TANGENT & NORMAL

1. If θ denotes the acute angle between the curves, $y = 10 - x^2$ and $y = 2 + x^2$ at a point of their intersection, then $|\tan \theta|$ is equal to :

(1) 4/9 (2) 7/17 (3) 8/17 (4) 8/15

2. The tangent to the curve $y = x^2 - 5x + 5$, parallel to the line $2y = 4x + 1$, also passes through the point.

(1) $\left(\frac{1}{4}, \frac{7}{2}\right)$ (2) $\left(\frac{7}{2}, \frac{1}{4}\right)$

(3) $\left(-\frac{1}{8}, 7\right)$ (4) $\left(\frac{1}{8}, -7\right)$

3. A helicopter is flying along the curve given by $y - x^{3/2} = 7$, $(x \geq 0)$. A soldier positioned at the point $\left(\frac{1}{2}, 7\right)$ wants to shoot down the helicopter when it is nearest to him. Then this nearest distance is:

(1) $\frac{1}{2}$ (2) $\frac{1}{3}\sqrt{\frac{7}{3}}$ (3) $\frac{1}{6}\sqrt{\frac{7}{3}}$ (4) $\frac{\sqrt{5}}{6}$

4. Let S be the set of all values of x for which the tangent to the curve $y = f(x) = x^3 - x^2 - 2x$ at (x, y) is parallel to the line segment joining the points $(1, f(1))$ and $(-1, f(-1))$, then S is equal to :

(1) $\left\{-\frac{1}{3}, -1\right\}$ (2) $\left\{\frac{1}{3}, -1\right\}$

(3) $\left\{-\frac{1}{3}, 1\right\}$ (4) $\left\{\frac{1}{3}, 1\right\}$

5. If the tangent to the curve, $y = x^3 + ax - b$ at the point $(1, -5)$ is perpendicular to the line, $-x + y + 4 = 0$, then which one of the following points lies on the curve ?

(1) (–2, 2) (2) (2, –2)

(3) (2, –1) (4) (–2, 1)

6. A water tank has the shape of an inverted right circular cone, whose semi-vertical angle is $\tan^{-1}\left(\frac{1}{2}\right)$. Water is poured into it at a constant rage of 5 cubic meter per minute. The the rate (in m/min.), at which the level of water is rising at the instant when the depth of water in the tank is 10m; is :-

(1) $2/\pi$ (2) $1/5\pi$ (3) $1/10\pi$ (4) $1/15\pi$

7. A spherical iron ball of radius 10 cm is coated with a layer of ice of uniform thickness that melts at a rate of 50 cm^3/min. When the thickness of the ice is 5cm, then the rate at which the thickness (in cm/min) of the ice decreases, is :

(1) $\frac{1}{9\pi}$ (2) $\frac{5}{6\pi}$ (3) $\frac{1}{18\pi}$ (4) $\frac{1}{36\pi}$

8. If the tangent to the curve $y = \frac{x}{x^2 - 3}$, $x \in \mathbb{R}$, $\left(x \neq \pm\sqrt{3}\right)$, at a point $(\alpha, \beta) \neq (0, 0)$ on it is parallel to the line $2x + 6y - 11 = 0$, then :

(1) $|6\alpha + 2\beta| = 19$ (2) $|2\alpha + 6\beta| = 11$

(3) $|6\alpha + 2\beta| = 9$ (4) $|2\alpha + 6\beta| = 19$ XZ

SOLUTION

1. Ans. (4)

Point of intersection is P(2,6).

Also, $m_1 = \left(\frac{dy}{dx}\right)_{P(2,6)} = -2x = -4$

$m_2 = \left(\frac{dy}{dx}\right)_{P(2,6)} = 2x = 4$

$\therefore |\tan\theta| = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| = \frac{8}{15}$

2. Ans (4)

$y = x^2 - 5x + 5$

$\frac{dy}{dx} = 2x - 5 = 2 \Rightarrow x = \frac{7}{2}$

at $x = \frac{7}{2}$, $y = \frac{-1}{4}$

Equation of tangent at $\left(\frac{7}{2}, \frac{-1}{4}\right)$ is $2x - y - \frac{29}{4} = 0$

Now check options

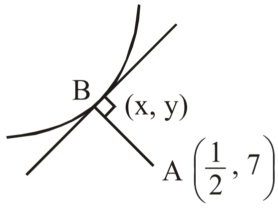
$x = \frac{1}{8}$, $y = -7$

3. Ans. (3)

$y - x^{3/2} = 7 \ (x \geq 0)$

$\frac{dy}{dx} = \frac{3}{2}x^{1/2}$

$\left(\frac{3}{2}\sqrt{x}\right)\left(\frac{7-y}{\frac{1}{2}-x}\right) = -1$



$\left(\frac{3}{2}\sqrt{x}\right)\left(\frac{-x^{3/2}}{\frac{1}{2}-x}\right) = -1$

$\frac{3}{2}\cdot x^2 = \frac{1}{2} - x$

$3x^2 = 1 - 2x$

$3x^2 + 2x - 1 = 0$

$3x^2 + 3x - x - 1 = 0$

$(x + 1)(3x - 1) = 0$

$\therefore \ x = -1$ (rejected)

$x = \frac{1}{3}$

$y = 7 + x^{3/2} = 7 + \left(\frac{1}{3}\right)^{3/2}$

$\ell_{AB} = \sqrt{\left(\frac{1}{2} - \frac{1}{3}\right)^2 + \left(\frac{1}{3}\right)^3} = \sqrt{\frac{1}{36} + \frac{1}{27}}$

$= \sqrt{\frac{3+4}{9\times 12}}$

$= \sqrt{\frac{7}{108}} = \frac{1}{6}\sqrt{\frac{7}{3}}$

Option (3)

4. Official Ans. by NTA (3)

Sol. $f(1) = 1 - 1 - 2 = -2$

$f(-1) = -1 - 1 + 2 = 0$

$$m = \frac{f(1) - f(-1)}{1+1} = \frac{-2-0}{2} = -1$$

$$\frac{dy}{dx} = 3x^2 - 2x - 2$$

$3x^2 - 2x - 2 = -1$

$\Rightarrow 3x^2 - 2x - 1 = 0$

$\Rightarrow (x - 1)(3x + 1) = 0$

$\Rightarrow x = 1,\ -\frac{1}{3}$

5. Official Ans. by NTA (2)

Sol. $y = x^3 + ax - b$

(1, –5) lies on the curve

$\Rightarrow -5 = 1 + a - b \Rightarrow a - b = -6$ ... (i)

Also, $y' = 3x^2 + a$

$y'_{(1,-5)} = 3 + a$ (slope of tangent)

$\because$ this tangent is $\perp$ to $-x + y + 4 = 0$

$\Rightarrow (3 + a)(1) = -1$

$\Rightarrow a = -4$ ....(ii)

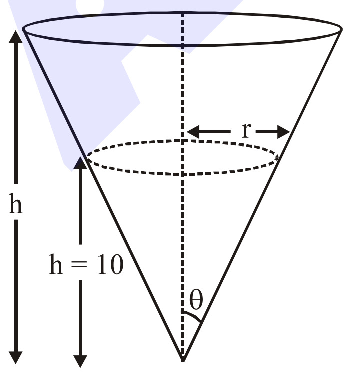
By (i) and (ii) : $a = -4$, $b = 2$

$\therefore\ y = x^3 - 4x - 2.$

(2,–2) lies on this curve.

6. Official Ans. by NTA (2)

Sol. $\tan\theta = \frac{1}{2} = \frac{r}{h}$



$$r = \frac{h}{2}$$

$$V = \frac{1}{3}\pi r^2 h$$

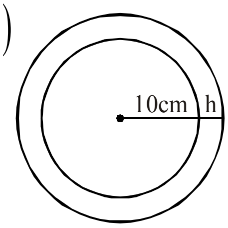
$$V = \frac{1}{3}\pi.\frac{h^3}{4}$$

$$\frac{dV}{dt} = \frac{\pi}{12}(3h)^2\left(\frac{dh}{dt}\right)$$

$$5 = \frac{\pi}{4}.(100)\frac{dh}{dt} \Rightarrow \frac{dh}{dt} = \frac{1}{5\pi}$$

7. Official Ans. by NTA (3)

Sol. $V = \frac{4}{3}\pi\left((10+h)^3 - 10^3\right)$



$$\frac{dV}{dt} = 4\pi(10+h)^2\frac{dh}{dt}$$

$$-50 = 4\pi\ (10 + 5)^2\frac{dh}{dt}$$

$$\Rightarrow \frac{dh}{dt} = -\frac{1}{18}\frac{cm}{min}$$

8. Official Ans. by NTA (1)

Sol. $\left.\frac{dy}{dx}\right|_{(\alpha,\beta)} = \frac{-\alpha^2 - 3}{\left(\alpha^2 - 3\right)^2}$

Given that :

$$\frac{-\alpha^2 - 3}{\left(\alpha^2 - 3\right)^2} = -\frac{1}{3}$$

$\Rightarrow \alpha = 0, \pm 3$ $(\alpha \neq 0)$

$\Rightarrow \beta = \pm\frac{1}{2}.$ $(\beta \neq 0)$

$|6\alpha + 2\beta| = 19$