

INDEFINITE INTEGRATION

1. $x^2 \neq n\pi + 1, n \in \mathbb{N}$ (प्राकृत संख्याओं का समुच्चय) के लिये, समाकल

$$\int x \sqrt{\frac{2\sin(x^2-1) - \sin 2(x^2-1)}{2\sin(x^2-1) + \sin 2(x^2-1)}} dx$$

होगा :

(जहाँ c समाकलन अचर है)

(1) $\log_e \left| \sec \left(\frac{x^2-1}{2} \right) \right| + c$

(2) $\log_e \left| \frac{1}{2} \sec^2(x^2-1) \right| + c$

(3) $\frac{1}{2} \log_e \left| \sec^2 \left(\frac{x^2-1}{2} \right) \right| + c$

(4) $\frac{1}{2} \log_e |\sec(x^2-1)| + c$

2. यदि $f(x) = \int \frac{5x^8 + 7x^6}{(x^2 + 1 + 2x^7)^2} dx, (x \geq 0)$ तथा

$f(0) = 0$ है, तो $f(1)$ का मान है :

(1) $-\frac{1}{2}$ (2) $\frac{1}{2}$ (3) $-\frac{1}{4}$ (4) $\frac{1}{4}$

3. माना $n \geq 2$ एक प्राकृत संख्या है तथा $0 < \theta < \pi/2$ है,

तो $\int \frac{(\sin^n \theta - \sin \theta)^{\frac{1}{n}} \cos \theta}{\sin^{n+1} \theta} d\theta$ बराबर है :

(जहाँ C एक समाकलन अचर है)

(1) $\frac{n}{n^2-1} \left(1 - \frac{1}{\sin^{n+1} \theta} \right)^{\frac{n+1}{n}} + C$

(2) $\frac{n}{n^2+1} \left(1 - \frac{1}{\sin^{n-1} \theta} \right)^{\frac{n+1}{n}} + C$

(3) $\frac{n}{n^2-1} \left(1 - \frac{1}{\sin^{n-1} \theta} \right)^{\frac{n+1}{n}} + C$

(4) $\frac{n}{n^2-1} \left(1 + \frac{1}{\sin^{n-1} \theta} \right)^{\frac{n+1}{n}} + C$

4. यदि $\int x^5 e^{-4x^3} dx = \frac{1}{48} e^{-4x^3} f(x) + C$, जहाँ C एक समाकलन अचर है, तो $f(x)$ होगा

(1) $-4x^3 - 1$ (2) $4x^3 + 1$
 (3) $-2x^3 - 1$ (4) $-2x^3 + 1$

5. उपयुक्त पूर्णांक m तथा $A(x)$ के लिये

$$\int \frac{\sqrt{1-x^2}}{x^4} dx = A(x) (\sqrt{1-x^2})^m + C,$$

जहाँ C एक समाकलन अचर है, तो $(A(x))^m$ बराबर है :

(1) $\frac{-1}{3x^3}$ (2) $\frac{-1}{27x^9}$ (3) $\frac{1}{9x^4}$ (4) $\frac{1}{27x^6}$

6. यदि $\int \frac{x+1}{\sqrt{2x-1}} dx = f(x) \sqrt{2x-1} + C$ है, जहाँ C एक समाकलन अचर है, तो $f(x)$ बराबर है :

(1) $\frac{1}{3}(x+4)$ (2) $\frac{1}{3}(x+1)$

(3) $\frac{2}{3}(x+2)$ (4) $\frac{2}{3}(x-4)$

7. समाकल $\int \cos(\log_e x) dx$ बराबर है :

(जहाँ C एक समाकलन अचर है)

(1) $\frac{x}{2} [\sin(\log_e x) - \cos(\log_e x)] + C$

(2) $\frac{x}{2} [\cos(\log_e x) + \sin(\log_e x)] + C$

(3) $x [\cos(\log_e x) + \sin(\log_e x)] + C$

(4) $x [\cos(\log_e x) - \sin(\log_e x)] + C$

8. समाकल $\int \frac{3x^{13} + 2x^{11}}{(2x^4 + 3x^2 + 1)^4} dx$ होगा :

(जहाँ C समाकलन अचर है)

(1) $\frac{x^4}{(2x^4 + 3x^2 + 1)^3} + C$

(2) $\frac{x^{12}}{6(2x^4 + 3x^2 + 1)^3} + C$

(3) $\frac{x^4}{6(2x^4 + 3x^2 + 1)^3} + C$

(4) $\frac{x^{12}}{(2x^4 + 3x^2 + 1)^3} + C$

9. $\int \frac{\sin \frac{5x}{2}}{\sin \frac{x}{2}} dx$ बराबर है -

(जहाँ c एक समाकलन अचर है)

(1) $2x + \sin x + 2\sin 2x + c$

(2) $x + 2\sin x + 2\sin 2x + c$

(3) $x + 2\sin x + \sin 2x + c$

(4) $2x + \sin x + \sin 2x + c$

10. यदि $\int \frac{dx}{x^3(1+x^6)^{2/3}} = xf(x)(1+x^6)^{1/3} + C$

जहाँ C एक समाकलन अचर है, तो फलन $f(x)$ बराबर है :-

(1) $-\frac{1}{6x^3}$ (2) $\frac{3}{x^2}$

(3) $-\frac{1}{2x^2}$ (4) $-\frac{1}{2x^3}$

11. समाकल $\int \sec^{2/3} x \operatorname{cosec}^{4/3} x dx$ बराबर है :
(यहाँ C एक समाकलन अचर है)

(1) $3\tan^{-1/3} x + C$ (2) $-\frac{3}{4}\tan^{-4/3} x + C$

(3) $-3\cot^{-1/3} x + C$ (4) $-3\tan^{-1/3} x + C$

12. यदि $\int \frac{dx}{(x^2 - 2x + 10)^2}$

$= A \left(\tan^{-1} \left(\frac{x-1}{3} \right) + \frac{f(x)}{x^2 - 2x + 10} \right) + C$

जहाँ C एक समाकलन अचर है, तो :

(1) $A = \frac{1}{27}$ तथा $f(x) = 9(x-1)$

(2) $A = \frac{1}{81}$ तथा $f(x) = 3(x-1)$

(3) $A = \frac{1}{54}$ तथा $f(x) = 9(x-1)^2$

(4) $A = \frac{1}{54}$ तथा $f(x) = 3(x-1)$

13. यदि $\int x^5 e^{-x^2} dx = g(x)e^{-x^2} + c$ है, जहाँ c एक समाकलन अचर है, तो $g(-1)$ बराबर है:

(1) $-\frac{5}{2}$ (2) 1

(3) $-\frac{1}{2}$ (4) -1

14. समाकल $\int \frac{2x^3 - 1}{x^4 + x} dx$ बराबर है :

(यहाँ C समाकलन अचर है)

(1) $\log_e \left| \frac{x^3 + 1}{x} \right| + C$

(2) $\frac{1}{2} \log_e \frac{(x^3 + 1)^2}{|x^3|} + C$

(3) $\frac{1}{2} \log_e \frac{|x^3 + 1|}{x^2} + C$

(4) $\log_e \frac{|x^3 + 1|}{x^2} + C$

15. माना $\alpha \in (0, \pi/2)$ दिया है। यदि समाकल

$$\int \frac{\tan x + \tan \alpha}{\tan x - \tan \alpha} dx =$$

$A(x) \cos 2\alpha + B(x) \sin 2\alpha + C$, जहाँ C एक समाकलन अचर है, तो फलन A(x) तथा B(x) क्रमशः हैं :

(1) $x - \alpha$ और $\log_e |\cos(x - \alpha)|$

(2) $x + \alpha$ और $\log_e |\sin(x - \alpha)|$

(3) $x - \alpha$ और $\log_e |\sin(x - \alpha)|$

(4) $x + \alpha$ और $\log_e |\sin(x + \alpha)|$

16. यदि $\int e^{\sec x} (\sec x \tan x f(x) + (\sec x \tan x + \sec^2 x)) dx = e^{\sec x} f(x) + C$, तो f(x) का एक संभव विकल्प (choice) है :-

(1) $\sec x - \tan x - \frac{1}{2}$

(2) $x \sec x + \tan x + \frac{1}{2}$

(3) $\sec x + x \tan x - \frac{1}{2}$

(4) $\sec x + \tan x + \frac{1}{2}$

SOLUTION

1. (1 or 3)

$$\text{Put } (x^2 - 1) = 1$$

$$\Rightarrow 2x dx = dt$$

$$\therefore I = \frac{1}{2} \int \sqrt{\frac{1 - \cos t}{1 + \cos t}} dt$$

$$= \frac{1}{2} \int \tan\left(\frac{t}{2}\right) dt$$

$$= \ln \left| \sec \frac{t}{2} \right| + c$$

$$I = \ln \left| \sec \left(\frac{x^2 - 1}{2} \right) \right| + c$$

2. Ans. (4)

$$\int \frac{5x^8 + 7x^6}{(x^2 + 1 + 2x^7)^2} dx$$

$$= \int \frac{5x^{-6} + 7x^{-8}}{\left(\frac{1}{x^7} + \frac{1}{x^5} + 2\right)^2} dx = \frac{1}{2 + \frac{1}{x^5} + \frac{1}{x^7}} + C$$

$$\text{As } f(0) = 0, f(x) = \frac{x^7}{2x^7 + x^2 + 1}$$

$$f(1) = \frac{1}{4}$$

3. Ans. (3)

$$\int \frac{(\sin^n \theta - \sin \theta)^{1/n} \cos \theta}{\sin^{n+1} \theta} d\theta$$

$$= \int \frac{\sin \theta \left(1 - \frac{1}{\sin^{n-1} \theta}\right)^{1/n}}{\sin^{n+1} \theta} d\theta$$

$$\text{Put } 1 - \frac{1}{\sin^{n-1} \theta} = t$$

$$\text{So } \frac{(n-1)}{\sin^n \theta} \cos \theta d\theta = dt$$

$$\text{Now } \frac{1}{n-1} \int (t)^{1/n} dt$$

$$= \frac{1}{(n-1)} \frac{(t)^{\frac{1}{n}+1}}{\frac{1}{n}+1} + C$$

$$= \frac{1}{(n-1)} \left(1 - \frac{1}{\sin^{n-1} \theta}\right)^{\frac{1}{n}+1} + C$$

4. Ans. (1)

$$\int x^5 \cdot e^{-4x^3} dx = \frac{1}{48} e^{-4x^3} f(x) + c$$

$$\text{Put } x^3 = t$$

$$3x^2 dx = dt$$

$$\int x^3 \cdot e^{-4x^3} \cdot x^2 dx$$

$$\frac{1}{3} \int t \cdot e^{-4t} dt$$

$$\frac{1}{3} \left[t \cdot \frac{e^{-4t}}{-4} - \int \frac{e^{-4t}}{-4} dt \right]$$

$$- \frac{e^{-4t}}{48} [4t + 1] + c$$

$$- \frac{e^{-4x^3}}{48} [4x^3 + 1] + c$$

$$\therefore f(x) = -1 - 4x^3$$

Option (1)

(From the given options (1) is most suitable)

5. Ans. (2)

$$\int \frac{\sqrt{1-x^2}}{x^4} dx = A(x)(\sqrt{1-x^2})^m + C$$

$$\int \frac{|x|\sqrt{\frac{1}{x^2}-1}}{x^4} dx,$$

Put $\frac{1}{x^2}-1 = t \Rightarrow \frac{dt}{dx} = \frac{-2}{x^3}$

Case-I $x \geq 0$

$$-\frac{1}{2} \int \sqrt{t} dt \Rightarrow -\frac{t^{3/2}}{3} + C$$

$$\Rightarrow -\frac{1}{3} \left(\frac{1}{x^2} - 1 \right)^{3/2} \Rightarrow \frac{(\sqrt{1-x^2})^3}{-3x^2} + C$$

$$A(x) = -\frac{1}{3x^3} \text{ and } m = 3$$

$$(A(x))^m = \left(-\frac{1}{3x^3} \right)^3 = -\frac{1}{27x^9}$$

Case-II $x \leq 0$

We get $\frac{(\sqrt{1-x^2})^3}{-3x^3} + C$

$$A(x) = \frac{1}{-3x^3}, \quad m = 3$$

$$(A(x))^m = \frac{-1}{27x^9}$$

6. Ans. (1)

$$\sqrt{2x-1} = t \Rightarrow 2x-1 = t^2 \Rightarrow 2dx = 2t \cdot dt$$

$$\int \frac{x+1}{\sqrt{2x-1}} dx = \int \frac{\frac{t^2+1}{2} + 1}{t} t dt = \int \frac{t^2+3}{2} dt$$

$$= \frac{1}{2} \left(\frac{t^3}{3} + 3t \right) = \frac{t}{6} (t^2 + 9) + c$$

$$= \sqrt{2x-1} \left(\frac{2x-1+9}{6} \right) + c = \sqrt{2x-1} \left(\frac{x+4}{3} \right) + c$$

$$\Rightarrow f(x) = \frac{x+4}{3}$$

7. Ans. (2)

$$I = \int \cos(\ell n x) dx$$

$$I = \cos(\ell n x) \cdot x + \int \sin(\ell n x) dx$$

$$\cos(\ell n x) x + [\sin(\ell n x) \cdot x - \int \cos(\ell n x) dx]$$

$$I = \frac{x}{2} [\sin(\ell n x) + \cos(\ell n x)] + C$$

8. Ans. (2)

$$\int \frac{3x^{13} + 2x^{11}}{(2x^4 + 3x^2 + 1)^4} dx$$

$$\int \frac{\left(\frac{3}{x^3} + \frac{2}{x^5} \right) dx}{\left(2 + \frac{3}{x^2} + \frac{1}{x^4} \right)^4}$$

Let $\left(2 + \frac{3}{x^2} + \frac{1}{x^4} \right) = t$

$$-\frac{1}{2} \int \frac{dt}{t^4} = \frac{1}{6t^3} + C \Rightarrow \frac{x^{12}}{6(2x^4 + 3x^2 + 1)^3} + C$$

9. Official Ans. by NTA (3)

Sol. $\int \frac{\sin \frac{5x}{2}}{\sin \frac{x}{2}} dx = \int \frac{2 \sin \frac{5x}{2} \cos \frac{x}{2}}{2 \sin \frac{x}{2} \cos \frac{x}{2}} dx$

$$= \int \frac{\sin 3x + \sin 2x}{\sin x} dx$$

$$= \int \frac{3 \sin x - 4 \sin^3 x - 2 \sin x \cos x}{\sin x} dx$$

$$= \int (3 - 4 \sin^2 x + 2 \cos x) dx$$

$$= \int (3 - 2(1 - \cos 2x) + 2 \cos x) dx$$

$$= \int (1 + 2 \cos 2x + 2 \cos x) dx$$

$$= x + \sin 2x + 2 \sin x + c$$

10. Official Ans. by NTA (4)

$$\text{Sol. } \int \frac{dx}{x^3(1+x^6)^{2/3}} = xf(x)(1+x^6)^{1/3} + c$$

$$\int \frac{dx}{x^7 \left(\frac{1}{x^6} + 1\right)^{2/3}} = xf(x)(1+x^6)^{1/3} + c$$

$$\text{Let } t = \frac{1}{x^6} + 1$$

$$dt = \frac{-6}{x^7} dx$$

$$-\frac{1}{6} \int \frac{dt}{t^{2/3}} = -\frac{1}{2} t^{1/3}$$

$$= -\frac{1}{2} \left(\frac{1}{x^6} + 1\right)^{1/3} = -\frac{1}{2} \frac{(1+x^6)^{1/3}}{x^2}$$

$$\therefore f(x) = -\frac{1}{2x^3}$$

11. Official Ans. by NTA (4)

$$\text{Sol. } I = \int \frac{dx}{(\sin x)^{4/3} \cdot (\cos x)^{2/3}}$$

$$I = \int \frac{dx}{\left(\frac{\sin x}{\cos x}\right)^{4/3} \cdot \cos^2 x}$$

$$\Rightarrow I = \int \frac{\sec^2 x}{(\tan x)^{4/3}} dx$$

$$\text{put } \tan x = t \Rightarrow \sec^2 x dx = dt$$

$$\therefore I = \int \frac{dt}{t^{4/3}} \Rightarrow I = \frac{-3}{t^{1/3}} + c$$

$$\Rightarrow I = \frac{-3}{(\tan x)^{1/3}} + c$$

12. Official Ans. by NTA (4)

$$\text{Sol. } \int \frac{dx}{((x-1)^2 + 9)^2} = \frac{1}{27} \int \cos^2 \theta d\theta$$

$$(\text{Put } x-1 = 3 \tan \theta)$$

$$= \frac{1}{54} \int (1 + \cos 2\theta) d\theta = \frac{1}{54} \left(\theta + \frac{\sin 2\theta}{2} \right) + C$$

$$= \frac{1}{54} \left(\tan^{-1} \left(\frac{x-1}{3} \right) + \frac{3(x-1)}{x^2 - 2x + 10} \right) + C$$

13. Official Ans. by NTA (1)

$$\text{Sol. Let } x^2 = t \quad 2x dx = dt$$

$$\Rightarrow \frac{1}{2} \int t^2 \cdot e^{-t} dt = \frac{1}{2} \left[-t^2 \cdot e^{-t} + \int 2t \cdot e^{-t} dt \right]$$

$$= \frac{1}{2} \left(-t^2 \cdot e^{-t} \right) + \left(-t \cdot e^{-t} + \int 1 \cdot e^{-t} dt \right)$$

$$= -\frac{t^2 e^{-t}}{2} - t e^{-t} - e^{-t} = \left(-\frac{t^2}{2} - t - 1 \right) e^{-t}$$

$$= \left(-\frac{x^4}{2} - x^2 - 1 \right) e^{-x^2} + C$$

$$g(x) = -1 - x^2 - \frac{x^4}{2} + k e^{x^2}$$

$$\text{for } k = 0$$

$$g(-1) = -1 - 1 - \frac{1}{2} = -\frac{5}{2}$$

14. Official Ans. by NTA (1)

Sol. $\int \frac{2x^3 - 1}{x^4 + x} dx$

$$\int \frac{2x - \frac{1}{x^2}}{x^2 + \frac{1}{x}} dx$$

$$x^2 + \frac{1}{x} = t$$

$$\left(2x - \frac{1}{x^2}\right) dx = dt$$

$$\int \frac{dt}{t} = \ln(t) + C$$

$$= \ln\left(x^2 + \frac{1}{x}\right) + C$$

15. Official Ans. by NTA (3)

Sol. $\int \frac{\tan x + \tan \alpha}{\tan x - \tan \alpha} dx = \int \frac{\sin(x + \alpha)}{\sin(x - \alpha)} dx$

Let $x - \alpha = t$

$$\Rightarrow \int \frac{\sin(t + 2\alpha)}{\sin t} dt = \int \cos 2\alpha dt + \int \cot(t) \sin 2\alpha dt$$

$$= t \cdot \cos 2\alpha + \ln|\sin t| \cdot \sin 2\alpha + C$$

$$= (x - \alpha) \cos 2\alpha + \ln|\sin(x - \alpha)| \cdot \sin 2\alpha + C$$

16. Official Ans. by NTA (4)

Sol. $\int e^{\sec x} (\sec x \tan x f(x) + (\sec x \tan x + \sec^2 x)) dx$
 $= e^{\sec x} f(x) + C$

Diff. both sides w.r.t. 'x'

$$e^{\sec x} (\sec x \tan x f(x) + (\sec x \tan x + \sec^2 x))$$

$$= e^{\sec x} \cdot \sec x \tan x f(x) + e^{\sec x} f'(x)$$

$$f'(x) = \sec^2 x + \tan x \sec x$$

$$\Rightarrow f(x) = \tan x + \sec x + c$$