

METHOD OF DIFFERENTIATION

- यदि $x = 3 \tan t$ तथा $y = 3 \sec t$ है, तो $t = \frac{\pi}{4}$ पर $\frac{d^2y}{dx^2}$ का मान है :
 (1) $\frac{3}{2\sqrt{2}}$ (2) $\frac{1}{3\sqrt{2}}$ (3) $\frac{1}{6}$ (4) $\frac{1}{6\sqrt{2}}$
- माना फलन $f : \mathbb{R} \rightarrow \mathbb{R}$ इस प्रकार है कि $f(x) = x^3 + x^2 f'(1) + x f''(2) + f'''(3)$, $x \in \mathbb{R}$ तो $f(2)$ बराबर है
 (1) 8 (2) -2 (3) -4 (4) 30
- यदि $x \log_e(\log_e x) - x^2 + y^2 = 4$ ($y > 0$), तो $x = e$ पर dy/dx बराबर है :
 (1) $\frac{e}{\sqrt{4+e^2}}$ (2) $\frac{(1+2e)}{2\sqrt{4+e^2}}$
 (3) $\frac{(2e-1)}{2\sqrt{4+e^2}}$ (4) $\frac{(1+2e)}{\sqrt{4+e^2}}$
- यदि $x > 1$ के लिये $(2x)^{2y} = 4e^{2x-2y}$ है, तो $(1 + \log_e 2x)^2 \frac{dy}{dx}$ बराबर है
 (1) $\log_e 2x$ (2) $\frac{x \log_e 2x + \log_e 2}{x}$
 (3) $x \log_e 2x$ (4) $\frac{x \log_e 2x - \log_e 2}{x}$
- माना f एक अवकलनीय फलन इस प्रकार है कि $f(1) = 2$ तथा सभी $x \in \mathbb{R}$ के लिये $f'(x) = f(x)$ है। यदि $h(x) = f(f(x))$ हो, तो $h'(1)$ होगा :
 (1) $4e$ (2) $4e^2$ (3) $2e$ (4) $2e^2$

- यदि $2y = \left(\cot^{-1} \left(\frac{\sqrt{3} \cos x + \sin x}{\cos x - \sqrt{3} \sin x} \right) \right)^2$, $x \in \left(0, \frac{\pi}{2} \right)$ है, तो $\frac{dy}{dx}$ बराबर है -
 (1) $2x - \frac{\pi}{3}$ (2) $\frac{\pi}{3} - x$
 (3) $\frac{\pi}{6} - x$ (4) $x - \frac{\pi}{6}$
- यदि $f(1) = 1, f'(1) = 3$ है तो $f(f(f(x))) + (f(x))^2$ का $x = 1$ पर अवकलज है :-
 (1) 12 (2) 33 (3) 9 (4) 15
- माना $f(x) = \log_e(\sin x)$, ($0 < x < \pi$) तथा $g(x) = \sin^{-1}(e^{-x})$, ($x \geq 0$) हैं। यदि एक धनात्मक वास्तविक संख्या α के लिए $a = (f \circ g)'(\alpha)$ तथा $b = (g \circ f)'(\alpha)$ है, तो :
 (1) $a\alpha^2 - b\alpha - a = 0$
 (2) $a\alpha^2 + b\alpha - a = -2\alpha^2$
 (3) $a\alpha^2 + b\alpha + a = 0$
 (4) $a\alpha^2 - b\alpha - a = 1$
- यदि $e^y + xy = e$, तो $x = 0$ पर क्रमित युग्म $\left(\frac{dy}{dx}, \frac{d^2y}{dx^2} \right)$ बराबर हैं :
 (1) $\left(-\frac{1}{e}, \frac{1}{e^2} \right)$ (2) $\left(\frac{1}{e}, \frac{1}{e^2} \right)$
 (3) $\left(\frac{1}{e}, -\frac{1}{e^2} \right)$ (4) $\left(-\frac{1}{e}, -\frac{1}{e^2} \right)$
- $\frac{x}{2}$ के सापेक्ष $\tan^{-1} \left(\frac{\sin x - \cos x}{\sin x + \cos x} \right)$, जहाँ $\left(x \in \left(0, \frac{\pi}{2} \right) \right)$ का अवकलज है :
 (1) $\frac{1}{2}$ (2) $\frac{2}{3}$ (3) 1 (4) 2

SOLUTION

1. **Ans. (4)**

$$\frac{dx}{dt} = 3\sec^2 t$$

$$\frac{dy}{dt} = 3\sec t \tan t$$

$$\frac{dy}{dx} = \frac{\tan t}{\sec t} = \sin t$$

$$\frac{d^2y}{dx^2} = \cos t \frac{dt}{dx}$$

$$= \frac{\cos t}{3\sec^2 t} = \frac{\cos^3 t}{3} = \frac{1}{3 \cdot 2\sqrt{2}} = \frac{1}{6\sqrt{2}}$$

2. **Ans. (2)**

$$f(x) = x^3 + x^2 f'(1) + x f''(2) + f'''(3)$$

$$\Rightarrow f'(x) = 3x^2 + 2x f'(1) + f''(x) \quad \dots(1)$$

$$\Rightarrow f''(x) = 6x + 2f'(1) \quad \dots(2)$$

$$\Rightarrow f'''(x) = 6 \quad \dots(3)$$

put $x = 1$ in equation (1) :

$$f'(1) = 3 + 2f'(1) + f''(2) \quad \dots(4)$$

put $x = 2$ in equation (2) :

$$f''(2) = 12 + 2f'(1) \quad \dots(5)$$

from equation (4) & (5) :

$$-3 - f'(1) = 12 + 2f'(1)$$

$$\Rightarrow 3f'(1) = -15$$

$$\Rightarrow f'(1) = -5 \Rightarrow f''(2) = 2 \quad \dots(2)$$

put $x = 3$ in equation (3) :

$$f'''(3) = 6$$

$$\therefore f(x) = x^3 - 5x^2 + 2x + 6$$

$$f(2) = 8 - 20 + 4 + 6 = -2$$

3. **Ans. (3)**Differentiating with respect to x ,

$$x \cdot \frac{1}{\ln x} \cdot \frac{1}{x} + \ln(\ln x) - 2x + 2y \cdot \frac{dy}{dx} = 0$$

at $x = e$ we get

$$1 - 2e + 2y \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{2e - 1}{2y}$$

$$\Rightarrow \frac{dy}{dx} = \frac{2e - 1}{2\sqrt{4 + e^2}} \text{ as } y(e) = \sqrt{4 + e^2}$$

4. **Ans. (4)**

$$(2x)^{2y} = 4e^{2x-2y}$$

$$2y \ln 2x = \ln 4 + 2x - 2y$$

$$y = \frac{x + \ln 2}{1 + \ln 2x}$$

$$y' = \frac{(1 + \ln 2x) - (x + \ln 2) \frac{1}{x}}{(1 + \ln 2x)^2}$$

$$y'(1 + \ln 2x)^2 = \left[\frac{x \ln 2x - \ln 2}{x} \right]$$

5. **Ans (1)**

$$\frac{f'(x)}{f(x)} = 1 \quad \forall x \in \mathbb{R}$$

Integrate & use $f(1) = 2$

$$f(x) = 2e^{x-1} \Rightarrow f'(x) = 2e^{x-1}$$

$$h(x) = f(f(x)) \Rightarrow h'(x) = f'(f(x)) f'(x)$$

$$h'(1) = f'(f(1)) f'(1)$$

$$= f'(2) f'(1)$$

$$= 2e \cdot 2 = 4e$$

6. Official Ans. by NTA (4)

Sol. Consider $\cot^{-1} \left(\frac{\frac{\sqrt{3}}{2} \cos x + \frac{1}{2} \sin x}{\frac{1}{2} \sin x - \frac{\sqrt{3}}{2} \cos x} \right)$

$$= \cot^{-1} \left(\frac{\sin \left(x + \frac{\pi}{3} \right)}{\cos \left(x + \frac{\pi}{3} \right)} \right)$$

$$= \cot^{-1} \left(\tan \left(x + \frac{\pi}{3} \right) \right) = \frac{\pi}{2} - \tan^{-1} \left(\tan \left(x + \frac{\pi}{3} \right) \right)$$

$$\begin{cases} \frac{\pi}{2} - \left(x + \frac{\pi}{3} \right) = \left(\frac{\pi}{6} - x \right); & 0 < x < \frac{\pi}{6} \\ \frac{\pi}{2} - \left(\left(x - \frac{\pi}{3} \right) - \pi \right) = \left(\frac{7\pi}{6} - x \right); & \frac{\pi}{6} < x < \frac{\pi}{2} \end{cases}$$

$$\therefore 2y = \begin{cases} \left(\frac{\pi}{6} - x \right)^2; & 0 < x < \frac{\pi}{6} \\ \left(\frac{7\pi}{6} - x \right)^2; & \frac{\pi}{6} < x < \frac{\pi}{2} \end{cases}$$

$$\therefore 2 \frac{dy}{dx} = \begin{cases} 2 \left(\frac{\pi}{6} - x \right) \cdot (-1); & 0 < x < \frac{\pi}{6} \\ 2 \left(\frac{7\pi}{6} - x \right) \cdot (-1); & \frac{\pi}{6} < x < \frac{\pi}{2} \end{cases}$$

7. Official Ans. by NTA (2)

Sol. $y = f(f(f(x))) + (f(x))^2$

$$\begin{aligned} \frac{dy}{dx} &= f'(f(f(x)))f'(f(x))f'(x) + 2f(x)f'(x) \\ &= f'(1)f'(1)f'(1) + 2f(1)f'(1) \\ &= 3 \times 5 \times 3 + 2 \times 1 \times 3 \\ &= 27 + 6 \\ &= 33 \end{aligned}$$

8. Official Ans. by NTA (4)

Sol. $f \circ g(x) = (-x) \Rightarrow (f \circ g(\alpha)) = -\alpha = b$
 $(f \circ g(x))' = -1 \Rightarrow (f \circ g(\alpha))' = -1 = a$

9. Official Ans. by NTA (1)

Sol. $e^y = xy = e$
 differentiate w.r.t. x

$$e^y \frac{dy}{dx} + x \frac{dy}{dx} + y = 0$$

$$\frac{dy}{dx}(x + e^y) = -y, \quad \left. \frac{dy}{dx} \right|_{(0,1)} = -\frac{1}{e}$$

again differentiate w.r.t. x

$$e^y \cdot \frac{d^2y}{dx^2} + \frac{dy}{dx} \cdot e^y \cdot \frac{dy}{dx} + x \cdot \frac{d^2y}{dx^2} + \frac{dy}{dx} + \frac{dy}{dx} = 0$$

$$(x + e^y) \frac{d^2y}{dx^2} + \left(\frac{dy}{dx} \right)^2 \cdot e^y + 2 \frac{dy}{dx} = 0$$

$$e \frac{d^2y}{dx^2} + \frac{1}{e^2} e + 2 \left(-\frac{1}{e} \right) = 0$$

$$\therefore \frac{d^2y}{dx^2} = \frac{1}{e^2}$$

10. Official Ans. by NTA (4)

Sol. $f(x) = \tan^{-1} \left(\frac{\sin x - \cos x}{\sin x + \cos x} \right)$

$$= \tan^{-1} \left(\frac{\tan x - 1}{\tan x + 1} \right) = \tan^{-1} \left(\tan \left(x - \frac{\pi}{4} \right) \right)$$

$$\therefore x - \frac{\pi}{4} \in \left(-\frac{\pi}{4}, \frac{\pi}{4} \right)$$

$$\therefore f(x) = x - \frac{\pi}{4}$$

$$\Rightarrow \text{its derivative w.r.t. } \frac{x}{2} \text{ is } \frac{1}{1/2} = 2$$