

**COMPOUND ANGLE**

1. किसी  $\theta \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$  के लिये, व्यंजक  $3(\sin\theta - \cos\theta)^4 + 6(\sin\theta + \cos\theta)^2 + 4\sin^6\theta$  होगा:
  - (1)  $13 - 4 \cos^6\theta$
  - (2)  $13 - 4 \cos^4\theta + 2 \sin^2\theta \cos^2\theta$
  - (3)  $13 - 4 \cos^2\theta + 6 \cos^4\theta$
  - (4)  $13 - 4 \cos^2\theta + 6 \sin^2\theta \cos^2\theta$
  
2.  $\cos \frac{\pi}{2^2} \cdot \cos \frac{\pi}{2^3} \cdot \dots \cdot \cos \frac{\pi}{2^{10}} \cdot \sin \frac{\pi}{2^{10}}$  का मान होगा
  - (1)  $\frac{1}{256}$
  - (2)  $\frac{1}{2}$
  - (3)  $\frac{1}{512}$
  - (4)  $\frac{1}{1024}$
  
3. माना  $k = 1, 2, 3, \dots$  के लिये  $f_k(x) = \frac{1}{k}(\sin^k x + \cos^k x)$  तो सभी  $x \in \mathbb{R}$  के लिये,  $f_4(x) - f_6(x)$  का मान बराबर है :-
  - (1)  $\frac{5}{12}$
  - (2)  $\frac{-1}{12}$
  - (3)  $\frac{1}{4}$
  - (4)  $\frac{1}{12}$
  
4.  $3\cos\theta + 5\sin\left(\theta - \frac{\pi}{6}\right)$  का  $\theta$  के किसी भी वास्तविक मान के लिये अधिकतम मान है :
  - (1)  $\sqrt{19}$
  - (2)  $\frac{\sqrt{79}}{2}$
  - (3)  $\sqrt{31}$
  - (4)  $\sqrt{34}$

5. यदि  $\cos(\alpha + \beta) = \frac{3}{5}, \sin(\alpha - \beta) = \frac{5}{13}$  तथा  $0 < \alpha, \beta < \frac{\pi}{4}$  है, तो  $\tan(2\alpha)$  बराबर है -
  - (1)  $\frac{21}{16}$
  - (2)  $\frac{63}{52}$
  - (3)  $\frac{33}{52}$
  - (4)  $\frac{63}{16}$
  
6.  $\cos^2 10^\circ - \cos 10^\circ \cos 50^\circ + \cos^2 50^\circ$  का मान है :
  - (1)  $\frac{3}{2}(1 + \cos 20^\circ)$
  - (2)  $\frac{3}{4}$
  - (3)  $\frac{3}{4} + \cos 20^\circ$
  - (4)  $\frac{3}{2}$
  
7.  $\sin 10^\circ \sin 30^\circ \sin 50^\circ \sin 70^\circ$  का मान है :-
  - (1)  $\frac{1}{36}$
  - (2)  $\frac{1}{32}$
  - (3)  $\frac{1}{18}$
  - (4)  $\frac{1}{16}$

## SOLUTION

## 1. Ans. (1)

We have,

$$\begin{aligned} & 3(\sin \theta - \cos \theta)^4 + 6(\sin \theta + \cos \theta)^2 + 4 \sin^6 \theta \\ &= 3(1 - \sin 2\theta)^2 + 6(1 + \sin 2\theta) + 4 \sin^6 \theta \\ &= 3(1 - 2 \sin 2\theta + \sin^2 2\theta) + 6 + 6 \sin 2\theta + 4 \sin^6 \theta \\ &= 9 + 12 \sin^2 \theta \cdot \cos^2 \theta + 4(1 - \cos^2 \theta)^3 \\ &= 13 - 4 \cos^6 \theta \end{aligned}$$

## 2. Ans. (3)

$$2 \sin \frac{\pi}{2^{10}} \cos \frac{\pi}{2^{10}} \dots \dots \cos \frac{\pi}{2^2}$$

$$\frac{1}{2^9} \sin \frac{\pi}{2} = \frac{1}{512}$$

Option (3)

## 3. Ans. (4)

$$\begin{aligned} & f_4(x) - f_6(x) \\ &= \frac{1}{4}(\sin^4 x + \cos^4 x) - \frac{1}{6}(\sin^6 x + \cos^6 x) \\ &= \frac{1}{4}\left(1 - \frac{1}{2} \sin^2 2x\right) - \frac{1}{6}\left(1 - \frac{3}{4} \sin^2 2x\right) = \frac{1}{12} \end{aligned}$$

## 4. Ans. (1)

$$y = 3 \cos \theta + 5 \left( \sin \theta \frac{\sqrt{3}}{2} - \cos \theta \frac{1}{2} \right)$$

$$\frac{5\sqrt{3}}{2} \sin \theta + \frac{1}{2} \cos \theta$$

$$y_{\max} = \sqrt{\frac{75}{4} + \frac{1}{4}} = \sqrt{19}$$

## 5. Official Ans. by NTA (4)

$$\text{Sol. } 0 < \alpha + \beta = \frac{\pi}{2} \text{ and } \frac{-\pi}{4} < \alpha - \beta < \frac{\pi}{4}$$

$$\text{if } \cos(\alpha + \beta) = \frac{3}{5} \text{ then } \tan(\alpha + \beta) = \frac{4}{3}$$

$$\text{and if } \sin(\alpha - \beta) = \frac{5}{13} \text{ then } \tan(\alpha - \beta) = \frac{5}{12}$$

(since  $\alpha - \beta$  here lies in the first quadrant)

$$\text{Now } \tan(2\alpha) = \tan\{(\alpha + \beta) + (\alpha - \beta)\}$$

$$= \frac{\tan(\alpha + \beta) + \tan(\alpha - \beta)}{1 - \tan(\alpha + \beta) \cdot \tan(\alpha - \beta)} = \frac{\frac{4}{3} + \frac{5}{12}}{1 - \frac{4}{3} \cdot \frac{5}{12}} = \frac{63}{16}$$

## 6. Official Ans. by NTA (2)

$$\text{Sol. } \frac{1}{2}(2 \cos^2 10^\circ - 2 \cos 10^\circ \cos 50^\circ + 2 \cos^2 50^\circ)$$

$$\Rightarrow \frac{1}{2}(1 + \cos 20^\circ - (\cos 60^\circ + \cos 40^\circ) + 1 + \cos 100^\circ)$$

$$\Rightarrow \frac{1}{2}\left(\frac{3}{2} + \cos 20^\circ + 2 \sin 70^\circ \sin(-30^\circ)\right)$$

$$\Rightarrow \frac{1}{2}\left(\frac{3}{2} + \cos 20^\circ - \sin 70^\circ\right)$$

$$\Rightarrow \frac{3}{4} \text{ Ans. (2)}$$

## 7. Official Ans. by NTA (4)

$$\text{Sol. } (\sin 10^\circ \sin 30^\circ \sin 70^\circ) \sin 30^\circ$$

$$\frac{1}{4}(\sin 30^\circ)^2 = \frac{1}{4} \cdot \frac{1}{4} = \frac{1}{16}$$