STATISTICS

1. 5 students of a class have an average height 150 cm and variance 18 cm^2. A new student, whose height is 156 cm, joined them. The variance (in cm^2) of the height of these six students is:

(1) 22 (2) 20
(3) 16 (4) 18

2. A data consists of n observations:

$x_1, x_2, \ldots, x_n$. If $\sum_{i=1}^{n}(x_i+1)^2 = 9n$ and $\sum_{i=1}^{n}(x_i-1)^2 = 5n$, then the standard deviation of this data is :

(1) 5 (2) $\sqrt{5}$ (3) $\sqrt{7}$ (4) 2

3. The mean of five observations is 5 and their variance is 9.20. If three of the given five observations are 1, 3 and 8, then a ratio of other two observations is :

(1) 4 : 9 (2) 6 : 7
(3) 5 : 8 (4) 10 : 3

4. If mean and standard deviation of 5 observations x_1, x_2, x_3, x_4, x_5 are 10 and 3, respectively, then the variance of 6 observations $x_1, x_2, \ldots, x_5$ and -50 is equal to:

(1) 582.5 (2) 507.5 (3) 586.5 (4) 509.5

5. The outcome of each of 30 items was observed; 10 items gave an outcome $\frac{1}{2} - d$ each, 10 items gave outcome $\frac{1}{2}$ each and the remaining 10 items gave outcome $\frac{1}{2} + d$ each. If the variance of this outcome data is $\frac{4}{3}$ then $|d|$ equals :-

(1) 2 (2) $\frac{\sqrt{5}}{2}$
(3) $\frac{2}{3}$ (4) $\sqrt{2}$

6. If the sum of the deviations of 50 observations from 30 is 50, then the mean of these observation is :

(1) 50 (2) 51 (3) 30 (4) 31

7. The mean and the variance of five observation are 4 and 5.20, respectively. If three of the observations are 3, 4 and 4; then then absolute value of the difference of the other two observations, is :

(1) 1 (2) 3
(3) 7 (4) 5

8. The mean and variance of seven observations are 8 and 16, respectively. If 5 of the observations are 2, 4, 10, 12, 14, then the product of the remaining two observations is :

(1) 40 (2) 49

(3) 48 (4) 45

9. A student scores the following marks in five tests : 45,54,41,57,43. His score is not known for the sixth test. If the mean score is 48 in the six tests, then the standard deviation of the marks in six tests is

(1) $\frac{10}{\sqrt{3}}$ (2) $\frac{100}{\sqrt{3}}$

(3) $\frac{100}{3}$ (4) $\frac{10}{3}$

10. If the standard deviation of the numbers –1, 0, 1, k is $\sqrt{5}$ where k > 0, then k is equal to

(1) $2\sqrt{\frac{10}{3}}$ (2) $2\sqrt{6}$

(3) $4\sqrt{\frac{5}{3}}$ (4) $\sqrt{6}$

11. The mean and the median of the following ten numbers in increasing order 10, 22, 26, 29, 34, x 42, 67, 70, y are 42 and 35 respectively, then $\frac{y}{x}$ is equal to :-

(1) 7/3 (2) 9/4

(3) 7/2 (4) 8/3

12. If for some $x \in R$, the frequency distribution of the marks obtained by 20 students in a test is :

Marks	2	3	5	7
Frequencey	$(x+1)^2$	$2x-5$	x^2-3x	x

then the mean of the marks is :

(1) 2.8 (2) 3.2

(3) 3.0 (4) 2.5

13. If both the mean and the standard deviation of 50 observations $x_1, x_2,....,x_{50}$ are equal to 16, then the mean of $(x_1 - 4)^2, (x_2 - 4)^2,.....(x_{50} - 4)^2$ is :

(1) 525 (2) 380

(3) 480 (4) 400

14. If the data $x_1, x_2, ..., x_{10}$ is such that the mean of first four of these is 11, the mean of the remaining six is 16 and the sum of squares of all of these is 2,000; then the standard deviation of this data is :

(1) 4 (2) 2

(3) $\sqrt{2}$ (4) $2\sqrt{2}$

SOLUTION

1. Ans. (2)

Given $\vec{x} = \frac{\Sigma x_i}{5} = 150$

$\Rightarrow \sum_{i=1}^{5} x_i = 750$ .....(i)

$\frac{\Sigma x_i^2}{5} - (\bar{x})^2 = 18$

$\frac{\Sigma x_i^2}{5} - (150)^2 = 18$

$\Sigma x_i^2 = 112590$ ....(ii)

Given height of new student

$x_6 = 156$

Now, $\vec{x}_{new} = \frac{\sum_{i=1}^{6} x_i}{6} = \frac{750+156}{6} = 151$

Also, New variance $= \frac{\sum_{i=1}^{6} x_i^2}{6} - (\bar{x}_{new})^2$

$= \frac{112590 + (156)^2}{6} - (151)^2$

$= 22821 - 22801 = 20$

2. Ans. (2)

$\sum (x_i + 1)^2 = 9n$ ...(1)

$\sum (x_i - 1)^2 = 5n$ ...(2)

(1) + (2) $\Rightarrow \sum (x_i^2 + 1) = 7n$

$\Rightarrow \frac{\sum x_i^2}{n} = 6$

(1) - (2) $\Rightarrow 4\Sigma x_i = 4n$

$\Rightarrow \Sigma x_i = n$

$\Rightarrow \frac{\Sigma x_i}{n} = 1$

$\Rightarrow$ variance = 6 – 1 = 5

$\Rightarrow$ Standard diviation $= \sqrt{5}$

3. Ans. (1)

Let two observations are x_1 & x_2

mean $= \frac{\sum x_i}{5} = 5 \Rightarrow 1 + 3 + 8 + x_1 + x_2 = 25$

$\Rightarrow x_1 + x_2 = 13$ ....(1)

variance $(\sigma^2) = \frac{\sum x_i^2}{5} - 25 = 9.20$

$\Rightarrow \sum x_i^2 = 171$

$\Rightarrow x_1^2 + x_2^2 = 97$ .....(2)

by (1) & (2)

$(x_1 + x_2)^2 - 2x_1x_2 = 97$

or $x_1x_2 = 36$

$\therefore x_1 : x_2 = 4 : 9$

4. Ans. (2)

$\bar{x} = 10 \Rightarrow \sum_{i=1}^{5} x_i = 50$

$\text{S.D.} = \sqrt{\frac{\sum_{i=1}^{5} x_i^2}{5} - (\bar{x})^2} = 8$

$\Rightarrow \sum_{i=1}^{5} (x_i)^2 = 109$

$\text{variance} = \frac{\sum_{i=1}^{5} (x_i)^2 + (-50)^2}{6} - \left(\frac{\sum_{i=1}^{5} x_i - 50}{6}\right)$

= 507.5

Option (2)

5. Ans. (4)

Variance is independent of origin. So we shift the given data by $\frac{1}{2}$.

so, $\frac{10d^2 + 10 \times 0^2 + 10d^2}{30} - (0)^2 = \frac{4}{3}$

$\Rightarrow d^2 = 2 \Rightarrow |d| = \sqrt{2}$

6. Ans. (4)

$$\sum_{i=1}^{50}(x_i-30)=50$$

$$\Sigma x_i=50\times30=50$$

$$\Sigma x_i=50+50+30$$

$$\text{Mean}=\bar{x}=\frac{\Sigma x_i}{n}=\frac{50\times30+50}{50}=30+1=31$$

7. Ans. (3)

mean $\bar{x}=4,\ \sigma^2=5.2,\ n=5,.\ x_1=3\ x_2=4=x_3$

$\sum x_i=20$

$x_4+x_5=9$ ....(i)

$\frac{\sum x_i^2}{x}-(\bar{x})^2=\sigma^2\ \Rightarrow\ \sum x_i^2=106$

$x_4^2+x_5^2=65$ ....(ii)

Using (i) and (ii) $(x_4-x_5)^2=49$

$|x_4-x_5|=7$

8. Official Ans. by NTA (3)

Sol. Let 7 observations be $x_1, x_2, x_3, x_4, x_5, x_6, x_7$

$\bar{x}=8\Rightarrow\sum_{i=1}^{7}x_i=56$ ......(1)

Also $\sigma^2=16$

$$\Rightarrow\quad 16=\frac{1}{7}\left(\sum_{i=1}^{7}x_i^2\right)-(\bar{x})^2$$

$$\Rightarrow\quad 16=\frac{1}{7}\left(\sum_{i=1}^{7}x_i^2\right)-64$$

$\Rightarrow\quad\left(\sum_{i=1}^{7}x_i^2\right)=560$ .....(2)

Now, $x_1=2,\ x_2=4,\ x_3=10,\ x_4=12,\ x_5=14$

$\Rightarrow\quad x_6+x_7=14$ (from (1))

& $\quad x_6^2+x_7^2=100$ (from (2))

$\therefore\quad x_6^2+x_7^2=(x_6+x_7)^2-2x_6.x_7\Rightarrow x_6.x_7=48$

9. Official Ans. by NTA (1)

Sol. Let x be the 6th observation

$\Rightarrow 45+54+41+57+43+x=48\times6=288$

$\Rightarrow x=48$

$$\text{variance}=\left(\frac{\Sigma x_i^2}{6}-(\bar{x})^2\right)$$

$$\Rightarrow\text{variance}=\frac{14024}{6}-(48)^2$$

$$=\frac{100}{3}$$

$$\Rightarrow\text{standard deviation}=\frac{10}{\sqrt{3}}$$

10. Official Ans. by NTA (2)

Sol. $$\text{S.D}=\sqrt{\frac{\Sigma(x-\bar{x})^2}{n}}$$

$$\bar{x}=\frac{\Sigma x}{4}=\frac{-1+0+1+k}{4}=\frac{k}{4}$$

$$\text{Now}\quad\sqrt{5}=\sqrt{\frac{\left(-1-\frac{k}{4}\right)^2+\left(0-\frac{k}{4}\right)^2+\left(1-\frac{k}{4}\right)^2+\left(k-\frac{k}{4}\right)^2}{4}}$$

$$\Rightarrow 5\times4=2\left(1+\frac{k}{16}\right)^2+\frac{5k^2}{8}$$

$$\Rightarrow 18=\frac{3k^2}{4}$$

$\Rightarrow k^2=24$

$\Rightarrow k=2\sqrt{6}$

11. Official Ans. by NTA (1)

Sol. $$\frac{34+x}{2}=35$$

$x=36$

$$42=\frac{10+22+26+29+34+36+42+67+70+y}{10}$$

$420-336=y\Rightarrow y=84$

$$\frac{y}{x}=\frac{84}{36}=\frac{7}{3}$$

12. Official Ans. by NTA (1)

Sol. $\sum f_i = 20 = 2x^2 + 2x - 4$

$\Rightarrow x^2 + 2x - 24 = 0$

$x = 3, -4$ (rejected)

$$\bar{x} = \frac{\sum x_i f_i}{\sum f_i} = 2.8$$

13. Official Ans. by NTA (4)

Sol. Mean (μ) = $\frac{\sum x_i}{50} = 16$

standard deviation (σ) = $\sqrt{\frac{\sum x_i^2}{50} - (\mu)^2} = 16$

$\Rightarrow (256) \times 2 = \frac{\sum x_i^2}{50}$

$\Rightarrow$ New mean

$$= \frac{\sum (x_i - 4)^2}{50} = \frac{\sum x_i^2 + 16 \times 50 - 8\sum x_i}{50}$$

$= (256) \times 2 + 16 - 8 \times 16 = 400$

14. Official Ans. by NTA (2)

Sol. $x_1 + ... + x_4 = 44$

$x_5 + ... + x_{10} = 96$

$\bar{x} = 14, \Sigma x_i = 140$

Variance = $\frac{\Sigma x_i^2}{n} - \bar{x}^2 = 4$

Standard deviation = 2