2u\sin(90°-\theta)}{g}$

and $R = \frac{u^2\sin 2\theta}{g}$

$$t_1t_2 = \frac{4u^2\sin\theta\cos\theta}{g^2} = \frac{2}{g}\left[\frac{2u^2\sin\theta\cos\theta}{g}\right]$$

$$= \frac{2R}{g}$$