# INVERSE TRIGONOMETRIC FUNCTION

**1.** यदि $\cos^{-1}\left(\frac{2}{3x}\right)+\cos^{-1}\left(\frac{3}{4x}\right)=\frac{\pi}{2}\left(x>\frac{3}{4}\right)$ हो, तो x का मान होगा :

(1) $\frac{\sqrt{145}}{12}$ (2) $\frac{\sqrt{145}}{10}$

(3) $\frac{\sqrt{146}}{12}$ (4) $\frac{\sqrt{145}}{11}$

**2.** यदि $x = \sin^{-1}(\sin 10)$ तथा $y=\cos^{-1}(\cos 10)$ है, तो $y-x$ बराबर है :

(1) $\pi$ (2) $7\pi$ (3) 0 (4) 10

**3.** $\cot\left(\sum_{n=1}^{19}\cot^{-1}\left(1+\sum_{p=1}^{n}2p\right)\right)$ का मान होगा

(1) $\frac{22}{23}$ (2) $\frac{23}{22}$

(3) $\frac{21}{19}$ (4) $\frac{19}{21}$

**4.** वे सभी x जो समीकरण

$(\cot^{-1} x)^2 - 7 (\cot^{-1} x) + 10 > 0$ को संतुष्ट करते हैं, निम्न में से किस अंतराल में है?

(1) $(-\infty, \cot 5) \cup (\cot 4, \cot 2)$

(2) $(\cot 5, \cot 4)$

(3) $(\cot 2, \infty)$

(4) $(-\infty, \cot 5) \cup (\cot 2, \infty)$

**5.** प्रतिलोम फलनों (inverse functions) के केवल मुख्य मान (principal values) लेते हुए, समुच्चय

$A=\left\{x\geq 0:\tan^{-1}(2x)+\tan^{-1}(3x)=\frac{\pi}{4}\right\}$

(1) एक रिक्त समुच्चय

(2) दो से अधिक अवयव है।

(3) में दो अवयव हैं।

(4) एक एकल समुच्चय है।

**6.** यदि $\alpha=\cos^{-1}\left(\frac{3}{5}\right), \beta=\tan^{-1}\left(\frac{1}{3}\right)$ है, जहाँ $0<\alpha, \beta<\frac{\pi}{2}$ तो $\alpha - \beta$ बराबर है -

(1) $\sin^{-1}\left(\frac{9}{5\sqrt{10}}\right)$ (2) $\tan^{-1}\left(\frac{9}{14}\right)$

(3) $\cos^{-1}\left(\frac{9}{5\sqrt{10}}\right)$ (4) $\tan^{-1}\left(\frac{9}{5\sqrt{10}}\right)$

**7.** यदि $\cos^{-1}x - \cos^{-1}\frac{y}{2} = \alpha$,

जहाँ $-1 \le x \le 1, -2 \le y \le 2, x \le \frac{y}{2}$

है, तो सभी x, y के लिए, $4x^2 - 4xy \cos\alpha + y^2$ बराबर है -

(1) $4\sin^2\alpha - 2x^2y^2$ (2) $4\cos^2\alpha + 2x^2y^2$

(3) $4\sin^2\alpha$ (4) $2\sin^2\alpha$

**8.** $\sin^{-1}\left(\frac{12}{13}\right)-\sin^{-1}\left(\frac{3}{5}\right)$ का मान है :

(1) $\pi-\sin^{-1}\left(\frac{63}{65}\right)$ (2) $\pi-\cos^{-1}\left(\frac{33}{65}\right)$

(3) $\frac{\pi}{2}-\sin^{-1}\left(\frac{56}{65}\right)$ (4) $\frac{\pi}{2}-\cos^{-1}\left(\frac{9}{65}\right)$

## SOLUTION

**1. Ans. (1)**

$$\cos^{-1}\left(\frac{2}{3x}\right)+\cos^{-1}\left(\frac{3}{4x}\right)=\frac{\pi}{2}\quad\left(x>\frac{3}{4}\right)$$

$$\cos^{-1}\left(\frac{3}{4x}\right)=\frac{\pi}{2}-\cos^{-1}\left(\frac{2}{3x}\right)$$

$$\cos^{-1}\left(\frac{3}{4x}\right)=\sin^{-1}\left(\frac{2}{3x}\right)$$

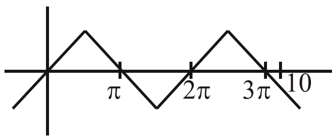
$$\cos\left(\cos^{-1}\left(\frac{3}{4x}\right)\right)=\cos\left(\sin^{-1}\frac{2}{3x}\right)$$

$$\frac{3}{4x}=\frac{\sqrt{9x^2-4}}{3x}$$

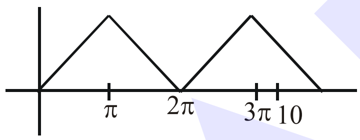
$$\frac{81}{16}+4=9x^2$$

$$x^2=\frac{145}{16\times 9}\quad\Rightarrow\quad x=\frac{\sqrt{145}}{12}$$

**2. Ans. (1)**




$x = \sin^{-1}(\sin 10) = 3\pi - 10$




$y = \cos^{-1}(\cos 10) = 4\pi - 10$

$y - x = \pi$

**3. Ans. (3)**

$$\cot\left(\sum_{n=1}^{19}\cot^{-1}(1+n(n+1))\right)$$

$$\cot\left(\sum_{n=1}^{19}\cot^{-1}(n^2+n+1)\right)=\cot\left(\sum_{n=1}^{19}\tan^{-1}\frac{1}{1+n(n+1)}\right)$$

$$\sum_{n=1}^{19}(\tan^{-1}(n+1)-\tan^{-1}n)$$

$$\cot(\tan^{-1}20-\tan^{-1}1)=\frac{\cot A\cot\beta+1}{\cot\beta-\cot A}$$

(Where tanA=20, tanB=1) $\dfrac{1\left(\frac{1}{20}\right)+1}{1-\frac{1}{20}}=\dfrac{21}{19}$

$\therefore$ Option (3)

**4. Ans. (3)**

$\cot^{-1}x > 5$ (reject), $\cot^{-1}x < 2$

$\therefore\ x > \cot 2$

$\therefore\ x \in (\cot 2, \infty)$

**5. Ans. (4)**

$\tan^{-1}(2x) + \tan^{-1}(3x) = \pi/4$

$$\Rightarrow \frac{5x}{1-6x^2}=1$$

$\Rightarrow 6x^2 + 5x - 1 = 0$

$x = -1$ or $x=\dfrac{1}{6}$

$x=\dfrac{1}{6}\quad \because x>0$

**6. Official Ans. by NTA (1)**

**Sol.** $\cos\alpha = \frac{3}{5}, \tan\beta = \frac{1}{3}$

$\Rightarrow \quad \tan\alpha = \frac{4}{3}$

$\Rightarrow \quad \tan(\alpha - \beta) = \frac{\frac{4}{3} - \frac{1}{3}}{1 + \frac{4}{3}\cdot\frac{1}{3}} = \frac{9}{13}$

$\Rightarrow \quad \sin(\alpha - \beta) = \frac{9}{5\sqrt{10}}$

$\Rightarrow \quad \alpha - \beta = \sin^{-1}\left(\frac{9}{5\sqrt{10}}\right)$

**7. Official Ans. by NTA (3)**

**Sol.** $\cos^{-1}x - \cos^{-1}\frac{y}{2} = \alpha$

$\cos(\cos^{-1}x - \cos^{-1}\frac{y}{2}) = \cos\alpha$

$\Rightarrow x \times \frac{y}{2} + \sqrt{1-x^2}\sqrt{1-\frac{y^2}{4}} = \cos\alpha$

$\Rightarrow \left(\cos\alpha - \frac{xy}{2}\right)^2 = (1-x^2)\left(1 - \frac{y^2}{4}\right)$

$x^2 + \frac{y^2}{4} - xy\cos\alpha = 1 - \cos^2\alpha = \sin^2\alpha$

**8. Official Ans. by NTA (3)**

**Sol.** $\sin^{-1}\left(\frac{12}{13}\right) - \sin^{-1}\left(\frac{3}{5}\right)$

$\sin^{-1}\left(x\sqrt{1-y^2} - y\sqrt{1-x^2}\right)$

$= \sin^{1}\left(\frac{33}{65}\right) = \cos^{-1}\left(\frac{56}{65}\right) = \frac{\pi}{2} - \sin^{1}\left(\frac{56}{65}\right)$