TRIGONOMETRY EQUATION

1. If $0 \le x < \frac{\pi}{2}$, then the number of values of x for which sin x-sin2x+sin3x = 0, is

(1) 2 (2) 1 (3) 3 (4) 4

2. The sum of all values of $\theta \in \left(0, \frac{\pi}{2}\right)$ satisfying $\sin^2 2\theta + \cos^4 2\theta = \frac{3}{4}$ is :

(1) $\frac{\pi}{2}$ (2) π

(3) $\frac{3\pi}{8}$ (4) $\frac{5\pi}{4}$

3. Let $S = \{\theta \in [-2\pi, 2\pi] : 2\cos^2\theta + 3\sin\theta = 0\}$. Then the sum of the elements of S is

(1) $\frac{13\pi}{6}$ (2) π (3) 2π (4) $\frac{5\pi}{3}$

4. All the pairs (x, y) that satisfy the inequality $2\sqrt{\sin^2 x - 2\sin x + 5} \cdot \frac{1}{4^{\sin^2 y}} \le 1$ also satisfy the eauation.

(1) sin x = |siny| (2) sin x = 2 sin y

(3) 2|sinx| = 3siny (4) 2sin x = siny

5. The number of solutions of the equation $1 + \sin^4 x = \cos^2 3x,\ x \in \left[-\frac{5\pi}{2}, \frac{5\pi}{2}\right]$ is :

(1) 5 (2) 4 (3) 7 (4) 3

6. Let S be the set of all $\alpha \in R$ such that the equation, $\cos 2x + \alpha \sin x = 2\alpha - 7$ has a solution. Then S is equal to :

(1) [2, 6] (2) [3,7] (3) R (4) [1,4]

SOLUTION

1. Ans. (1)

sinx – sin2x + sin3x = 0

$\Rightarrow$ (sinx + sin3x) – sin2x = 0

$\Rightarrow$ 2sinx. cosx – sin2x = 0

$\Rightarrow$ sin2x(2 cosx – 1) = 0

$\Rightarrow$ sin2x = 0 or $\cos x = \frac{1}{2}$

$\Rightarrow x = 0, \frac{\pi}{3}$

2. Ans. (1)

$\sin^2 2\theta + \cos^4 2\theta = \frac{3}{4},\ \theta \in \left(0, \frac{\pi}{2}\right)$

$\Rightarrow 1 - \cos^2 2\theta + \cos^4 2\theta = \frac{3}{4}$

$\Rightarrow 4\cos^4 2\theta - 4\cos^2 2\theta + 1 = 0$

$\Rightarrow (2\cos^2 2\theta - 1)^2 = 0$

$\Rightarrow \cos^2 2\theta = \frac{1}{2} = \cos^2 \frac{\pi}{4}$

$\Rightarrow 2\theta = n\pi \pm \frac{\pi}{4}$, n $\in$ I

$\Rightarrow \theta = \frac{n\pi}{2} \pm \frac{\pi}{8}$

$\Rightarrow \theta = \frac{\pi}{8}, \frac{\pi}{2} - \frac{\pi}{8}$

Sum of solutions $\frac{\pi}{2}$

3. Official Ans. by NTA (3)

Sol. $2\left(1 - \sin^2\theta\right) + 3\sin\theta = 0$

$\Rightarrow 2\sin^2\theta - 3\sin\theta - 2 = 0$

$\Rightarrow \left(2\sin\theta + 1\right)\left(\sin\theta - 2\right) = 0$

$\Rightarrow \sin\theta = -\frac{1}{2}; \sin\theta = 2\left(\text{reject}\right)$

roots : $\pi + \frac{\pi}{6}, 2\pi - \frac{\pi}{6}, -\frac{\pi}{6}, -\pi + \frac{\pi}{6}$

$\Rightarrow$ sum of values $= 2\pi$

4. Official Ans. by NTA (1)

Sol. $2^{\sqrt{\sin^2 x - 2\sin x + 5}} \cdot 4^{-\sin^2 y} \le 1$

$\Rightarrow 2^{\sqrt{(\sin x - 1)^2 + 4}} \le 2^{2\sin^2 y}$

$\Rightarrow \sqrt{(\sin x - 1)^2 + 4} \le 2\sin^2 y$

$\Rightarrow$ sinx=1 and |siny| =1

5. Official Ans. by NTA (1)

Sol. $1 + \sin^4 x = \cos^2 3x$

sinx = 0 & cos3x = 1

0, 2π, –2π, –π, π

6. Official Ans. by NTA (1)

Sol. cos2x + αsinx = 2α – 7

$\Rightarrow 2\sin^2 x - \alpha \sin x + 2\alpha - 8 = 0$

$\sin^2 x - \frac{\alpha}{2}\sin x + \alpha - 4 = 0$

$\Rightarrow \sin x = 2$ (rejected) or $\sin x = \frac{\alpha - 4}{2}$

$\Rightarrow \left|\frac{\alpha - 4}{2}\right| \le 1$

$\Rightarrow \alpha \in [2, 6]$