CIRCULAR MOTION

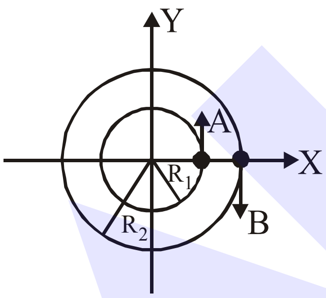
1. A body is projected at t = 0 with a velocity 10 ms^{-1} at an angle of 60° with the horizontal. The radius of curvature of its trajectory at t = 1s is R. Neglecting air resistance and taking acceleration due to gravity g = 10 ms^{-2}, the value of R is :

(1) 2.5 m (2) 10.3 m
(3) 2.8 m (4) 5.1 m

2. A particle is moving along a circular path with a constant speed of 10 ms^{-1}. What is the magnitude of the change is velocity of the particle, when it moves through an angle of 60° around the centre of the circle?

(1) zero (2) 10 m/s
(3) $10\sqrt{3}$ m/s (4) $10\sqrt{2}$ m/s

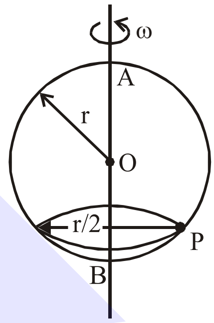
3. Two particles A, B are moving on two concentric circles of radii R_1 and R_2 with equal angular speed ω. At t = 0, their positions and direction of motion are shown in the figure :



The relative velocity $\vec{\upsilon}_A - \vec{\upsilon}_B$ at $t = \frac{\pi}{2\omega}$ is given by :

(1) $-\omega(R_1 + R_2)\hat{i}$ (2) $\omega(R_1 + R_2)\hat{i}$
(3) $\omega(R_1 - R_2)\hat{i}$ (4) $\omega(R_2 - R_1)\hat{i}$

4. A smooth wire of length 2πr is bent into a circle and kept in a vertical plane. A bead can slide smoothly on the wire. When the circle is rotating with angular speed ω about the vertical diameter AB, as shown in figure, the bead is at rest with respect to the circular ring at position P as shown. Then the value of ω^2 is equal to :



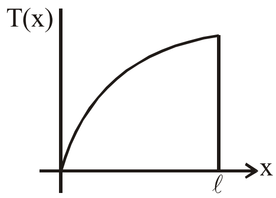
(1) $(g\sqrt{3})/r$

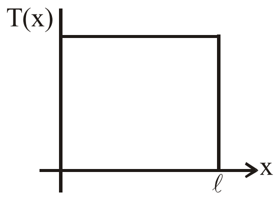
(2) $\frac{\sqrt{3}g}{2r}$

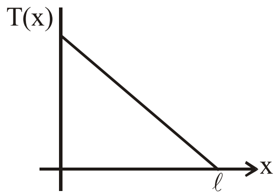
(3) 2g/r

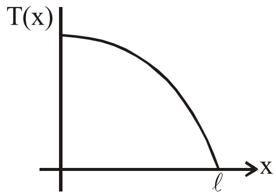
(4) $2g/(r\sqrt{3})$

5. A uniform rod of length ℓ is being rotated in a horizontal plane with a constant angular speed about an axis passing through one of its ends. If the tension generated in the rod due to rotation is T(x) at a distance x from the axis, then which of the following graphs depicts it most closely?

(1)


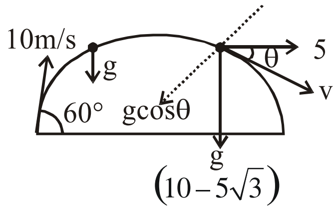
(2)


(3)


(4)


SOLUTION

1. Ans. (3)



$v_x = 10\cos 60° = 5$ m/s

$v_y = 10\cos 30° = 5\sqrt{3}$ m/s

velocity after t = 1 sec.

$v_x = 5$ m/s

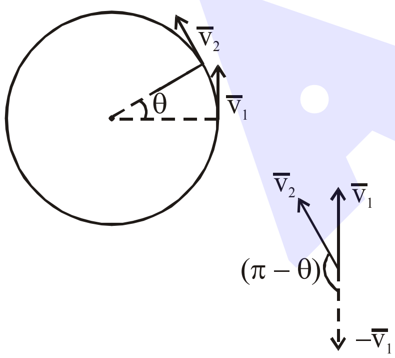
$v_y = \left|\left(5\sqrt{3}-10\right)\right| m/s = 10 - 5\sqrt{3}$

$$a_n = \frac{v^2}{R} \Rightarrow R = \frac{v_x^2+v_y^2}{a_n} = \frac{25+100+75-100\sqrt{3}}{10\cos\theta}$$

$$\tan\theta = \frac{10-5\sqrt{3}}{5} = 2-\sqrt{3} \Rightarrow \theta = 15°$$

$$R = \frac{100\left(2-\sqrt{3}\right)}{10\cos 15} = 2.8\text{m}$$

2. Ans. (2)



$$|\Delta\overline{v}| = \sqrt{v_1^2+v_2^2+2v_1v_2\cos(\pi-\theta)}$$

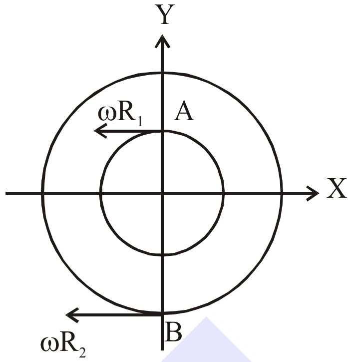
$$= 2v\sin\frac{\theta}{2} \quad \text{since}\left[|\overline{v}_1| = |\overline{v}_2|\right]$$

$= (2 \times 10) \times \sin(30°)$

$= 10$ m/s

3. Ans. (4)

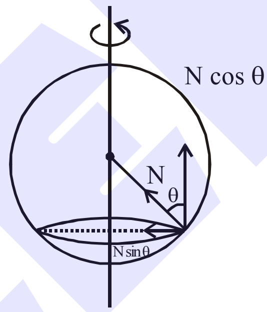
$$\theta = \omega t = \omega\frac{\pi}{2\omega} = \frac{\pi}{2}$$



$$\vec{V}_A - \vec{V}_S = \omega R_1\left(-\hat{i}\right) - \omega R_2\left(-i\right)$$

4. Ans. (4)

Sol.



$N\sin\theta = m\dfrac{r}{2}\omega^2$ ......(1)

$N\cos\theta = mg$ ....(2)

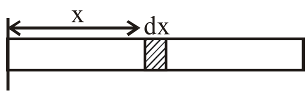
$$\tan\theta = \frac{r\omega^2}{2g}$$

$$\frac{r}{2\dfrac{\sqrt{3}r}{2}} = \frac{r\omega^2}{2g}$$

$$\omega^2 = \frac{2g}{\sqrt{3}r}$$

5. Ans. (4)

Sol.



$$T = \int_{x=x}^{x=\ell} dm\omega^2 x = \int_{x=x}^{x=\ell}\frac{m}{\ell}dx\,\omega^2 x$$

$$= \frac{m\omega^2}{2\ell}\left(\ell^2 - x^2\right)$$

$$\boxed{T = \frac{m\omega^2}{2\ell}\left(\ell^2 - x^2\right)}$$