VECTORS

1. Let $\vec{a}=\hat{i}-\hat{j}, \vec{b}=\hat{i}+\hat{j}+\hat{k}$ and $\vec{c}$ be a vector such that $\vec{a}\times\vec{c}+\vec{b}=\vec{0}$ and $\vec{a}.\vec{c}=4$, then $|\vec{c}|^2$ is equal to:-

(1) $\frac{19}{2}$ (2) 8
(3) $\frac{17}{2}$ (4) 9

2. Let $\vec{a}=\hat{i}+\hat{j}+\sqrt{2}\hat{k}$, $\vec{b}=b_1\hat{i}+b_2\hat{j}+\sqrt{2}\hat{k}$ and $\vec{c}=5\hat{i}+\hat{j}+\sqrt{2}\hat{k}$ be three vectors such that the projection vector of $\vec{b}$ on $\vec{a}$ is $\vec{a}$. If $\vec{a}+\vec{b}$ is perpendicular to $\vec{c}$, then $|\vec{b}|$ is equal to:

(1) $\sqrt{22}$ (2) 4 (3) $\sqrt{32}$ (4) 6

3. Let $\vec{a}=2\hat{i}+\lambda_1\hat{j}+3\hat{k}$, $\vec{b}=4\hat{i}+(3-\lambda_2)\hat{j}+6\hat{k}$ and $\vec{c}=3\hat{i}+6\hat{j}+(\lambda_3-1)\hat{k}$ be three vectors such that $\vec{b}=2\vec{a}$ and $\vec{a}$ is perpendicular to $\vec{c}$. Then a possible value of $(\lambda_1,\lambda_2,\lambda_3)$ is :-

(1) $\left(\frac{1}{2},4,-2\right)$ (2) $\left(-\frac{1}{2},4,0\right)$
(3) (1,3,1) (4) (1,5,1)

4. Let $\vec{\alpha}=(\lambda-2)\vec{a}+\vec{b}$ and $\vec{\beta}=(4\lambda-2)\vec{a}+3\vec{b}$ be two given vectors where vectors $\vec{a}$ and $\vec{b}$ are non-collinear. The value of λ for which vectors $\vec{\alpha}$ and $\vec{\beta}$ are collinear, is :

(1) –3 (2) 4 (3) 3 (4) –4

5. Let $\vec{a}=\hat{i}+2\hat{j}+4\hat{k}$, $\vec{b}=\hat{i}+\lambda\hat{j}+4\hat{k}$ and $\vec{c}=2\hat{i}+4\hat{j}+(\lambda^2-1)\hat{k}$ be coplanar vectors. Then the non-zero vector $\vec{a}\times\vec{c}$ is :

(1) $-14\hat{i}-5\hat{j}$ (2) $-10\hat{i}-5\hat{j}$
(3) $-10\hat{i}+5\hat{j}$ (4) $-14\hat{i}+5\hat{j}$

6. Let $\sqrt{3}\hat{i}+\hat{j}$, $\hat{i}+\sqrt{3}\hat{j}$ and $\beta\hat{i}+(1-\beta)\hat{j}$ respectively be the position vectors of the points A, B and C with respect to the origin O. If the distance of C from the bisector of the acute angle between OA and OB is $\frac{3}{\sqrt{2}}$, then the sum of all possible values of β is :-

(1) 2 (2) 1
(3) 3 (4) 4

7. The sum of the distinct real values of μ, for which the vectors, $\mu\hat{i}+\hat{j}+\hat{k}$, $\hat{i}+\mu\hat{j}+\hat{k}$, $\hat{i}+\hat{j}+\mu\hat{k}$ are co-planer, is :

(1) 2 (2) 0 (3) –1 (4) 1

8. Let $\vec{a},\vec{b}$ and $\vec{c}$ be three unit vectors, out of which vectors $\vec{b}$ and $\vec{c}$ are non-parallel. If α and β are the angles which vector $\vec{a}$ makes with vectors $\vec{b}$ and $\vec{c}$ respectively and $\vec{a}\times(\vec{b}\times\vec{c})=\frac{1}{2}\vec{b}$, then $|\alpha-\beta|$ is equal to :

(1) 60° (2) 30° (3) 90° (4) 45°

9. The magnitude of the projection of the vector $2\hat{i}+3\hat{j}+\hat{k}$ on the vector perpendicular to the plane containing the vectors $\hat{i}+\hat{j}+\hat{k}$ and $\hat{i}+2\hat{j}+3\hat{k}$, is :

(1) $\frac{\sqrt{3}}{2}$ (2) $\sqrt{\frac{3}{2}}$
(3) $\sqrt{6}$ (4) $3\sqrt{6}$

10. Let $\vec{a}=3\hat{i}+2\hat{j}+x\hat{k}$ and $\vec{b}=\hat{i}-\hat{j}+\hat{k}$, for some real x. Then $|\vec{a}\times\vec{b}|=r$ is possible if :

(1) $3\sqrt{\frac{3}{2}}<r<5\sqrt{\frac{3}{2}}$ (2) $0<r\le\sqrt{\frac{3}{2}}$
(3) $\sqrt{\frac{3}{2}}<r\le 3\sqrt{\frac{3}{2}}$ (4) $r\ge 5\sqrt{\frac{3}{2}}$

11. Let $\vec{\alpha} = 3\hat{i} + \hat{j}$ and $\vec{\beta} = 2\hat{i} - \hat{j} + 3\hat{k}$. If $\vec{\beta} = \vec{\beta}_1 - \vec{\beta}_2$, where $\vec{\beta}_1$ is parallel to $\vec{\alpha}$ and $\vec{\beta}_2$ is perpendicular to $\vec{\alpha}$, then $\vec{\beta}_1 \times \vec{\beta}_2$ is equal to

(1) $-3\hat{i} + 9\hat{j} + 5\hat{k}$ (2) $3\hat{i} - 9\hat{j} - 5\hat{k}$

(3) $\frac{1}{2}\left(-3\hat{i} + 9\hat{j} + 5\hat{k}\right)$ (4) $\frac{1}{2}\left(3\hat{i} - 9\hat{j} + 5\hat{k}\right)$

12. If a unit vector $\vec{a}$ makes angles $\pi/3$ with $\hat{i}, \pi/4$ with $\hat{j}$ and $\theta \in (0, \pi)$ with $\hat{k}$, then a value of θ is :-

(1) $\frac{5\pi}{12}$ (2) $\frac{5\pi}{6}$

(3) $\frac{2\pi}{3}$ (4) $\frac{\pi}{4}$

13. The distance of the point having position vector $-\hat{i} + 2\hat{j} + 6\hat{k}$ from the straight line passing through the point (2, 3, –4) and parallel to the vector, $6\hat{i} + 3\hat{j} - 4\hat{k}$ is :

(1) 7 (2) $4\sqrt{3}$

(3) $2\sqrt{13}$ (4) 6

14. If the volume of parallelopiped formed by the vectors $\hat{i} + \lambda\hat{j} + \hat{k}$, $\hat{j} + \lambda\hat{k}$ and $\lambda\hat{i} + \hat{k}$ is minimum, then λ is equal to :

(1) $\sqrt{3}$ (2) $-\frac{1}{\sqrt{3}}$

(3) $\frac{1}{\sqrt{3}}$ (4) $-\sqrt{3}$

15. Let $\vec{a} = 3\hat{i} + 2\hat{j} + 2\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$ be two vectors. If a vector perpendicular to both the vectors $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ has the magnitude 12 then one such vector is

(1) $4(2\hat{i} + 2\hat{j} - \hat{k})$

(2) $4(-2\hat{i} - 2\hat{j} + \hat{k})$

(3) $4(2\hat{i} - 2\hat{j} - \hat{k})$

(4) $4(2\hat{i} + 2\hat{j} + \hat{k})$

16. Let $\alpha \in$ R and the three vectors $\vec{a} = \alpha\hat{i} + \hat{j} + 3\hat{k}$, $\vec{b} = 2\hat{i} + \hat{j} - \alpha\hat{k}$ and $\vec{c} = \alpha\hat{i} - 2\hat{j} + 3\hat{k}$. Then the set S = {$\alpha$: $\vec{a}$, $\vec{b}$ and $\vec{c}$ are coplanar}

(1) is singleton

(2) Contains exactly two numbers only one of which is positive

(3) Contains exactly two positive numbers

(4) is empty

SOLUTION

1. **Ans. (1)**

$\vec{a}\times\vec{c}=-\vec{b}$

$(\vec{a}\times\vec{c})\times\vec{a}=-\vec{b}\times\vec{a}$

$\Rightarrow (\vec{a}\times\vec{c})\times\vec{a}=\vec{a}\times\vec{b}$

$\Rightarrow (\vec{a}.\vec{a})\vec{c}-(\vec{c}.\vec{a})\vec{a}=\vec{a}\times\vec{b}$

$\Rightarrow 2\vec{c}-4\vec{a}=\vec{a}\times\vec{b}$

Now $\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -1 & 0\\ 1 & 1 & 1\end{vmatrix}=-\hat{i}-\hat{j}+2\hat{k}$

So, $2\vec{c}=4\hat{i}-4\hat{j}-\hat{i}-\hat{j}+2\hat{k}$

$=3\hat{i}-5\hat{j}+2\hat{k}$

$\Rightarrow \vec{c}=\frac{3}{2}\hat{i}-\frac{5}{2}\hat{j}+\hat{k}$

$|\vec{c}|=\sqrt{\frac{9}{4}+\frac{25}{4}+1}=\sqrt{\frac{38}{4}}=\sqrt{\frac{19}{2}}$

$|\vec{c}|^2=\frac{19}{2}$

2. **Ans. (4)**

Projection of $\vec{b}$ on $\vec{a}=\frac{\vec{a}.\vec{b}}{|\vec{a}|}=|\vec{a}|$

$\Rightarrow b_1 + b_2 = 2$ ...(1)

and $(\vec{a}+\vec{b})\perp\vec{c}\Rightarrow(\vec{a}+\vec{b}).\vec{c}=0$

$\Rightarrow 5b_1 + b_2 = -10$ ...(2)

from (1) and (2) $\Rightarrow b_1 = -3$ and $b_2 = 5$

then $|\vec{b}|=\sqrt{b_1^2+b_2^2+2}=6$

3. **Ans. (2)**

$4\hat{i}+(3-\lambda_2)\hat{j}+6\hat{k}=4\hat{i}+2\lambda_1\hat{j}+6\hat{k}$

$\Rightarrow 3-\lambda_2=2\lambda_1 \Rightarrow 2\lambda_1+\lambda_2=3$ ....(1)

Given $\vec{a}.\vec{c}=0$

$\Rightarrow 6+6\lambda_1+3(\lambda_3-1)=0$

$\Rightarrow 2\lambda_1+\lambda_3=-1$ .....(2)

Now $(\lambda_1, \lambda_2, \lambda_3) = (\lambda_1, 3-2\lambda_1, -1-2\lambda_1)$

Now check the options, option (2) is correct

4. **Ans. (4)**

$\vec{\alpha}=(\lambda-2)\vec{\alpha}+\vec{b}$

$\vec{\beta}=(4\lambda-2)\vec{\alpha}+3\vec{b}$

$\frac{\lambda-2}{4\lambda-2}=\frac{1}{3}$

$3\lambda-6=4\lambda-2$

$\boxed{\lambda=-4}$

$\therefore$ Option (4)

5. **Ans. (3)**

$[\vec{a}\ \ \vec{b}\ \ \vec{c}]=0$

$\Rightarrow \begin{vmatrix}1 & 2 & 4\\ 1 & \lambda & 4\\ 2 & 4 & \lambda^2-1\end{vmatrix}=0$

$\Rightarrow \lambda^3-2\lambda^2-9\lambda+18=0$

$\Rightarrow \lambda^2(\lambda-2)-9(\lambda-2)=0$

$\Rightarrow (\lambda-3)(\lambda+3)(\lambda-2)=0$

$\Rightarrow \lambda = 2, 3, -3$

So, $\lambda = 2$ (as $\vec{a}$ is parallel to $\vec{c}$ for $\lambda = \pm 3$)

Hence $\vec{a}\times\vec{c}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 2 & 4\\ 2 & 4 & 3\end{vmatrix}=-10\hat{i}+5\hat{j}$

6. **Ans. (2)**

Angle bisector is $x - y = 0$

$\Rightarrow \frac{|\beta-(1-\beta)|}{\sqrt{2}}=\frac{3}{\sqrt{2}}$

$\Rightarrow |2\beta-1|=3$

$\Rightarrow \beta=2 \text{ or } -1$

7. **Ans. (3)**

$\begin{vmatrix}\mu & 1 & 1\\ 1 & \mu & 1\\ 1 & 1 & \mu\end{vmatrix}=0$

$\mu(\mu^2-1)-1(\mu-1)+1(1-\mu)=0$

$\mu^3-\mu-\mu+1+1\ \mu=0$

$\mu^3-3\mu+2=0$

$\mu^3-1-3(\mu-1)=0$

$\mu=1,\ \mu^2+\mu-2=0$

$\mu=1,\ \mu=-2$

sum of distinct solutions $=-1$

8. Ans. (2)

$(\vec{a}.\vec{c})\vec{b}-(\vec{a}.\vec{b}).\vec{c}=\frac{1}{2}\vec{b}$

$\because$ $\vec{b}$ & $\vec{c}$ are linearly independent

$\therefore$ $\vec{a}.\vec{c}=\frac{1}{2}$ & $\vec{a}.\vec{b}=0$

(All given vectors are unit vectors)

$\therefore$ $\vec{a}\wedge\vec{c}=60^\circ$ & $\vec{a}\wedge\vec{b}=90^\circ$

$\therefore$ $|\alpha-\beta| = 30^\circ$

9. Official Ans. by NTA (2)

Sol. Vector perpendicular to plane containing the vectors $\hat{i}+\hat{j}+\hat{k}$ & $\hat{i}+2\hat{j}+3\hat{k}$ is parallel to vector

$$=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 1 & 1\\ 1 & 2 & 3\end{vmatrix}=\hat{i}-2\hat{j}+\hat{k}$$

$\therefore$ Required magnitude of projection

$$=\frac{|(2\hat{i}+3\hat{j}+\hat{k}).(\hat{i}-2\hat{j}+\hat{k})|}{|\hat{i}-2\hat{j}+\hat{k}|}$$

$$=\frac{|2-6+1|}{|\sqrt{6}|}=\frac{3}{\sqrt{6}}=\sqrt{\frac{3}{2}}$$

10. Official Ans. by NTA (4)

$$\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & 2 & x\\ 1 & -1 & 1\end{vmatrix}$$

$$=(2+x)\hat{i}+(x-3)\hat{j}-5k$$

$$|\vec{a}\times\vec{b}|=\sqrt{4+x^2+4x+x^2+9-6x+25}$$

$$=\sqrt{2x^2-2x+38}$$

$$\Rightarrow|\vec{a}\times\vec{b}|\geq\sqrt{\frac{75}{2}}$$

$$\Rightarrow|\vec{a}\times\vec{b}|\geq 5\sqrt{\frac{3}{2}}$$

11. Official Ans. by NTA (3)

Sol. $\vec{\alpha}=3\hat{i}+\hat{j}$

$\vec{\beta}=2\hat{i}-\hat{j}+3\hat{k}$

$\vec{\beta}=\vec{\beta}_1-\vec{\beta}_2$

$\vec{\beta}_1=\lambda(3\hat{i}+\hat{j}),\vec{\beta}_2=\lambda(3\hat{i}+\hat{j})-2\hat{i}+j-3\hat{k}$

$\vec{\beta}_2.\vec{\alpha}=0$

$(3\lambda-2).3+(\lambda+1)=0$

$9\lambda-6+\lambda+1=0$

$\lambda=\frac{1}{2}$

$\Rightarrow \vec{\beta}_1=\frac{3}{2}\hat{i}+\frac{1}{2}\hat{j}$

$\Rightarrow \vec{\beta}_2=-\frac{1}{2}\hat{i}+\frac{3}{2}\hat{j}-3\hat{k}$

Now $\vec{\beta}_1\times\vec{\beta}_2=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ \frac{3}{2} & \frac{1}{2} & 0\\ -\frac{1}{2} & \frac{3}{2} & -3\end{vmatrix}$

$$=\hat{i}\left(-\frac{3}{2}-0\right)-\hat{j}\left(-\frac{9}{2}-0\right)+\hat{k}\left(\frac{9}{4}+\frac{1}{4}\right)$$

$$=-\frac{3}{2}\hat{i}+\frac{9}{2}\hat{j}+\frac{5}{2}\hat{k}$$

$$=\frac{1}{2}(-3\hat{i}+9\hat{j}+5\hat{k})$$

Aliter :

$\vec{\beta}=\vec{\beta}_1-\vec{\beta}_2 \Rightarrow \vec{\beta}.\hat{\alpha}=\vec{\beta}_1.\hat{\alpha}=|\vec{\beta}_1|$

$\Rightarrow \vec{\beta}_1=(\vec{\beta}.\hat{\alpha})\hat{\alpha}$

$\Rightarrow \vec{\beta}_2=(\vec{\beta}.\hat{\alpha})\hat{\alpha}-\vec{\beta}$

$\Rightarrow \vec{\beta}_1\times\vec{\beta}_2=-(\vec{\beta}.\hat{\alpha})\hat{\alpha}\times\vec{\beta}$

$$=\frac{-5}{10}(3\hat{i}+\hat{j})\times(2\hat{i}-\hat{j}+3\hat{k})$$

$$=\frac{1}{2}(-3\hat{i}+9\hat{j}+5\hat{k})$$

12. Official Ans. by NTA (3)

Sol. $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

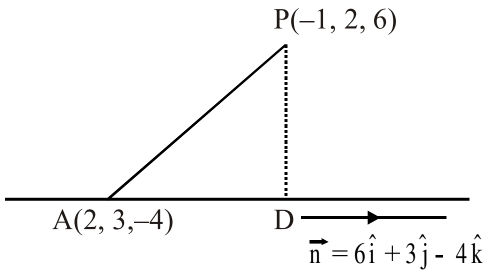
$$\frac{1}{4}+\frac{1}{2}+\cos^2\gamma = 1$$

$$\cos^2\gamma = 1-\frac{3}{4}=\frac{1}{4}$$

$$\cos^2\gamma = \pm\frac{1}{2} \Rightarrow \gamma = \frac{\pi}{3} \text{ or } \frac{2\pi}{3}$$

13. Official Ans. by NTA (1)

Sol.



$$AD = \left|\frac{\overrightarrow{AP}.\vec{n}}{|\vec{n}|}\right| = \sqrt{61}$$

$$\Rightarrow \quad PD = \sqrt{AP^2 - AD^2} = \sqrt{110-61} = 7$$

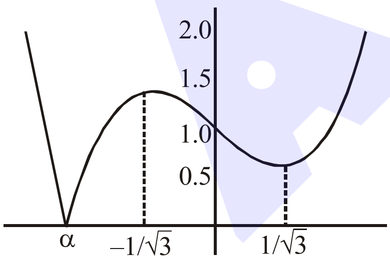
14. Official Ans. by NTA (3)

ALLEN Ans. Bonus

Sol. Volume of parallelopiped $= \left\|\begin{matrix} 1 & \lambda & 1 \\ 0 & 1 & \lambda \\ \lambda & 0 & 1 \end{matrix}\right\|$

$f(\lambda) = |\lambda^3 - \lambda + 1|$

Its graph as follows



where $\alpha \approx -1.32$

$\because$ Question is asking minimum value of volume of parallelopiped & corresponding value of λ; the minimum value is zero, $\because$ cubic always has atleast one real root.

Hence answer to the question must be root of cubic $\lambda^3 - \lambda + 1 = 0$. None of the options satisfies the cubic.

Hence Question must be Bonus.

In JEE (Screening) 2003 same Question was asked and answer was given to be none of these, where the options were :

(A) –3 (B) 3

(C) $\frac{1}{\sqrt{3}}$ (D) none of these

15. Official Ans. by NTA (3)

Sol. $(\vec{a}+\vec{b})\times(\vec{a}-\vec{b}) = 2(\vec{b}\times\vec{a})$

$$= 2\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -2 \\ 3 & 2 & 2 \end{vmatrix} = 2(8\hat{i}-8\hat{j}+4\hat{k})$$

Required vector $= \pm 12\frac{(2\hat{i}-2\hat{j}-\hat{k})}{3}$

$= \pm 4(2\hat{i}-2\hat{j}-\hat{k})$

16. Official Ans. by NTA (4)

Sol.
$$\begin{vmatrix} \alpha & 1 & 3 \\ 2 & 1 & -4 \\ \alpha & -2 & 3 \end{vmatrix} = 0$$

$$\Rightarrow 3\alpha^2 + 18 = 0$$

$$\Rightarrow \alpha \in \phi$$