

MATRIX

1. यदि $A = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$ हो, तो आव्यूह A^{-50} होगा,

जब $\theta = \frac{\pi}{12}$ हो :

(1) $\begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$ (2) $\begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$

(3) $\begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$ (4) $\begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$

2. यदि

$$A = \begin{bmatrix} e^t & e^{-t} \cos t & e^{-t} \sin t \\ e^t & -e^{-t} \cos t - e^{-t} \sin t & -e^{-t} \sin t + e^{-t} \cos t \\ e^t & 2e^{-t} \sin t & -2e^{-t} \cos t \end{bmatrix}$$

है, तो A -

- (1) व्युत्क्रमणीय है, केवल तब, जब $t = \frac{\pi}{2}$ है।
- (2) किसी भी $t \in \mathbb{R}$ के लिए व्युत्क्रमणीय नहीं है।
- (3) सभी $t \in \mathbb{R}$ के लिए व्युत्क्रमणीय है।
- (4) व्युत्क्रमणीय (invertible) है, केवल तब, जब $t = \pi$ है।

3. माना $A = \begin{bmatrix} 2 & b & 1 \\ b & b^2 + 1 & b \\ 1 & b & 2 \end{bmatrix}$ जहाँ $b > 0$ है। तब $\frac{\det(A)}{b}$

का न्यूनतम मान होगा :

(1) $\sqrt{3}$ (2) $-\sqrt{3}$ (3) $-2\sqrt{3}$ (4) $2\sqrt{3}$

4. माना $A = \begin{pmatrix} 0 & 2q & r \\ p & q & -r \\ p & -q & r \end{pmatrix}$ । यदि $AA^T = I_3$,

तो $|p|$ बराबर है :

(1) $\frac{1}{\sqrt{2}}$ (2) $\frac{1}{\sqrt{5}}$ (3) $\frac{1}{\sqrt{6}}$ (4) $\frac{1}{\sqrt{3}}$

5. माना A तथा B, 3×3 कोटि के दो व्युत्क्रमणीय आव्यूह हैं। यदि $\det(ABA^T) = 8$ तथा $\det(AB^{-1}) = 8$, तो $\det(BA^{-1}B^T)$ बराबर है

(1) 16 (2) $\frac{1}{16}$ (3) $\frac{1}{4}$ (4) 1

6. यदि $P = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 9 & 3 & 1 \end{bmatrix}$ तथा $Q = [q_{ij}]$ दो ऐसे 3×3 आव्यूह

हैं, कि $Q - P^5 = I_3$ है, तो $\frac{q_{21} + q_{31}}{q_{32}}$ बराबर है :

(1) 15 (2) 9 (3) 135 (4) 10

7. यदि $A = \begin{bmatrix} 1 & \sin \theta & 1 \\ -\sin \theta & 1 & \sin \theta \\ -1 & -\sin \theta & 1 \end{bmatrix}$ हो, तो सही

$\theta \in \left(\frac{3\pi}{4}, \frac{5\pi}{4}\right)$ के लिये $\det(A)$ किस अन्तराल में स्थित होगा :

(1) $\left[\frac{5}{2}, 4\right)$ (2) $\left(\frac{3}{2}, 3\right]$
 (3) $\left(0, \frac{3}{2}\right]$ (4) $\left(1, \frac{5}{2}\right]$

8. माना संख्याएं 2, b, c एक समान्तर श्रेणी में है तथा

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & b & c \\ 4 & b^2 & c^2 \end{bmatrix}$$

यदि $\det(A) \in [2, 16]$,

तो c निम्न में से किस अन्तराल में है :-

(1) [2, 3) (2) $(2 + 2^{3/4}, 4)$
 (3) $[3, 2 + 2^{3/4}]$ (4) [4, 6]

9. माना $A = \begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix}$, ($\alpha \in \mathbb{R}$) इस प्रकार है कि

$$A^{32} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

तो α का एक मान है -

(1) $\frac{\pi}{16}$ (2) 0 (3) $\frac{\pi}{32}$ (4) $\frac{\pi}{64}$

10. यदि $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} \cdots \begin{bmatrix} 1 & n-1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 78 \\ 0 & 1 \end{bmatrix}$ है, तो

$\begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix}$ का व्युत्क्रम है :

(1) $\begin{bmatrix} 1 & -13 \\ 0 & 1 \end{bmatrix}$ (2) $\begin{bmatrix} 1 & 0 \\ 12 & 1 \end{bmatrix}$

(3) $\begin{bmatrix} 1 & -12 \\ 0 & 1 \end{bmatrix}$ (4) $\begin{bmatrix} 1 & 0 \\ 13 & 1 \end{bmatrix}$

11. आव्यूहों $A = \begin{pmatrix} 0 & 2y & 1 \\ 2x & y & -1 \\ 2x & -y & 1 \end{pmatrix}$, $(x, y \in \mathbb{R}, x \neq y)$

जिनके लिए $A^T A = 3I_3$ है, की कुल संख्या है :-

(1) 6 (2) 2 (3) 3 (4) 4

12. यदि एक 3×3 के आव्यूह A का व्युत्क्रम (inverse)

$$B = \begin{bmatrix} 5 & 2\alpha & 1 \\ 0 & 2 & 1 \\ \alpha & 3 & -1 \end{bmatrix}$$
 है, तो α के उन सभी मानों का

योग, जिनके लिए $\det(A) + 1 = 0$ है, हैं :

(1) 0 (2) 2 (3) 1 (4) -1

13. यदि एक सममित (symmetric) आव्यूह A तथा एक विषम सममित (skew-symmetric) आव्यूह B इस प्रकार हैं कि

$$A + B = \begin{bmatrix} 2 & 3 \\ 5 & -1 \end{bmatrix},$$
 तो AB बराबर है :

(1) $\begin{bmatrix} -4 & 2 \\ 1 & 4 \end{bmatrix}$ (2) $\begin{bmatrix} -4 & -2 \\ -1 & 4 \end{bmatrix}$

(3) $\begin{bmatrix} 4 & -2 \\ -1 & -4 \end{bmatrix}$ (4) $\begin{bmatrix} 4 & -2 \\ 1 & -4 \end{bmatrix}$

SOLUTION

1. **Ans. (1)**

Here, $AA^T = I$

$$\Rightarrow A^{-1} = A^T = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$$

$$\text{Also, } A^{-n} = \begin{bmatrix} \cos(n\theta) & \sin(n\theta) \\ -\sin(n\theta) & \cos(n\theta) \end{bmatrix}$$

$$\therefore A^{-50} = \begin{bmatrix} \cos(50)\theta & \sin(50)\theta \\ -\sin(50)\theta & \cos(50)\theta \end{bmatrix}$$

$$= \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$$

2. **Ans. (3)**

$$|A| = e^{-t} \begin{vmatrix} 1 & \cos t & \sin t \\ 1 & -\cos t - \sin t & -\sin t + \cos t \\ 1 & 2\sin t & -2\cos t \end{vmatrix}$$

$$= e^{-t}[5\cos^2 t + 5\sin^2 t] \quad \forall t \in \mathbb{R}$$

$$= 5e^{-t} \neq 0 \quad \forall t \in \mathbb{R}$$

3. **Ans. (4)**

$$A = \begin{bmatrix} 2 & b & 1 \\ b & b^2 + 1 & b \\ 1 & b & 2 \end{bmatrix} \quad (b > 0)$$

$$|A| = 2(2b^2 + 2 - b^2) - b(2b - b) + 1(b^2 - b^2 - 1)$$

$$|A| = 2(b^2 + 2) - b^2 - 1$$

$$|A| = b^2 + 3$$

$$\frac{|A|}{b} = b + \frac{3}{b} \Rightarrow \frac{b + \frac{3}{b}}{2} \geq \sqrt{3}$$

$$b + \frac{3}{b} \geq 2\sqrt{3}$$

Option (4)

4. **Ans. (1)**

A is orthogonal matrix

$$\Rightarrow 0^2 + p^2 + p^2 = 1 \Rightarrow |p| = \frac{1}{\sqrt{2}}$$

5. **Ans. (2)**

$$|A|^2 \cdot |B| = 8 \text{ and } \frac{|A|}{|B|} = 8 \Rightarrow |A| = 4 \text{ and } |B| = \frac{1}{2}$$

$$\therefore \det(BA^{-1} \cdot B^T) = \frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$$

6. **Ans. (4)**

$$P = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 9 & 3 & 1 \end{bmatrix}$$

$$P^2 = \begin{bmatrix} 1 & 0 & 0 \\ 3+3 & 1 & 0 \\ 9+9+9 & 3+3 & 1 \end{bmatrix}$$

$$P^3 = \begin{bmatrix} 1 & 0 & 0 \\ 3+3+3 & 1 & 0 \\ 6.9 & 3+3+3 & 1 \end{bmatrix}$$

$$P^n = \begin{bmatrix} 1 & 0 & 0 \\ 3n & 1 & 0 \\ \frac{n(n+1)}{2} 3^2 & 3n & 1 \end{bmatrix}$$

$$P^5 = \begin{bmatrix} 1 & 0 & 0 \\ 5.3 & 1 & 0 \\ 15.9 & 5.3 & 1 \end{bmatrix}$$

$$Q = P^5 + I_3$$

$$Q = \begin{bmatrix} 2 & 0 & 0 \\ 15 & 2 & 0 \\ 135 & 15 & 2 \end{bmatrix}$$

$$\frac{q_{21} + q_{31}}{q_{32}} = \frac{15 + 135}{15} = 10$$

Aliter

$$P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 3 & 0 & 0 \\ 9 & 3 & 0 \end{pmatrix}$$

$$P = I + X$$

$$X = \begin{pmatrix} 0 & 0 & 0 \\ 3 & 0 & 0 \\ 9 & 3 & 0 \end{pmatrix}$$

$$X^2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 9 & 0 & 0 \end{pmatrix}$$

$$X^3 = 0$$

$$P^5 = I + 5X + 10X^2$$

$$Q = P^5 + I = 2I + 5X + 10X^2$$

$$Q = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 15 & 0 & 0 \\ 15 & 15 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 90 & 0 & 0 \end{pmatrix}$$

$$\Rightarrow Q = \begin{pmatrix} 2 & 0 & 0 \\ 15 & 2 & 0 \\ 135 & 15 & 2 \end{pmatrix}$$

7. **Ans (2)**

$$|A| = \begin{vmatrix} 1 & \sin \theta & 1 \\ -\sin \theta & 1 & \sin \theta \\ -1 & -\sin \theta & 1 \end{vmatrix}$$

$$= 2(1 + \sin^2 \theta)$$

$$\theta \in \left(\frac{3\pi}{4}, \frac{5\pi}{4} \right) \Rightarrow \frac{1}{\sqrt{2}} < \sin \theta < \frac{1}{\sqrt{2}}$$

$$\Rightarrow 0 \leq \sin^2 \theta < \frac{1}{2}$$

$$\therefore |A| \in [2, 3)$$

8. **Official Ans. by NTA (4)**

Sol. put $b = \frac{2+c}{2}$ in determinant of A

$$|A| = \frac{c^3 - 6c^2 + 12c - 8}{4} \in [2, 16]$$

$$\Rightarrow (c-2)^3 \in [8, 64]$$

$$\Rightarrow c \in [4, 6]$$

9. Official Ans. by NTA (4)

Sol. $A = \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}$

$$A^2 = \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix} \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}$$

$$= \begin{bmatrix} \cos 2\alpha & -\sin 2\alpha \\ \sin 2\alpha & \cos 2\alpha \end{bmatrix}$$

$$A^3 = \begin{bmatrix} \cos 2\alpha & -\sin 2\alpha \\ \sin 2\alpha & \cos 2\alpha \end{bmatrix} \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}$$

$$= \begin{bmatrix} \cos 3\alpha & -\sin 3\alpha \\ \sin 3\alpha & \cos 3\alpha \end{bmatrix}$$

Similarly

$$A^{32} = \begin{bmatrix} \cos 32\alpha & -\sin 32\alpha \\ \sin 32\alpha & \cos 32\alpha \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$$\Rightarrow \cos 32\alpha = 0 \text{ \& \; } \sin 32\alpha = 1$$

$$\Rightarrow 32\alpha = (4n+1)\frac{\pi}{2}, n \in I$$

$$\alpha = (4n+1)\frac{\pi}{64}, n \in I$$

$$\alpha = \frac{\pi}{64} \text{ for } n = 0$$

10. Official Ans. by NTA (1)

Sol. $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} \dots \begin{bmatrix} 1 & n-1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 78 \\ 0 & 1 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} 1 & 1+2+3+\dots+n-1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 78 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \frac{n(n-2)}{2} = 78 \Rightarrow n = 13, -12(\text{reject})$$

\(\therefore\) We have to find inverse of $\begin{bmatrix} 1 & 13 \\ 0 & 1 \end{bmatrix}$

$$\therefore \begin{bmatrix} 1 & -13 \\ 0 & 1 \end{bmatrix}$$

11. Official Ans. by NTA (4)

Sol. $A^T A = 3I_3$

$$\begin{pmatrix} 0 & 2x & 2x \\ 2y & y & -y \\ 1 & -1 & 1 \end{pmatrix} \begin{pmatrix} 0 & 2y & 1 \\ 2x & y & -1 \\ 2x & -y & 1 \end{pmatrix} = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

$$\begin{pmatrix} 8x^2 & 0 & 0 \\ 0 & 6y^2 & 0 \\ 0 & 0 & 3 \end{pmatrix} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$$8x^2 = 3$$

$$6y^2 = 3$$

$$x^2 = 3/8$$

$$y^2 = 1/2$$

$$x = \pm \sqrt{\frac{3}{8}}; y = \pm \sqrt{\frac{1}{2}}$$

12. Official Ans. by NTA (3)

Sol. $|B| = 5(-5) - 2\alpha(-\alpha) - 2\alpha$
 $= 2\alpha^2 - 2\alpha - 25$

$$1 + |A| = 0$$

$$\alpha^2 - \alpha - 12 = 0$$

$$\text{Sum} = 1$$

13. Official Ans. by NTA (3)

Sol. $A = A', B = -B'$

$$A + B = \begin{bmatrix} 2 & 3 \\ 5 & -1 \end{bmatrix} \dots(1)$$

$$A' + B' = \begin{bmatrix} 2 & 5 \\ 3 & -1 \end{bmatrix}$$

$$A - B = \begin{bmatrix} 2 & 5 \\ 3 & -1 \end{bmatrix} \dots(2)$$

After adding Eq. (1) & (2)

$$A = \begin{bmatrix} 2 & 4 \\ 4 & -1 \end{bmatrix}, B = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$$AB = \begin{bmatrix} 4 & -2 \\ -1 & -4 \end{bmatrix}$$