

DEFINITE INTEGRATION

- $\int_0^{\pi} |\cos x|^3 dx$ का मान होगा
 (1) 2/3 (2) 0 (3) -4/3 (4) 4/3
- यदि $\int_0^{\frac{\pi}{3}} \frac{\tan \theta}{\sqrt{2k \sec \theta}} d\theta = 1 - \frac{1}{\sqrt{2}}$, ($k > 0$) है, तो k का मान है :
 (1) 2 (2) $\frac{1}{2}$ (3) 4 (4) 1
- माना $I = \int_a^b (x^4 - 2x^2) dx$ है। यदि I न्यूनतम है, तो क्रमित युग्म (a, b) है :
 (1) $(-\sqrt{2}, 0)$ (2) $(-\sqrt{2}, \sqrt{2})$
 (3) $(0, \sqrt{2})$ (4) $(\sqrt{2}, -\sqrt{2})$
- $\int_{-\pi/2}^{\pi/2} \frac{dx}{[x] + [\sin x] + 4}$, का मान होगा, जहाँ $[t]$, t से कम या समान महत्तम पूर्णांक फलन को दर्शाता है :
 (1) $\frac{1}{12}(7\pi + 5)$ (2) $\frac{3}{10}(4\pi - 3)$
 (3) $\frac{1}{12}(7\pi - 5)$ (4) $\frac{3}{20}(4\pi - 3)$
- यदि $\int_0^x f(t) dt = x^2 + \int_x^1 t^2 f(t) dt$ हो, तो $f(1/2)$ का मान होगा
 (1) $\frac{6}{25}$ (2) $\frac{24}{25}$ (3) $\frac{18}{25}$ (4) $\frac{4}{5}$
- समाकल $\int_{-2}^2 \frac{\sin^2 x}{\left[\frac{x}{\pi}\right] + \frac{1}{2}} dx$
 (जहाँ $[x]$, x के समान या उससे कम महत्तम पूर्णांक को दर्शाता है) का मान है:
 (1) 4 (2) $4 - \sin 4(3) \sin 4$ (4) 0

- समाकल $\int_{\pi/6}^{\pi/4} \frac{dx}{\sin 2x (\tan^5 x + \cot^5 x)}$ बराबर हैं :-
 (1) $\frac{1}{10} \left(\frac{\pi}{4} - \tan^{-1} \left(\frac{1}{9\sqrt{3}} \right) \right)$
 (2) $\frac{1}{5} \left(\frac{\pi}{4} - \tan^{-1} \left(\frac{1}{3\sqrt{3}} \right) \right)$
 (3) $\frac{\pi}{10}$
 (4) $\frac{1}{20} \tan^{-1} \left(\frac{1}{9\sqrt{3}} \right)$
- माना f तथा g , $[0, a]$ पर ऐसे संतत फलन हैं कि $f(x) = f(a-x)$ तथा $g(x) + g(a-x) = 4$ है, तो $\int_0^a f(x)g(x) dx$ बराबर है :-
 (1) $4 \int_0^a f(x) dx$ (2) $2 \int_0^a f(x) dx$
 (3) $-3 \int_0^a f(x) dx$ (4) $\int_0^a f(x) dx$
- समाकल $\int_1^e \left\{ \left(\frac{x}{e} \right)^{2x} - \left(\frac{e}{x} \right)^x \right\} \log_e x dx$ होगा :
 (1) $\frac{1}{2} - e - \frac{1}{e^2}$ (2) $\frac{3}{2} - \frac{1}{e} - \frac{1}{2e^2}$
 (3) $-\frac{1}{2} + \frac{1}{e} - \frac{1}{2e^2}$ (4) $\frac{3}{2} - e - \frac{1}{2e^2}$
- $\lim_{n \rightarrow \infty} \left(\frac{n}{n^2 + 1^2} + \frac{n}{n^2 + 2^2} + \frac{n}{n^2 + 3^2} + \dots + \frac{1}{5n} \right)$ होगा
 (1) $\frac{\pi}{4}$ (2) $\tan^{-1}(2)$
 (3) $\tan^{-1}(3)$ (4) $\frac{\pi}{2}$
- यदि $f(x) = \frac{2 - x \cos x}{2 + x \cos x}$ तथा $g(x) = \log_e x$, ($x > 0$) हैं, तो समाकल $\int_{-\pi/4}^{\pi/4} g(f(x)) dx$ का मान है -
 (1) $\log_e 3$ (2) $\log_e 2$

- (3) $\log_e e$ (4) $\log_e 1$
12. माना $f(x) = \int_0^x g(t) dt$ है, जहाँ g एक शून्येतर सम

फलन है। यदि $f(x+5) = g(x)$ है, तो $\int_0^x f(t) dt$

बराबर है :-

(1) $\int_{x+5}^5 g(t) dt$ (2) $5 \int_{x+5}^5 g(t) dt$

(3) $\int_5^{x+5} g(t) dt$ (4) $2 \int_5^{x+5} g(t) dt$

13. $\int_0^{\pi/2} \frac{\sin^3 x}{\sin x + \cos x} dx$ का मान है

(1) $\frac{\pi-2}{4}$ (2) $\frac{\pi-2}{8}$ (3) $\frac{\pi-1}{4}$ (4) $\frac{\pi-1}{2}$

14. यदि $f: \mathbb{R} \rightarrow \mathbb{R}$ एक अवकलनीय फलन है तथा

$f(2) = 6$ है, तो $\lim_{x \rightarrow 2} \int_6^{f(x)} \frac{2t dt}{(x-2)}$ है :-

(1) 0 (2) $2f'(2)$
(3) $12f'(2)$ (4) $24f'(2)$

15. $\int_0^{2\pi} [\sin 2x(1 + \cos 3x)] dx$, का मान, जहाँ $[t]$ महत्तम

पूर्णांक फलन है, है :

(1) -2π (2) π (3) $-\pi$ (4) 2π

16. $\lim_{n \rightarrow \infty} \left(\frac{(n+1)^{1/3}}{n^{4/3}} + \frac{(n+2)^{1/3}}{n^{4/3}} + \dots + \frac{(2n)^{1/3}}{n^{4/3}} \right)$

बराबर है :

(1) $\frac{4}{3}(2)^{4/3}$ (2) $\frac{3}{4}(2)^{4/3} - \frac{4}{3}$

(3) $\frac{3}{4}(2)^{4/3} - \frac{3}{4}$ (4) $\frac{4}{3}(2)^{3/4}$

17. समाकल $\int_{\pi/6}^{\pi/3} \sec^{2/3} x \operatorname{cosec}^{4/3} x dx$ बराबर है :

(1) $3^{7/6} - 3^{5/6}$ (2) $3^{5/3} - 3^{1/3}$

(3) $3^{4/3} - 3^{1/3}$ (4) $3^{5/6} - 3^{2/3}$

18. यदि $\int_0^{\pi/2} \frac{\cot x}{\cot x + \operatorname{cosec} x} dx = m(\pi + n)$, तो $m \cdot n$ बराबर है :

(1) -1 (2) 1 (3) $\frac{1}{2}$ (4) $-\frac{1}{2}$

19. समाकल $\int_0^1 x \cot^{-1}(1-x^2+x^4) dx$ का मान है :-

(1) $\frac{\pi}{4} - \frac{1}{2} \log_e 2$ (2) $\frac{\pi}{2} - \log_e 2$

(3) $\frac{\pi}{2} - \frac{1}{2} \log_e 2$ (4) $\frac{\pi}{4} - \log_e 2$

20. माना $f: \mathbb{R} \rightarrow \mathbb{R}$ एक संतततः अवकलनीय (continuously differentiable) फलन इस प्रकार है

कि $f(2) = 6$ and $f'(2) = \frac{1}{48}$. यदि

$\int_6^{f(x)} 4t^3 dt = (x-2)g(x)$, तो $\lim_{x \rightarrow 2} g(x)$ बराबर है :

(1) 24 (2) 36 (3) 12 (4) 18

21. α का एक मान, जिसके लिये

$\int_{\alpha}^{\alpha+1} \frac{dx}{(x+\alpha)(x+\alpha+1)} = \log_e \left(\frac{9}{8} \right)$ है, है :

(1) $\frac{1}{2}$ (2) 2 (3) $-\frac{1}{2}$ (4) -2

SOLUTION

1. **Ans. (4)**

$$\begin{aligned} \int_0^{\pi} |\cos x|^3 dx &= \int_0^{\pi/2} \cos^3 x dx - \int_{\pi/2}^{\pi} \cos^3 x dx \\ &= \int_0^{\pi/2} \left(\frac{\cos 3x + 3 \cos x}{4} \right) dx - \int_{\pi/2}^{\pi} \left(\frac{\cos 3x + 3 \cos x}{4} \right) dx \\ &= \frac{1}{4} \left[\left(\frac{\sin 3x}{3} + 3 \sin x \right) \Big|_0^{\pi/2} - \left(\frac{\sin 3x}{3} + 3 \sin x \right) \Big|_{\pi/2}^{\pi} \right] \\ &= \frac{1}{4} \left[\left(\frac{-1}{3} + 3 \right) - (0+0) - \left\{ (0+0) - \left(\frac{-1}{3} + 3 \right) \right\} \right] \\ &= \frac{4}{3} \end{aligned}$$

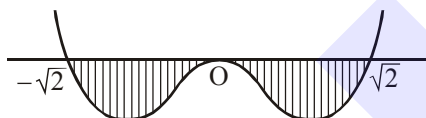
2. **Ans. (1)**

$$\begin{aligned} \frac{1}{\sqrt{2k}} \int_0^{\pi/3} \frac{\tan \theta}{\sqrt{\sec \theta}} d\theta &= \frac{1}{\sqrt{2k}} \int_0^{\pi/3} \frac{\sin \theta}{\sqrt{\cos \theta}} d\theta \\ &= -\frac{1}{\sqrt{2k}} 2\sqrt{\cos \theta} \Big|_0^{\pi/3} = -\frac{\sqrt{2}}{\sqrt{k}} \left(\frac{1}{\sqrt{2}} - 1 \right) \end{aligned}$$

given it is $1 - \frac{1}{\sqrt{2}} \Rightarrow k = 2$

3. **Ans. (2)**

Let $f(x) = x^2(x^2 - 2)$



As long as $f(x)$ lie below the x -axis, definite integral will remain negative, so correct value of (a, b) is $(-\sqrt{2}, \sqrt{2})$ for minimum of I

4. **Ans. (4)**

$$\begin{aligned} I &= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{dx}{[x] + [\sin x] + 4} \\ &= \int_{-\frac{\pi}{2}}^{-1} \frac{dx}{-2-1+4} + \int_{-1}^0 \frac{dx}{-1-1+4} + \int_0^1 \frac{dx}{0+0+4} + \int_1^{\frac{\pi}{2}} \frac{dx}{1+0+4} \\ &= \int_{-\frac{\pi}{2}}^{-1} \frac{dx}{1} + \int_{-1}^0 \frac{dx}{2} + \int_0^1 \frac{dx}{4} + \int_1^{\frac{\pi}{2}} \frac{dx}{5} \\ &= \left(-1 + \frac{\pi}{2} \right) + \frac{1}{2}(0+1) + \frac{1}{4} + \frac{1}{5} \left(\frac{\pi}{2} - 1 \right) \\ &= -1 + \frac{1}{2} + \frac{1}{4} - \frac{1}{5} + \frac{\pi}{2} + \frac{\pi}{10} \\ &= \frac{-20+10+5-4}{20} + \frac{6\pi}{10} \\ &= \frac{-9}{20} + \frac{3\pi}{5} \end{aligned}$$

Option (4)

5. **Ans. (2)**

$$\int_0^x f(t) dt = x^2 + \int_x^1 t^2 f(t) dt \quad f' \left(\frac{1}{2} \right) = ?$$

Differentiate w.r.t. 'x'

$$f(x) = 2x + 0 - x^2 f(x)$$

$$f(x) = \frac{2x}{1+x^2} \Rightarrow f'(x) = \frac{(1+x^2)2 - 2x(2x)}{(1+x^2)^2}$$

$$f'(x) = \frac{2x^2 - 4x^2 + 2}{(1+x^2)^2}$$

$$f' \left(\frac{1}{2} \right) = \frac{2 - 2 \left(\frac{1}{4} \right)}{\left(1 + \frac{1}{4} \right)^2} = \frac{\left(\frac{3}{2} \right)}{\frac{25}{16}} = \frac{48}{50} = \frac{24}{25}$$

Option (2)

6. Ans. (4)

$$I = \int_{-2}^2 \frac{\sin^2 x}{\left[\frac{x}{\pi}\right] + \frac{1}{2}} dx$$

$$I = \int_0^2 \left(\frac{\sin^2 x}{\left[\frac{x}{\pi}\right] + \frac{1}{2}} + \frac{\sin^2(-x)}{\left[-\frac{x}{\pi}\right] + \frac{1}{2}} \right) dx$$

$$\left(\left[\frac{x}{\pi}\right] + \left[-\frac{x}{\pi}\right] = -1 \text{ as } x \neq n\pi \right)$$

$$I = \int_0^2 \left(\frac{\sin^2 x}{\left[\frac{x}{\pi}\right] + \frac{1}{2}} + \frac{\sin^2 x}{-1 - \left[\frac{x}{\pi}\right] + \frac{1}{2}} \right) dx = 0$$

7. Ans. (1)

$$I = \int_{\pi/6}^{\pi/4} \frac{dx}{\sin 2x (\tan^5 x + \cot^5 x)}$$

$$I = \frac{1}{2} \int_{\pi/6}^{\pi/4} \frac{\tan^4 x \sec^2 x dx}{(1 + \tan^{10} x)} \text{ Put } \tan^5 x = t$$

$$I = \frac{1}{10} \int_{\left(\frac{1}{\sqrt{3}}\right)^5}^1 \frac{dt}{1+t^2} = \frac{1}{10} \left(\frac{\pi}{4} - \tan^{-1} \frac{1}{9\sqrt{3}} \right)$$

8. Ans. (2)

$$I = \int_0^a f(x)g(x)dx$$

$$I = \int_0^a f(a-x)g(a-x)dx$$

$$I = \int_0^a f(x)(4-g(x))dx$$

$$I = 4 \int_0^a f(x)dx - I$$

$$\Rightarrow I = 2 \int_0^a f(x)dx$$

9. Ans. (4)

$$\int_1^e \left(\frac{x}{e}\right)^{2x} \log_e x dx - \int_1^e \left(\frac{e}{x}\right) \log_e x dx$$

$$\text{Let } \left(\frac{x}{e}\right)^{2x} = t, \left(\frac{e}{x}\right)^x = v$$

$$= \frac{1}{2} \int_{\left(\frac{1}{e}\right)^2}^1 dt + \int_e^1 dv$$

$$= \frac{1}{2} \left(1 - \frac{1}{e^2} \right) + (1 - e) = \frac{3}{2} - \frac{1}{2e^2} - e$$

10. Ans. (2)

$$\lim_{x \rightarrow \infty} \sum_{r=1}^{2n} \frac{n}{n^2 + r^2}$$

$$\lim_{x \rightarrow \infty} \sum_{r=1}^{2n} \frac{1}{n \left(1 + \frac{r^2}{n^2} \right)} = \int_0^2 \frac{dx}{1+x^2} = \tan^{-1} 2$$

11. Official Ans. by NTA (4)

$$\text{Sol. } g(f(x)) = \ln(f(x)) = \ln \left(\frac{2-x \cos x}{2+x \cos x} \right)$$

$$\therefore I = \int_{-\pi/4}^{\pi/4} \ln \left(\frac{2-x \cos x}{2+x \cos x} \right) dx$$

$$= \int_0^{\pi/4} \left(\ln \left(\frac{2-x \cos x}{2+x \cos x} \right) + \ln \left(\frac{2+x \cos x}{2-x \cos x} \right) \right) dx$$

$$= \int_0^{\pi/2} (0) dx = 0 = \log_e (1)$$

12. Official Ans. by NTA (1)

Sol. $f(x) = \int_0^x g(t) dt$

$$f(-x) = \int_0^{-x} g(t) dt$$

put $t = -u$

$$= -\int_0^x g(-u) du$$

$$= -\int_0^x g(u) d(u) = -f(x)$$

$$\Rightarrow f(-x) = -f(x)$$

$\Rightarrow f(x)$ is an odd function

Also $f(5+x) = g(x)$

$$f(5-x) = g(-x) = g(x) = f(5+x)$$

$$\Rightarrow f(5-x) = f(5+x)$$

Now

$$I = \int_0^x f(t) dt$$

$$t = u + 5$$

$$I = \int_{-5}^{x-5} f(u+5) du$$

$$= \int_{-5}^{x-5} g(u) du$$

$$= \int_{-5}^{x-5} f'(u) du$$

$$= f(x-5) - f(-5)$$

$$= -f(5-x) + f(5)$$

$$= f(5) - f(5+x)$$

$$= \int_{5+x}^5 f'(t) dt = \int_{5+x}^5 g(t) dt$$

13. Official Ans. by NTA (3)

Sol. $I = \int_0^{\pi/2} \frac{\sin^3 x}{\sin x + \cos x} dx$

$$\Rightarrow I = \int_0^{\pi/4} \frac{\sin^3 x + \cos^3 x}{\sin x + \cos x} dx$$

$$= \int_0^{\pi/4} (1 - \sin x \cos x) dx$$

$$= \left(x - \frac{\sin^2 x}{2} \right)_0^{\pi/4}$$

$$= \frac{\pi}{4} - \frac{1}{4}$$

$$= \frac{\pi-1}{4}$$

14. Official Ans. by NTA (3)

$$\int_0^{f(x)} 2t dt$$

Sol. $\lim_{x \rightarrow 2} \frac{6}{x-2}$

L Hopital Rule

$$\lim_{x \rightarrow 2} \frac{2f(x)f'(x)}{1} = 2f(2) = f'(2) = 12f'(2)$$

15. Official Ans. by NTA (3)

Sol. $I = \int_0^{2\pi} [\sin 2x(1 + \cos 3x)] dx$

$$I = \int_0^{\pi} ([\sin 2x + \sin 2x \cos 3x] + [-\sin 2x - \sin 2x \cos 3x]) dx$$

$$= \int_0^{\pi} -dx = -\pi$$

16. Official Ans. by NTA (3)

Sol. $\lim_{n \rightarrow \infty} \sum_{r=1}^n \frac{1}{n} \left(\frac{n+r}{n} \right)^{1/3}$

$$= \int_0^1 (1+x)^{1/3} dx = \frac{3}{4} (2^{4/3} - 1)$$

17. Official Ans. by NTA (1)

$$\begin{aligned}
 \text{Sol. } I &= \int \frac{1}{\cos^{2/3} x \sin^{1/3} x \cdot \sin x} dx \\
 &= \int \frac{\tan^{2/3} x \cdot \sec^2 x \cdot dx}{\tan^2 x} \\
 &= \int \frac{\sec^2 x}{\tan^{4/3} x} \cdot dx \quad \{\tan x = t, \sec^2 x dx = dt\} \\
 &= \int \frac{dt}{t^{4/3}} = \frac{t^{-1/3}}{-1/3} = -3(t^{-1/3}) \\
 &\Rightarrow I = -3 \tan(x)^{-1/3} \\
 &\Rightarrow I = \frac{3}{(\tan x)^{1/3}} \Bigg|_{\pi/6}^{\pi/3} = -3 \left(\frac{1}{(\sqrt{3})^{1/3}} - (\sqrt{3})^{1/3} \right) \\
 &= 3 \left(3^{1/3} - \frac{1}{3^{1/6}} \right) = 3^{7/6} - 3^{5/6}
 \end{aligned}$$

18. Official Ans. by NTA (1)

$$\begin{aligned}
 \text{Sol. } \int_0^{\pi/2} \frac{\cot x dx}{\cot x + \operatorname{cosec} x} \\
 \int_0^{\pi/2} \frac{\cos x}{\cos x + 1} = \int \frac{2 \cos^2 \frac{x}{2} - 1}{2 \cos^2 \frac{x}{2}} \\
 \int_0^{\pi/2} \left(1 - \frac{1}{2} \sec^2 \frac{x}{2} \right) dx \\
 \left[x - \tan \frac{x}{2} \right]_0^{\pi/2} \\
 \frac{1}{2} [\pi - 2] \quad m = \frac{1}{2}, n = -2 \\
 mn = -1
 \end{aligned}$$

19. Official Ans. by NTA (1)

$$\begin{aligned}
 \text{Sol. } I &= \int_0^1 x \tan \left(\frac{1}{1+x^2(x^2-1)} \right) dx \\
 I &= \int_0^1 x \left(\tan^{-1} x^2 - \tan^{-1}(x^2-1) \right) dx \\
 x^2 = t &\Rightarrow 2x dx = dt \\
 I &= \frac{1}{2} \int_0^1 \left(\tan^{-1} t - \tan^{-1}(t-1) \right) dx \\
 &= \frac{1}{2} \int_0^1 \tan^{-1} t dt - \frac{1}{2} \int_0^1 \tan^{-1}(t-1) dt \\
 &= \frac{1}{2} \int_0^1 \tan^{-1} t dt - \frac{1}{2} \int_0^1 \tan^{-1} dt = \int_0^1 \tan^{-1} dt \\
 \tan^{-1} t = \theta &\Rightarrow t = \tan \theta \\
 dt &= \sec^2 \theta d\theta \\
 \int_0^{\pi/4} \theta \cdot \sec^2 \theta d\theta \\
 I &= (\theta \cdot \tan \theta) \Big|_0^{\pi/4} - \int_0^{\pi/4} \tan \theta d\theta \\
 &= \left(\frac{\pi}{4} - 0 \right) - \ln(\sec \theta) \Big|_0^{\pi/4} \\
 &= \frac{\pi}{4} - (\ell n \sqrt{2} - 0) \\
 &= \frac{\pi}{4} - \frac{1}{2} \ell n 2
 \end{aligned}$$

20. Official Ans. by NTA (4)

$$\begin{aligned}
 \text{Sol. } \lim_{x \rightarrow 2} g(x) &= \lim_{x \rightarrow 2} \frac{\int_6^{f(x)} 4t^3 dt}{x-2} \\
 &= \lim_{x \rightarrow 2} \frac{4 \cdot f^3(x) \cdot f'(x)}{1} \\
 &= 4f^3(2) f'(2) = 18
 \end{aligned}$$

21. Official Ans. by NTA (4)

$$\begin{aligned}\text{Sol. } \int_{\alpha}^{\alpha+1} \frac{(x+\alpha+1)-(x+\alpha)}{(x+\alpha)(x+\alpha+1)} dx &= (\ln|x+\alpha| - \ln|x+\alpha+1|)_{\alpha}^{\alpha+1} \\ &= \ln \left| \frac{2\alpha+1}{2\alpha+2} \times \frac{2\alpha+1}{2\alpha} \right| = \ln \frac{9}{8} \\ \Rightarrow \alpha &= -2, 1\end{aligned}$$