FLUIDS MECHANICS

1. The top of a water tank is open to air and its water level is maintained. It is giving out 0.74 m^3 water per minute through a circular opening of 2 cm radius in its wall. The depth of the centre of the opening from the level of water in the tank is close to :

(1) 9.6 m (2) 4.8 m

(3) 2.9 m (4) 6.0 m

2. A cylindrical plastic bottle of negligible mass is filled with 310 ml of water and left floating in a pond with still water. If pressed downward slightly and released, it starts performing simple harmonic motion at angular frequency ω. If the radius of the bottle is 2.5 cm then ω close to : (density of water = 10^3 kg / m^3)

(1) 5.00 rad s^{-1}

(2) 1.25 rad s^{-1}

(3) 3.75 rad s^{-1}

(4) 2.50 rad s^{-1}

3. Water flows into a large tank with flat bottom at the rate of 10^{-4} m^3s^{-1}. Water is also leaking out of a hole of area 1 cm^2 at its bottom. If the height of the water in the tank remains steady, then this height is:

(1) 4 cm (2) 2.9 cm

(3) 1.7 cm (4) 5.1 cm

4. A liquid of density ρ is coming out of a hose pipe of radius a with horizontal speed v and hits a mesh. 50% of the liquid passes through the mesh unaffected. 25% looses all of its momentum and 25% comes back with the same speed. The resultant pressure on the mesh will be :

(1) pv^2 (2) $\frac{3}{4}pv^2$

(3) $\frac{1}{2}pv^2$ (4) $\frac{1}{4}pv^2$

5. A load of mass M kg is suspended from a steel wire of length 2 m and radius 1.0 mm in Searle's apparatus experiment. The increase in length produced in the wire is 4.0 mm. Now the load is fully immersed in a liquid of relative density 2. The relative density of the material of load is 8. The new value of increase in length of the steel wire is :

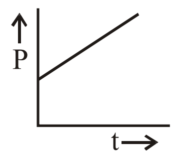
(1) 4.0mm (2) 3.0mm

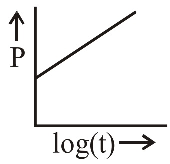
(3) 5.0mm (4) zero

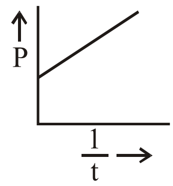
6. A long cylindrical vessel is half filled with a liquid. When the vessel is rotated about its own vertical axis, the liquid rises up near the wall. If the radius of vessel is 5 cm and its rotational speed is 2 rotations per second, then the difference in the heights between the centre and the sides, in cm, will be:

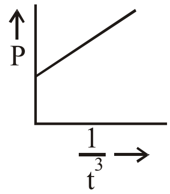
(1) 1.2 (2) 0.1 (3) 2.0 (4) 0.4

7. A soap bubble, blown by a mechanical pump at the mouth of a tube, increases in volume, with time, at a constant rate. The graph that correctly depicts the time dependence of pressure inside the bubble is given by :-

(1) 

(2) 

(3) 

(4) 

8. Water from a pipe is coming at a rate of 100 litres per minute. If the radius of the pipe is 5 cm, the Reynolds number for the flow is of the order of : (density of water = 1000 kg/m^3, coefficient of viscosity of water = 1mPas)

(1) 10^6 (2) 10^3

(3) 10^4 (4) 10^2

9. A wooden block floating in a bucket of water has $\frac{4}{5}$ of its volume submerged. When certain amount of an oil is poured into the bucket, it is found that the block is just under the oil surface with half of its volume under water and half in oil. The density of oil relative to that of water is :-

(1) 0.5 (2) 0.7 (3) 0.6 (4) 0.8

10. If 'M' is the mass of water that rises in a capillary tube of radius 'r', then mass of water which will rise in a capillary tube of radius '2r' is :

(1) 4M (2) M (3) 2M (4) $\frac{M}{2}$

11. A submarine experiences a pressure of 5.05×10^6 Pa at a depth of d_1 in a sea. When it goes further to a depth of d_2, it experiences a pressure of 8.08×10^6 Pa., Then $d_2 - d_1$ is approximately (density of water = 10^3 kg/m^3 and acceleration due to gravity = 10 ms^{-2})

(1) 500 m (2) 400 m
(3) 300 m (4) 600 m

12. Water from a tap emerges vertically downwards with an initial speed of 1.0 ms^{-1}. The cross-sectional area of the tap is 10^{-4} m^2. Assume that the pressure is constant throughout the stream of water and that the flow is streamlined. The cross-sectional area of the stream, 0.15 m below the tap would be: (Take g = 10 ms^{-2})

(1) 1×10^{-5} m^2 (2) 5×10^{-5} m^2
(3) 2×10^{-5} m^2 (4) 5×10^{-4} m^2

13. A cubical block of side 0.5 m floats on water with 30% of its volume under water. What is the maximum weight that can be put on the block without fully submerging it under water? (Take density of water = 10^3 kg/m^3)

(1) 65.4 kg (2) 87.5 kg
(3) 30.1 kg (4) 46.3 kg

14. The radtio of surface tensions of mercury and water is given to be 7.5 while the ratio of thier densities is 13.6. Their contact angles, with glass, are close to 135° and 0°, respectively. It is observed that mercury gets depressed by an amount h in a capillary tube of radius r_1, while water rises by the same amount h in a capillary tube of radius r_2. The ratio, (r_1/r_2), is then close to :

(1) 2/3 (2) 3/5
(3) 2/5 (4) 4/5

15. A solid sphere, of radius R acquires a terminal velocity ν_1 when falling (due to gravity) through a viscous fluid having a coefficient of viscosity η. The sphere is broken into 27 identical solid spheres. If each of these spheres acquires a terminal velocity, ν_2, when falling through the same fluid, the ratio (ν_1/ν_2) equals:

(1) 1/27 (2) 1/9 (3) 27 (4) 9

SOLUTION

1. Ans. (2)

In flow volume = outflow volume

$$\Rightarrow \frac{0.74}{60} = \left(\pi \times 4 \times 10^{-4}\right) \times \sqrt{2gh}$$

$$\Rightarrow \sqrt{2gh} = \frac{74 \times 100}{240\pi}$$

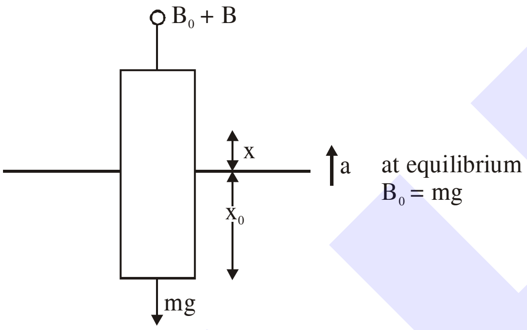
$$\Rightarrow \sqrt{2gh} = \frac{740}{24\pi}$$

$$\Rightarrow 2gh = \frac{740 \times 740}{24 \times 24 \times 10}\left(\pi^2 = 10\right)$$

$$\Rightarrow h = \frac{74 \times 74}{2 \times 24 \times 24}$$

$$\Rightarrow h \approx 4.8\text{m}$$

2. Ans. (Bonus)



Extra Boyant force = δAxg

$B_0 + B - mg = ma$

$B = ma$

$$a = \left(\frac{\delta Ag}{m}\right)x$$

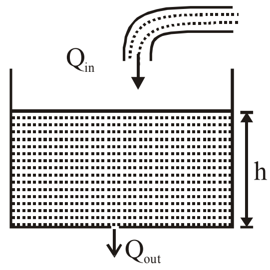
$$w^2 = \frac{\delta Ag}{m}$$

$$w = \sqrt{\frac{10^3 \times \pi(2.5)^2 \times 10^{-4} \times 10}{310 \times 10^{-6} \times 10^3}}$$

$$= \sqrt{63.30} = 7.95$$

No answer matches so Bonus by NTA

3. Ans. (4)



Since height of water column is constant therefore, water inflow rate (Q_{in})
= water outflow rate

$Q_{in} = 10^{-4}\ m^3s^{-1}$

$Q_{out} = Au = 10^{-4} \times \sqrt{2gh}$

$10^{-4} = 10^{-4}\ \sqrt{20 \times h}$

$$h = \frac{1}{20}\text{m}$$

h = 5cm

$\therefore$ correct answer is (4)

4. Ans. (2)

Momentum per second carried by liquid per second is ρav^2

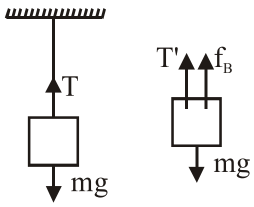
net force due to reflected liquid $= 2 \times \left[\frac{1}{4}\rho av^2\right]$

net force due to stopped liquid $= \frac{1}{4}\rho av^2$

Total force $= \frac{3}{4}\rho av^2$

net pressure $= \frac{3}{4}\rho v^2$

5. Ans. (2)



$$\frac{F}{A} = y.\frac{\Delta\ell}{\ell}$$

$\Delta\ell \propto F$ .....(i)

$T = mg$

$$T = mg - f_B = mg - \frac{m}{\rho_b}.\rho_\ell.g$$

$$= \left(1 - \frac{\rho_\ell}{\rho_b}\right)mg$$

$$= \left(1 - \frac{2}{8}\right) mg$$

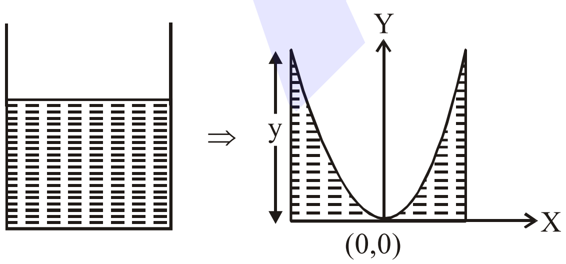
$$T' = \frac{3}{4}mg$$

From (i)

$$\frac{\Delta\ell'}{\Delta\ell} = \frac{T'}{T} = \frac{3}{4}$$

$$\Delta\ell' = \frac{3}{4} \cdot \Delta\ell = 3 \text{ mm}$$

6. Ans. (3)



$$y = \frac{\omega^2 x^2}{2g} = \frac{(2\times 2\pi)^2 \times (0.05)^2}{20} \simeq 2 \text{ cm}$$

7. Ans. (Bonus)

8. Ans. (3)

Sol. Reynolds Number $= \frac{\rho v d}{\eta}$

Volume flow rate = $v \times \pi r^2$

$$v = \frac{100\times10^{-3}}{60} \times \frac{1}{\pi \times 25 \times 10^{-4}}$$

$$v = \frac{2}{3\pi} m/s$$

$$\text{Reynolds Number} = \frac{10^3 \times 2 \times 10 \times 10^{-2}}{10^{-3} \times 3\pi}$$

$\simeq 2 \times 10^4$

order 10^4

9. Ans. (3)

Sol. In 1st situation

$V_b\rho_b g = V_s\rho_w g$

$$\frac{V_s}{V_b} = \frac{\rho_b}{\rho_w} = \frac{4}{5} \quad ...(i)$$

here V_b is volume of block

V_s is submerged volume of block

ρ_b is density of block

ρ_w is density of water

& Let ρ_o is density of oil

finally in equilibrium condition

$$V_b\rho_b\, g = \frac{V_b}{2}\rho_O g + \frac{V_b}{2}\rho_w g$$

$2\rho_b = \rho_0 + \rho_w$

$$\Rightarrow \frac{\rho_O}{\rho_w} = \frac{3}{5} = 0.6$$

10. Ans. (3)

Sol. Height of liquid rise in capillary tube $h = \frac{2T\cos\theta_C}{\rho r g}$

$$\Rightarrow h \propto \frac{1}{r}$$

when radius becomes double height become half

$$\therefore h' = \frac{h}{2}$$

Now, $M = \pi r^2 h \times \rho$

and $M' = \pi\,(2r)^2\,(h/2) \times \rho = 2M$

Option (3)

11. Ans. (3)

Sol. $P_0 + \rho g d_1 = P_1$

$P_0 + \rho g d_2 = P_2$

$\rho g(d_2 - d_1) = P_2 - P_1$

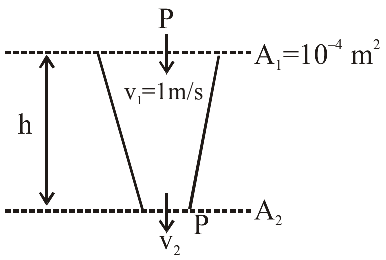
$10^3 \times 10\ (d_2 - d_1) = 3.03 \times 10^6$

$d_2 - d_1 = 303$ m

$\simeq 300$ m

12. Ans. (2)

Sol.



$A_1v_1 = A_2v_2$

$10^{-4} \times 1 = A_2v_2$

$A_2v_2 = 10^{-4}$ ....(1)

$P + \frac{1}{2}\rho(v_1^2 - v_2^2) + \rho gh = P$

$v_2^2 = v_1^2 + 2gh$

$v_2 = \sqrt{v_1^2 + 2gh}$

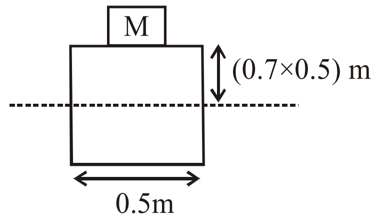
$= \sqrt{1 + 2 \times 10 \times 0.15}$

$\frac{10^{-4}}{A_2} = 2$

$A_2 = 5 \times 10^{-5}\ m^2$

13. Ans. (2)

Sol.



$M = \rho_L\ [0.5 \times 0.5 \times 0.35]$

$= 10^3\ [0.0875]$

$M = 87.5$ kg

14. Ans. (3)

Sol. $h = \frac{2S_1 \cos\theta_1}{r_1\rho_1 g}$

$h = \frac{2S_2 \cos\theta_2}{r_2\rho_2 g}$

$\Rightarrow \frac{r_1}{r_2} = \frac{2}{5}$

15. Ans. (4)

Sol. We have

$$V_T = \frac{2}{9}\frac{r^2}{\eta}(\rho_0 - \rho_\ell)g \Rightarrow v_T \propto r^2$$

since mass of the sphere will be same

$$\therefore\ \rho\frac{4}{3}\pi R^3 = 27.\frac{4}{3}\pi r^3\rho \Rightarrow r = \frac{R}{3}$$

$$\therefore\ \frac{v_1}{v_2} = \frac{R^2}{r^2} = 9$$