STRAIGHT LINE

1. Consider the set of all lines $px + qy + r = 0$ such that $3p + 2q + 4r = 0$. Which one of the following statements is true ?
(1) The lines are all parallel.
(2) Each line passes through the origin.
(3) The lines are not concurrent
(4) The lines are concurrent at the point $\left(\frac{3}{4}, \frac{1}{2}\right)$

2. Let the equations of two sides of a triangle be $3x - 2y + 6 = 0$ and $4x + 5y - 20 = 0$. If the orthocentre of this triangle is at (1,1), then the equation of its third side is :
(1) $122y - 26x - 1675 = 0$
(2) $26x + 61y + 1675 = 0$
(3) $122y + 26x + 1675 = 0$
(4) $26x - 122y - 1675 = 0$

3. Let S be the set of all triangles in the xy-plane, each having one vertex at the origin and the other two vertices lie on coordinate axes with integral coordinates. If each triangle in S has area 50sq. units, then the number of elements in the set S is:
(1) 9 (2) 18 (3) 32 (4) 36

4. If the line $3x + 4y - 24 = 0$ intersects the x-axis at the point A and the y-axis at the point B, then the incentre of the triangle OAB, where O is the origin, is
(1) (3, 4) (2) (2, 2) (3) (4, 4) (4) (4, 3)

5. A point P moves on the line $2x - 3y + 4 = 0$. If Q(1,4) and R(3,–2) are fixed points, then the locus of the centroid of ΔPQR is a line :
(1) parallel to x-axis (2) with slope $\frac{2}{3}$
(3) with slope $\frac{3}{2}$ (4) parallel to y-axis

6. Two vertices of a triangle are (0,2) and (4,3). If its orthocentre is at the origin, then its third vertex lies in which quadrant ?
(1) Fourth (2) Second (3) Third (4) First

7. Two sides of a parallelogram are along the lines, $x + y = 3$ and $x - y + 3 = 0$. If its diagonals intersect at (2,4), then one of its vertex is :
(1) (2,6) (2) (2,1) (3) (3,5) (4) (3,6)

8. If in a parallelogram ABDC, the coordinates of A, B and C are respectively (1, 2), (3, 4) and (2, 5), then the equation of the diagonal AD is:-
(1) $5x + 3y - 11 = 0$ (2) $3x - 5y + 7 = 0$
(3) $3x + 5y - 13 = 0$ (4) $5x - 3y + 1 = 0$

9. If the straight line, $2x-3y+17 = 0$ is perpendicular to the line passing through the points (7, 17) and (15, β), then β equals :-
(1) –5 (2) $-\frac{35}{3}$ (3) $\frac{35}{3}$ (4) 5

10. If a straight line passing thourgh the point P(–3, 4) is such that its intercepted portion between the coordinate axes is bisected at P, then its equation is :
(1) $x - y + 7 = 0$ (2) $3x - 4y + 25 = 0$
(3) $4x + 3y = 0$ (4) $4x - 3y + 24 = 0$

11. If a circle of radius R passes through the origin O and intersects the coordinate axes at A and B, then the locus of the foot of perpendicular from O on AB is :
(1) $(x^2 + y^2)^2 = 4Rx^2y^2$
(2) $(x^2 + y^2)(x + y) = R^2xy$
(3) $(x^2 + y^2)^3 = 4R^2x^2y^2$
(4) $(x^2 + y^2)^2 = 4R^2x^2y^2$

12. A point on the straight line, $3x + 5y = 15$ which is equidistant from the coordinate axes will lie only in :
(1) 1^{st} and 2^{nd} quadrants
(2) 4^{th} quadrant
(3) 1^{st}, 2^{nd} and 4^{th} quadrant
(4) 1^{st} quadrant

13. Suppose that the points (h,k), (1,2) and (–3,4) lie on the line L_1. If a line L_2 passing through the points (h,k) and (4,3) is perpendicular to L_1, then $\frac{k}{h}$ equals :

(1) 3 (2) $-\frac{1}{7}$ (3) $\frac{1}{3}$ (4) 0

14. Slope of a line passing through P(2, 3) and intersecting the line, x + y = 7 at a distance of 4 units from P, is

(1) $\frac{\sqrt{5}-1}{\sqrt{5}+1}$ (2) $\frac{1-\sqrt{5}}{1+\sqrt{5}}$

(3) $\frac{1-\sqrt{7}}{1+\sqrt{7}}$ (4) $\frac{\sqrt{7}-1}{\sqrt{7}+1}$

15. If the two lines x + (a – 1) y = 1 and $2x + a^2y = 1 (a \in R - \{0, 1\})$ are perpendicular, then the distance of their point of intersection from the origin is :-

(1) $\frac{2}{5}$ (2) $\frac{2}{\sqrt{5}}$ (3) $\frac{\sqrt{2}}{5}$ (4) $\sqrt{\frac{2}{5}}$

16. A rectangle is inscribed in a circle with a diameter lying along the line 3y = x + 7. If the two adjacent vertices of the rectangle are (–8, 5) and (6, 5), then the area of the rectangle (in sq. units) is :-

(1) 72 (2) 84 (3) 98 (4) 56

17. The region represented by $|x-y| \le 2$ and $|x+y| \le 2$ is bounded by a :

(1) square of side length $2\sqrt{2}$ units

(2) rhombus of side length 2 units

(3) square of area 16 sq, units

(4) rhombus of area 8 $\sqrt{2}$ sq. units

18. Lines are drawn parallel to the line $4x - 3y + 2 = 0$, at a distance $\frac{3}{5}$ from the origin.

Then which one of the following points lies on any of these lines ?

(1) $\left(-\frac{1}{4},\frac{2}{3}\right)$ (2) $\left(\frac{1}{4},\frac{1}{3}\right)$

(3) $\left(-\frac{1}{4},-\frac{2}{3}\right)$ (4) $\left(\frac{1}{4},-\frac{1}{3}\right)$

19. The equation y = sinx sin(x + 2) – $\sin^2(x+1)$ represents a straight line lying in :

(1) second and third quadrants only

(2) third and fourth quadrants only

(3) first, third and fourth quadrants

(4) first, second and fourth quadrants

20. A straight line L at a distance of 4 units from the origin makes positive intercepts on the coordinate axes and the perpendicular from the origin to this line makes an angle of 60° with the line x + y = 0. Then an equation of the line L is :

(1) $(\sqrt{3}+1)x+(\sqrt{3}-1)y=8\sqrt{2}$

(2) $(\sqrt{3}-1)x+(\sqrt{3}+1)y=8\sqrt{2}$

(3) $\sqrt{3}x+y=8$

(4) $x+\sqrt{3}y=8$

21. A triangle has a vertex at (1, 2) and the mid points of the two sides through it are (–1, 1) and (2,3). Then the centroid of this triangle is :

(1) $\left(\frac{1}{3},1\right)$ (2) $\left(\frac{1}{3},2\right)$

(3) $\left(1,\frac{7}{3}\right)$ (4) $\left(\frac{1}{3},\frac{5}{3}\right)$

SOLUTION

1. Ans. (4)

Given set of lines px + qy + r = 0

given condition 3p + 2q + 4r = 0

$\Rightarrow \frac{3}{4}p + \frac{1}{2}q + r = 0$

$\Rightarrow$ All lines pass through a fixed point $\left(\frac{3}{4}, \frac{1}{2}\right)$.

2. Ans. (4)

Equation of AB is

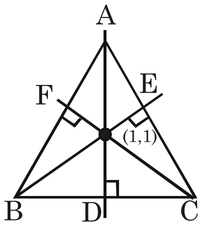
3x – 2y + 6 = 0

equation of AC is

4x + 5y – 20 = 0

Equation of BE is

2x + 3y – 5 = 0



Equation of CF is 5x – 4y – 1 = 0

$\Rightarrow$ Equation of BC is 26x – 122y = 1675

3. Ans. (4)

Let A(α,0) and B(0,β)

be the vertices of the given triangle AOB

$\Rightarrow |\alpha\beta| = 100$

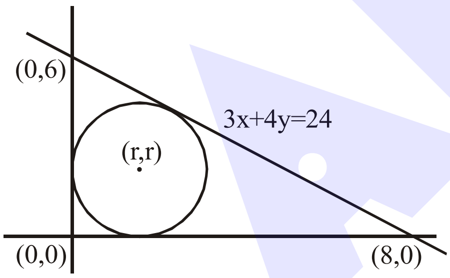
$\Rightarrow$ Number of triangles

= 4 × (number of divisors of 100)

= 4 × 9 = 36

4. Ans. (2)

$\left|\frac{3r + 4r - 24}{5}\right| = r$



7r – 24 = ±5r

2r = 24 or 12r + 24

r = 14, r = 2

then incentre is (2, 2)

5. Ans. (2)

Let the centroid of ΔPQR is (h, k) & P is (α, β), then

$\frac{\alpha + 1 + 3}{3} = h$ and $\frac{\beta + 4 - 2}{3} = k$

α = (3h – 4) β = (3k – 4)

Point P(α, β) lies on line 2x – 3y + 4 = 0

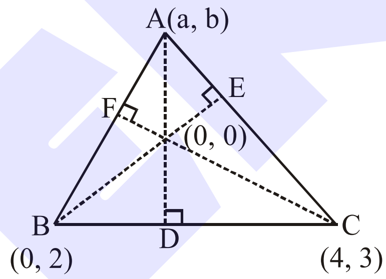
∴ 2(3h – 4) – 3(3k – 2) + 4 = 0

$\Rightarrow$ locus is 6x – 9y + 2 = 0

6. Ans. (2)

$m_{BD} \times m_{AD} = -1 \Rightarrow \left(\frac{3-2}{4-0}\right) \times \left(\frac{b-0}{a-0}\right) = -1$

$\Rightarrow$ b + 4a = 0 ……(i)



$m_{AB} \times m_{CF} = -1 \Rightarrow \left(\frac{(b-2)}{a-0}\right) \times \left(\frac{3}{4}\right) = -1$

$\Rightarrow$ 3b – 6 = –4a $\Rightarrow$ 4a + 3b = 6 ……(ii)

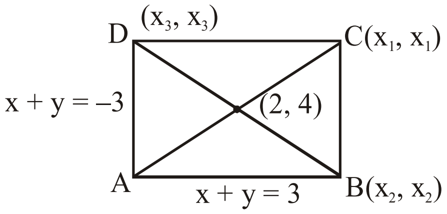
From (i) and (ii)

$a = \frac{-3}{4}$, b = 3

∴ II[nd] quadrant.

Option (2)

7. Ans. (4)



Solving $x + y = 3$
and $x - y = -3$ $\Big\rangle$ A(0, 3)

$\frac{x_1 + 0}{2} = 2;\ x_i = 4$ similarly $y_1 = 5$

C $\Rightarrow$ (4, 5)

Now equation of BC is $x - y = -1$

and equation of CD is $x + y = 9$

Solving $x + y = 9$ and $x - y = -3$

Point D is (3, 6)

Option (4)

8. Ans. (4)

co-ordinates of point D are (4,7)

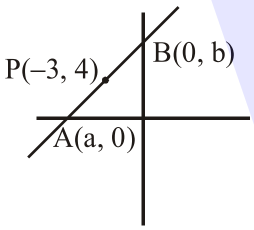
$\Rightarrow$ line AD is $5x - 3y + 1 = 0$

9. Ans. (4)

$$\frac{17-\beta}{-8} \times \frac{2}{3} = -1$$

$\beta = 5$

10. Ans (4)



Let the line be $\frac{x}{a} + \frac{y}{b} = 1$

$(-3, 4) = \left(\frac{a}{2}, \frac{b}{2}\right)$

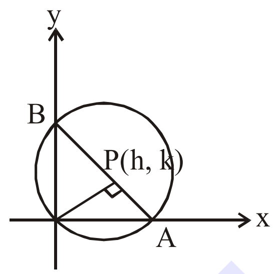
$a = -6$, $b = 8$

equation of line is $4x - 3y + 24 = 0$

11. Ans. (3)

Slope of AB $= \frac{-h}{k}$

Equation of AB is $hx + ky = h^2 + k^2$



$A\left(\frac{h^2+k^2}{h}, 0\right)$, $B\left(0, \frac{h^2+k^2}{k}\right)$

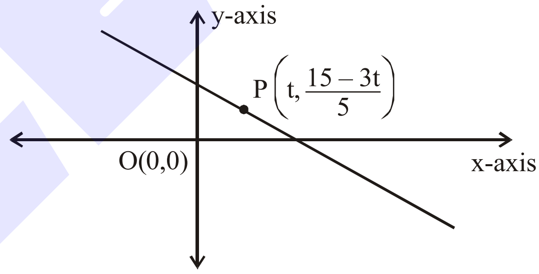
AB = 2R

$\Rightarrow (h^2 + k^2)^3 = 4R^2h^2k^2$

$\Rightarrow (x^2 + y^2)^3 = 4R^2x^2y^2$

12. Official Ans. by NTA (1)

Sol.



Now, $\left|\frac{15-3t}{5}\right| = |t|$

$\Rightarrow \frac{15-3t}{5} = t$ or $\frac{15-3t}{5} = -t$

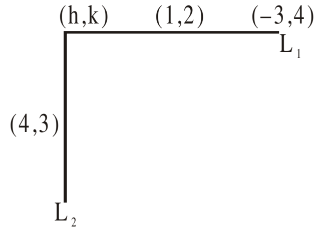
$\therefore t = \frac{15}{8}$ or $t = \frac{-15}{2}$

So, $P\left(\frac{15}{8}, \frac{15}{8}\right) \in I^{st}$ quadrant

or $P\left(\frac{-15}{2}, \frac{15}{2}\right) \in II^{nd}$ quadrant

13. Official Ans. by NTA (3)

Sol.



equation of L_1 is

$$y = -\frac{1}{2}x + \frac{5}{2} \quad ...(1)$$

equation of L_2 is

$y = 2x - 5$ ...(2)

by (1) and (2)

$x = 3$

$y = 1 \Rightarrow h = 3, k = 1$

$$\frac{k}{h} = \frac{1}{3}$$

14. Official Ans. by NTA (3)

Sol. $x = 2 + r\cos\theta$

$y = 3 + r\sin\theta$

$\Rightarrow 2 + r\cos\theta + 3 + r\sin\theta = 7$

$\Rightarrow r(\cos\theta + \sin\theta) = 2$

$$\Rightarrow \sin\theta + \cos\theta = \frac{2}{r} = \frac{2}{\pm 4} = \pm\frac{1}{2}$$

$$\Rightarrow 1 + \sin 2\theta = \frac{1}{4}$$

$$\Rightarrow \sin 2\theta = -\frac{3}{4}$$

$$\Rightarrow \frac{2m}{1+m^2} = -\frac{3}{4}$$

$\Rightarrow 3m^2 + 8m + 3 = 0$

$$\Rightarrow m = \frac{-4 \pm \sqrt{7}}{1-7}$$

$$\frac{1-\sqrt{7}}{1+\sqrt{7}} = \frac{\left(1-\sqrt{7}\right)^2}{1-7} = \frac{8-2\sqrt{7}}{-6} = \frac{-4+\sqrt{7}}{3}$$

15. Official Ans. by NTA (4)

Sol. $\left(\frac{-1}{a-1}\right)\left(\frac{-2}{a^2}\right) = -1$

$2 = -(a^2)(a - 1)$

$a^3 - a^2 + 2 = 0$

$(a + 1)(a^2 - 2a + 2) = 0$

$\therefore\ a = -1$

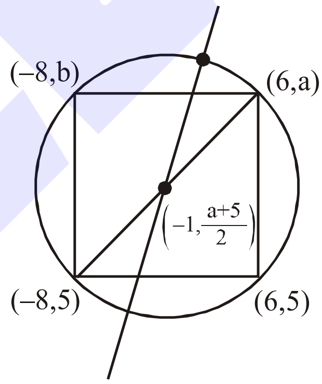
$$\left.\begin{aligned} L_1 &: x - 2y + 1 = 0 \\ L_2 &: 2x + y - 1 = 0 \end{aligned}\right\}$$

$0(0, 0) \quad P\left(\frac{1}{5}, \frac{3}{5}\right)$

$$OP = \sqrt{\frac{1}{25} + \frac{9}{25}} = \sqrt{\frac{10}{25}} = \sqrt{\frac{2}{5}}$$

16. Official Ans. by NTA (2)

Sol.



$$\frac{3(a+5)}{2} = -1 + 7$$

$$a + 5 = \frac{2(6)}{3}$$

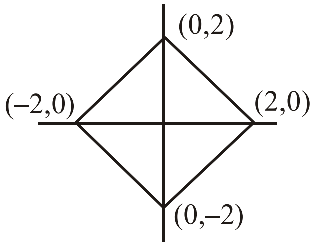
$a = -1$

sides = 6 and 14

$\Rightarrow A = 84$

17. Official Ans. by NTA (1)

Sol. $|x-y| \le 2$ and $|x+y| \le 2$

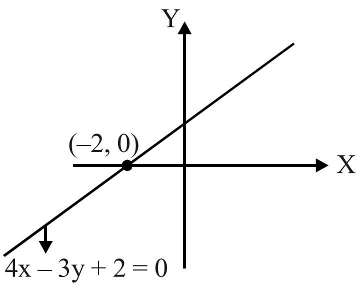


Square whose side is $2\sqrt{2}$

18. Official Ans. by NTA (1)

Sol. Required line is $4x - 3y + \lambda = 0$

$$\left|\frac{\lambda}{5}\right| = \frac{3}{5}$$



$\Rightarrow \lambda = \pm 3.$

So, required equation of line is

$4x - 3y + 3 = 0$ and $4x - 3y - 3 = 0$

$$(1)\ 4\left(-\frac{1}{4}\right) - 3\left(\frac{2}{3}\right) + 3 = 0$$

19. Official Ans. by NTA (2)

Sol. $2y = 2\sin x \sin(x+2) - 2\sin^2(x+1)$

$2y = \cos 2 - \cos(2x+2) - (1 - \cos(2x+2))$

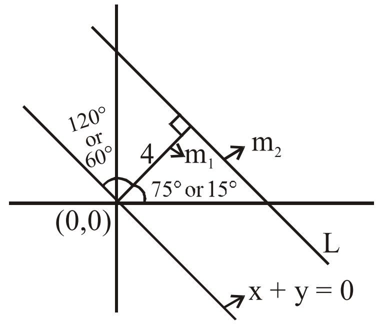
$= \cos 2 - 1$

$$2y = -2\sin^2\frac{1}{2}$$

$$y = -\sin^2\frac{1}{2} \le 0$$

20. Official Ans. by NTA (1) or (2)

Sol.



$$m_1 = \tan 75° = \frac{\sqrt{3}+1}{\sqrt{3}-1}$$

$$\text{or } m = \tan 15° = \frac{\sqrt{3}-1}{\sqrt{3}+1}$$

$$m_2 = \frac{-1}{m_1} = \frac{-\left(\sqrt{3}-1\right)}{\sqrt{3}+1}$$

$$\text{or } m_2 = \frac{-1}{m_1} = \frac{-\left(\sqrt{3}+1\right)}{\sqrt{3}-1}$$

$\Rightarrow y = m_2x + C$

$$\Rightarrow y = \frac{-\left(\sqrt{3}-1\right)x}{\sqrt{3}+1} + C \Rightarrow L$$

$$\text{or } y = \frac{-\left(\sqrt{3}+1\right)x}{\sqrt{3}-1} + C \Rightarrow L$$

Distance from origin = 4

$$\therefore \left|\frac{C}{\sqrt{1+\frac{\left(\sqrt{3}-1\right)^2}{\left(\sqrt{3}+1\right)^2}}}\right| = 4 \text{ or } \left|\frac{C}{\sqrt{1+\frac{\left(\sqrt{3}+1\right)^2}{\left(\sqrt{3}-1\right)^2}}}\right| = 4$$

$$\Rightarrow C = \frac{8\sqrt{2}}{\left(\sqrt{3}+1\right)} \text{ or } C = \frac{8\sqrt{2}}{\left(\sqrt{3}-1\right)}$$

$$\Rightarrow \left(\sqrt{3}-1\right)y + \left(\sqrt{3}+1\right)x = 8\sqrt{2}$$

$$\text{or } \left(\sqrt{3}-1\right)x + \left(\sqrt{3}+1\right)y = 8\sqrt{2}$$

21. Official Ans. by NTA (2)

Sol. Let B(α,β) and C(γ,δ)

$$\frac{\alpha+1}{2}=-1\Rightarrow\alpha=-3$$

$$\frac{\beta+2}{2}=1\Rightarrow\beta=0$$

$$\Rightarrow B(-3,0)$$

Now $\frac{\gamma+1}{2}=2\Rightarrow\gamma=3$

$$\frac{\delta+2}{2}=3\Rightarrow\delta=4$$

$\Rightarrow$ C(3,4)

$\Rightarrow$ centroid of triangle is $G\left(\frac{1}{3},2\right)$