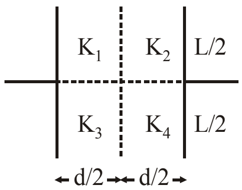
CAPACITOR

1. A parallel plate capacitor with square plates is filled with four dielectrics of dielectric constants K_1, K_2, K_3, K_4 arranged as shown in the figure. The effective dielectric constant K will be :



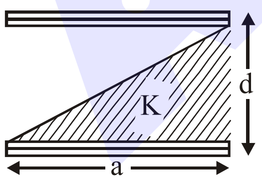
(1) $K = \dfrac{(K_1 + K_2)(K_3 + K_4)}{2(K_1 + K_2 + K_3 + K_4)}$

(2) $K = \dfrac{(K_1 + K_2)(K_3 + K_4)}{(K_1 + K_2 + K_3 + K_4)}$

(3) $K = \dfrac{(K_1 + K_4)(K_2 + K_3)}{2(K_1 + K_2 + K_3 + K_4)}$

(4) $K = \dfrac{(K_1 + K_3)(K_2 + K_4)}{K_1 + K_2 + K_3 + K_4}$

2. A parallel plate capacitor is made of two square plates of side 'a', separated by a distance d (d<<a). The lower triangular portion is filled with a dielectric of dielectric constant K, as shown in the figure.



Capacitance of this capacitor is :

(1) $\dfrac{1}{2}\dfrac{k \in_0 a^2}{d}$ (2) $\dfrac{k \in_0 a^2}{d}\ln K$

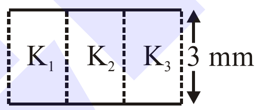
(3) $\dfrac{k \in_0 a^2}{d(K-1)}\ln K$ (4) $\dfrac{k \in_0 a^2}{2d(K+1)}$

3. A parallel plate capacitor having capacitance 12 pF is charged by a battery to a potential difference of 10 V between its plates. The charging battery is now disconnected and a porcelain slab of dielectric constant 6.5 is slipped between the plates the work done by the capacitor on the slab is :

(1) 692 pJ (2) 60 pJ

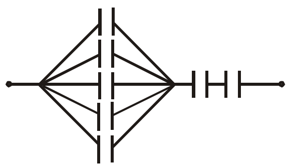
(3) 508 pJ (4) 560 pJ

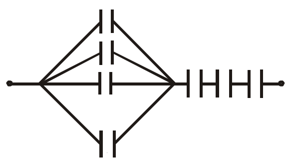
4. A parallel plate capacitor is of area 6 cm^2 and a separation 3 mm. The gap is filled with three dielectric materials of equal thickness (see figure) with dielectric constants K_1, = 10, K_2 = 12 and K_3 = 14. The dielectric constant of a material which when fully inserted in above capacitor, gives same capacitance would be :

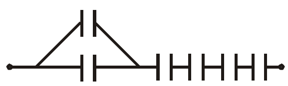


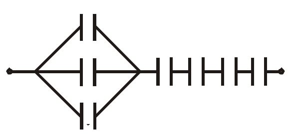
(1) 12 (2) 4 (3) 36 (4) 14

5. Seven capacitors, each of capacitance 2 μF, are to be connected in a configuration to obtain an effective capacitance of $\left(\dfrac{6}{13}\right)\mu F$. Which of the combinations, shown in figures below, will achieve the desired value ?

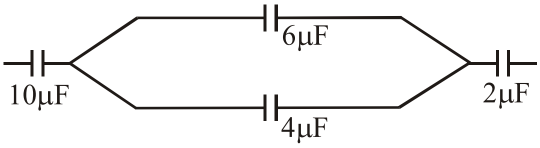
(1) 

(2) 

(3) 

(4) 

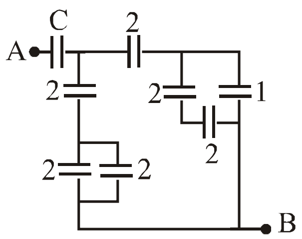
6. In the figure shown below, the charge on the left plate of the 10 μF capacitor is –30 μC. ? The charge on the right plate of the 6 μF capacitor is :



(1) –18 μC (2) –12 μC

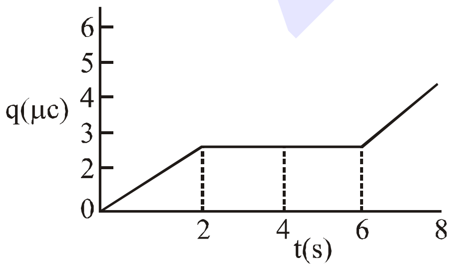
(3) +12 μC (4) +18 μC

7. In the circuit shown, find C if the effective capacitance of the whole circuit is to be 0.5 μF. All values in the circuit are in μF.



(1) $\frac{7}{10}$μF (2) $\frac{7}{11}$μF (3) $\frac{6}{5}$μF (4) 4μF

8. The charge on a capacitor plate in a circuit, as a function of time, is shown in the figure: What is the value of current at t = 4 s ?



(1) 3μA (2) 2μA

(3) zero (4) 1.5μA

9. A parallel plate capacitor with plates of area 1m^2 each, area t a separation of 0.1 m. If the electric field between the plates is 100 N/C, the magnitude of charge each plate is :-

(Take $\varepsilon_0 = 8.85\times10^{-12}\frac{C^2}{N-m^2}$)

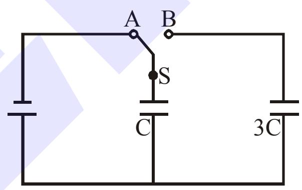
(1) 7.85×10^{-10} C

(2) 6.85×10^{-10} C

(3) 9.85×10^{-10} C

(4) 8.85×10^{-10} C

10. In the figure shown, after the switch 'S' is turned from position 'A' to position 'B', the energy dissipated in the circuit in terms of capacitance 'C' and total charge 'Q' is:



(1) $\frac{3}{8}\frac{Q^2}{C}$ (2) $\frac{3}{4}\frac{Q^2}{C}$

(3) $\frac{1}{8}\frac{Q^2}{C}$ (4) $\frac{5}{8}\frac{Q^2}{C}$

11. A parallel plate capacitor has 1μF capacitance. One of its two plates is given +2μC charge and the other plate, +4μC charge. The potential difference developed across the capacitor is:-

(1) 5V (2) 2V

(3) 3V (4) 1V

12. Voltage rating of a parallel plate capacitor is 500V. Its dielectric can withstand a maximum electric field of 10^6 V/m. The plate area is 10^{-4} m^2. What is the dielectric constant is the capacitance is 15 pF? (given $\epsilon_0 = 8.86 \times 10^{-12}$ C^2/Nm^2)

(1) 3.8 (2) 4.5

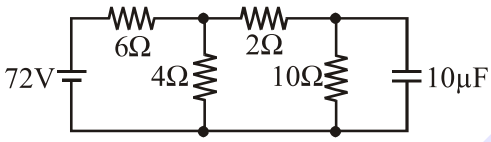
(3) 6.2 (4) 8.5

13. The parallel combination of two air filled parallel plate capacitors of capacitance C and nC is connected to a battery of voltage, V. When the capacitors are fully charged, the battery is removed and after that a dielectric material of dielectric constant K is placed between the two plates of the first capacitor. The new potential difference of the combined system is :-

(1) $\frac{V}{K+n}$ (2) V

(3) $\frac{(n+1)V}{(K+n)}$ (4) $\frac{nV}{K+n}$

14. Determine the charge on the capacitor in the following circuit :



(1) 2μC (2) 60μC

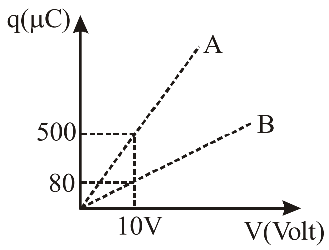
(3) 200μC (4) 10μC

15. A capacitor with capacitance 5μF is charged to 5μC. If the plates are pulled apart to reduce the capacitance to 2μF, how much work is done ?

(1) 3.75×10^{-6} J (2) 2.55×10^{-6} J

(3) 2.16×10^{-6} J (4) 6.25×10^{-6} J

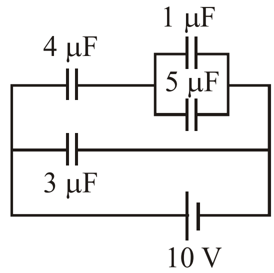
16. Figure shows charge (q) versus voltage (V) graph for series and parallel combination of two given capacitors. The capacitances are :



(1) 50 μF and 30 μF (2) 20 μF and 30 μF

(3) 60 μF and 40 μF (4) 40 μF and 10 μF

17. In the given circuit, the charge on 4 μF capacitor will be :

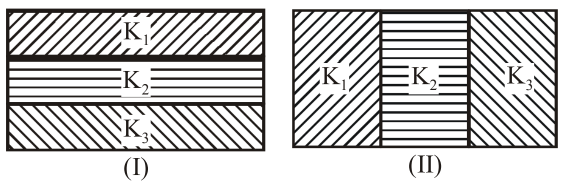


(1) 5.4 μC (2) 24 μC

(3) 13.4 μC (4) 9.6 μC

18. Two identical parallel plate capacitors, of capacitance C each, have plates of area A, separated by a distance d. The space between the plates of the two capacitors, is filled with three dielectrics, of equal thickness and dielectric constants K_1, K_2 and K_3. The first capacitor is filled as shown in fig. I, and the second one is filled as shown in fig. II.

If these two modified capacitors are charged by the same potential V, the ratio of the energy stored in the two, would be (E_1 refers to capacitor (I) and E_2 to capacitor (II)) :



(1) $\frac{E_1}{E_2} = \frac{9K_1K_2K_3}{(K_1+K_2+K_3)(K_2K_3+K_3K_1+K_1K_2)}$

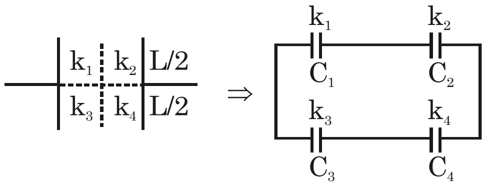
(2) $\frac{E_1}{E_2} = \frac{K_1K_2K_3}{(K_1+K_2+K_3)(K_2K_3+K_3K_1+K_1K_2)}$

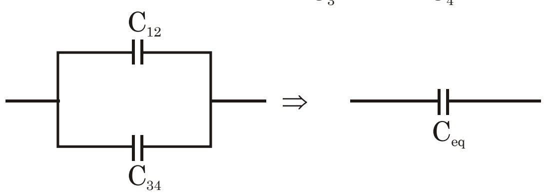
(3) $\frac{E_1}{E_2} = \frac{(K_1+K_2+K_3)(K_2K_3+K_3K_1+K_1K_2)}{K_1K_2K_3}$

(4) $\frac{E_1}{E_2} = \frac{(K_1+K_2+K_3)(K_2K_3+K_3K_1+K_1K_2)}{9K_1K_2K_3}$

SOLUTION

1. Ans. (Bonus)





$$C_{12} = \frac{C_1C_2}{C_1+C_2} = \frac{\frac{k_1 \in_0 \frac{L}{2}\times L}{d/2}\cdot\frac{k_2\left[\in_0 \frac{L}{2}\times L\right]}{d/2}}{(k_1+k_2)\left[\frac{\in_0 \cdot\frac{L}{2}\times L}{d/2}\right]}$$

$$C_{12} = \frac{k_1k_2}{k_1+k_2}\frac{\in_0 L^2}{d}$$

in the same way we get, $C_{34} = \frac{k_3k_4}{k_3+k_4}\frac{\in_0 L^2}{d}$

$$\therefore C_{eq} = C_{12} + C_{34} = \left[\frac{k_1k_2}{k_1+k_2}+\frac{k_3k_4}{k_3+k_4}\right]\frac{\in_0 L^2}{d} \quad ..(i)$$

Now if $k_{eq} = k$, $C_{eq} = \frac{k \in_0 L^2}{d}$ .....(ii)

on comparing equation (i) to equation (ii), we get

$$k_{eq} = \frac{k_1k_2(k_3+k_4)+k_3k_4(k_1+k_2)}{(k_1+k_2)(k_3+k_4)}$$

This does not match with any of the options so probably they have assumed the wrong combination

$$C_{13} = \frac{k_1 \in_0 L\frac{L}{2}}{d/2} + k_3 \in_0 \frac{L.\frac{L}{2}}{d/2}$$

$$= (k_1 + k_3)\frac{\in_0 L^2}{d}$$

$$C_{24} = (k_2 + k_4)\ \frac{\in_0 L^2}{d}$$

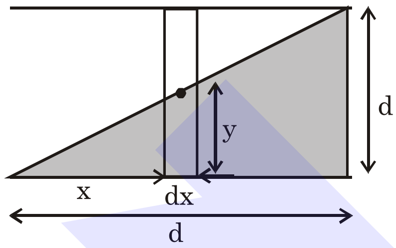
$$C_{eq} = \frac{C_{13}C_{24}}{C_{13}C_{24}} = \frac{(k_1+k_3)(k_2+k_4)}{(k_1+k_2+k_3+k_4)}\frac{\in_0 L^2}{d}$$

$$= \frac{k \in_0 L^2}{d}$$

$$k = \frac{(k_1+k_3)(k_2+k_4)}{(k_1+k_2+k_3+k_4)}$$

However this is one of the four options.
It must be a "Bonus" logically but of the given options probably they might go with (4)

2. Ans. (3)



$$\frac{y}{x} = \frac{d}{a}$$

$$y = \frac{d}{a}x$$

$$dy = \frac{d}{a}(dx)$$

$$\frac{1}{dc} = \frac{y}{KE\cdot adx} + \frac{(d-y)}{\in_0 adx}$$

$$\frac{1}{dc} = \frac{1}{\in_0 adx}\left(\frac{y}{k}+d-y\right)$$

$$\int dc = \int\frac{\in_0 adx}{\frac{y}{k}+d-y}$$

$$c = \in_0 a\cdot\frac{a}{d}\int_0^d\frac{dy}{d+y\left(\frac{1}{k}-1\right)}$$

$$= \frac{\in_0 a^2}{\left(\frac{1}{k}-1\right)d}\left[\ell n\left(d+y\left(\frac{1}{k}-1\right)\right)\right]_0^d$$

$$= \frac{k \in_0 a^2}{(1-k)d}\ell n\left(\frac{d+d\left(\frac{1}{k}-1\right)}{d}\right)$$

$$= \frac{k \in_0 a^2}{(1-k)d}\ell n\left(\frac{1}{k}\right) = \frac{k \in_0 a^2 \ell nk}{(k-1)d}$$

3. Ans. (3)

Initial energy of capacitor

$$U_i = \frac{1}{2}\frac{v^2}{c}$$

$$= \frac{1}{2} \times \frac{120 \times 120}{12} = 600 \text{ J}$$

Since battery is disconnected so charge remain same.

Final energy of capacitor

$$U_f = \frac{1}{2}\frac{v^2}{c}$$

$$= \frac{1}{2} \times \frac{120 \times 120}{12 \times 6.5} = 92$$

$W + U_f = U_i$

$W = 508$ J

4. Ans. (1)

Let dielectric constant of material used be K.

$$\therefore \frac{10 \in_0 A/3}{d} + \frac{12 \in_0 A/3}{d} + \frac{14 \in_0 A/3}{d} = \frac{K \in_0 A}{d}$$

$\Rightarrow K = 12$

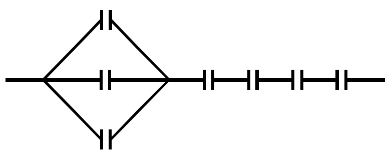
5. Ans. (4)

$$C_{eq} = \frac{6}{13}\mu F$$

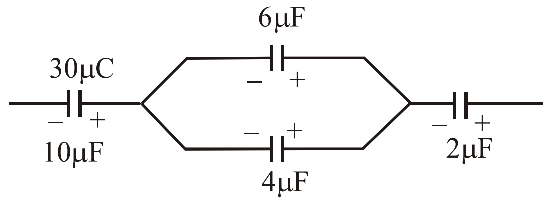
Therefore three capacitors most be in parallel to get 6 in

$$\frac{1}{C_{eq}} = \frac{1}{3C} + \frac{1}{C} + \frac{1}{C} + \frac{1}{C} + \frac{1}{C}$$

$$C_{eq} = \frac{3C}{13} = \frac{6}{13}\mu F$$



6. Ans. (4)



6μF & 4μF are in parallel & total charge on this combination is 30 μC

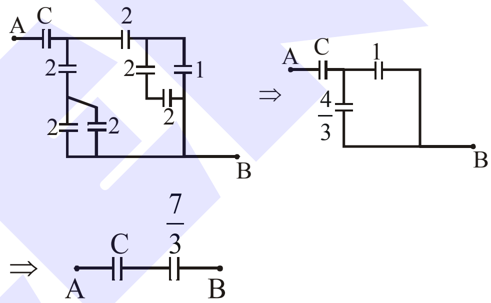
$\therefore$ Charge on 6μF capacitor = $\frac{6}{6+4} \times 30$

= 18 μC

Since charge is asked on right plate therefore is +18μC

Correct answer is (4)

7. Ans. (2)



From equs.

$$\frac{\frac{7C}{3}}{\frac{7}{3} + C} = \frac{1}{2}$$

$\Rightarrow 14\ C = 7 + 3\ C$

$\Rightarrow C = \frac{7}{11}$

8. Ans. (3)

9. Ans. (4)

$$E = \frac{\sigma}{\in_0} = \frac{Q}{A \in_0}$$

$Q = AE\in_0$

$Q = (1)(100)(8.85 \times 10^{-12})$

$Q = 8.85 \times 10^{-10}$C

10. Ans. (1)

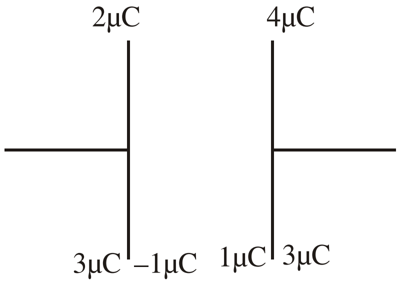
$$V_i = \frac{1}{2}CE^2$$

$$V_f = \frac{(CE)^2}{2 \times 4c} = \frac{1}{2}\frac{CE^2}{4}$$

$$\Delta E = \frac{1}{2}CE^2 \times \frac{3}{4} = \frac{3}{8}CE^2$$

11. Ans. (4)

Sol.



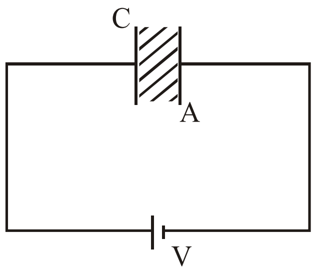
Charges at inner plates are 1μC and –1μC

∴ Potential difference across capacitor

$$= \frac{q}{c} = \frac{1\mu C}{1\mu F} = \frac{1\times10^{-6}C}{1\times10^{-6}\,\text{Farad}} = 1V$$

12. Ans. (4)

Sol.



$A = 10^{-4}$ m^2

$E_{max} = 10^6$ V/m

$C = 15$ μF

$$C = \frac{k\varepsilon_0 A}{d}$$

$$\frac{Cd}{\varepsilon_0 A} = k$$

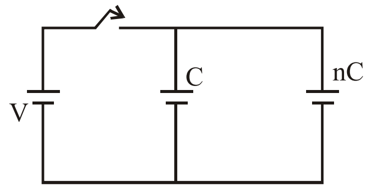
$$k = \frac{15\times10^{-12}\times500\times10^{-6}}{8.86\times10^{-12}\times10^{4}}$$

$$= \frac{15\times5}{8.86} = 8.465$$

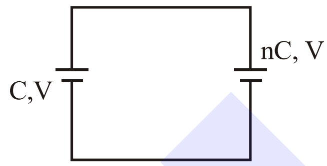
$k \approx 8.5$

13. Ans. (3)

Sol.



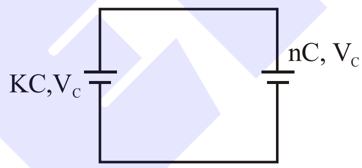
After fully charging, battery is disconnected



Total charge of the system = CV + nCV

= (n + 1)CV

After the insertion of dielectric of constant K



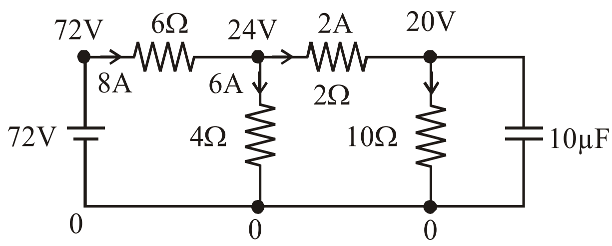
New potential (common)

$$V_C = \frac{\text{total charge}}{\text{total capacitance}}$$

$$= \frac{(n+1)CV}{KC+nC} = \frac{(n+1)V}{K+n}.$$

14. Ans. (3)

Sol. Applying point potential method



q = cV

q = 10μF × 20 = 200μC

Option (3)

15. Ans. (1)

Sol. Work done = ΔU

$$= U_f - U_i$$

$$= \frac{q^2}{2C_f} - \frac{q^2}{2C_i}$$

$$= \frac{\left(5\times10^{-6}\right)^2}{2}\left(\frac{1}{2\times10^{-6}} - \frac{1}{5\times10^{-6}}\right)$$

$$= \frac{15}{4}\times10^{-6}$$

$$= 3.75\times10^{-6}\ \text{J}$$

Option (1)

16. Ans. (4)

Sol. As q = CV

Hence slope of graph will give capacitance. Slope will be more in parallel combination. Hence capacitance in parallel should be 50μF & in series combination must be 8μF.

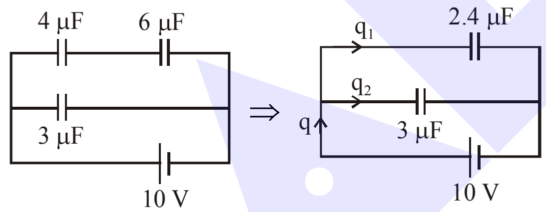
Only in option 40μF & 10 μF

$$C_{parallel} = 40 + 10 = 50\mu F$$

$$C_{series} = \frac{40\times10}{40+10} = 8\ \mu F$$

17. Ans. (2)

Sol.



So total charge flow = q = 5.4 μF × 10V

= 54 μC

The charge will be distributed in the ratio of capacitance

$$\Rightarrow \frac{q_1}{q_2} = \frac{2.4}{3} = \frac{4}{5}$$

∴ 9X = 54 μC ⇒ X = 6 μC

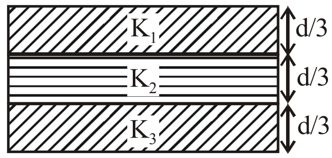
∴ charge on 4 μF capacitor

will be = 4X = 4 × 6 μC

= 24 μC

18. Ans. (1)

Sol. I.

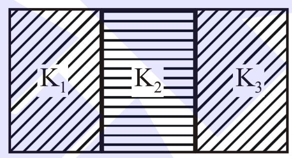


$$C_1 = \frac{3\varepsilon_0 AK_1}{d} \quad C_2 = \frac{3\varepsilon_0 AK_2}{d} \quad C_3 = \frac{3\varepsilon_0 AK_3}{d}$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$\Rightarrow C_{eq} = \frac{3\varepsilon_0 AK_1K_2K_3}{d(K_1K_2 + K_2K_3 + K_3K_1)} \qquad \text{........(1)}$$

II.



$$C_1 = \frac{\varepsilon_0 K_1 A}{3d} \quad C_2 = \frac{\varepsilon_0 K_2 A}{3d} \quad C_3 = \frac{\varepsilon_0 K_3 A}{3d}$$

$$C'_{eq} = C_1 + C_2 + C_3$$

$$= \frac{\varepsilon_0 A}{3d}(K_1 + K_2 + K_3) \qquad \text{........(2)}$$

Now,

$$\frac{E_1}{E_2} = \frac{\frac{1}{2}C_{eq}.V^2}{\frac{1}{2}C'_{eq}V^2} = \frac{9K_1K_2K_3}{(K_1+K_2+K_3)(K_1K_2+K_2K_3+K_3K_1)}$$