SEQUENCE & PROGRESSION

1. If a, b and c be three distinct real numbers in G. P. and a + b + c = xb, then x cannot be :
(1) 4 (2) –3 (3) –2 (4) 2

2. Let $a_1, a_2, \ldots\ldots\ldots, a_{30}$ be an A. P., $S = \sum_{i=1}^{30} a_i$ and $T = \sum_{i=1}^{15} a_{(2i-1)}$. If $a_5 = 27$ and $S - 2T = 75$, then a_{10} is equal to :
(1) 57 (2) 47 (3) 42 (4) 52

3. The sum of the follwing series
$$1+6+\frac{9(1^2+2^2+3^2)}{7}+\frac{12(1^2+2^2+3^2+4^2)}{9}+\frac{15(1^2+2^2+\ldots+5^2)}{11}+\ldots \text{ up to}$$
15 terms, is:
(1) 7820 (2) 7830 (3) 7520 (4) 7510

4. Let a, b and c be the 7th, 11th and 13th terms respectively of a non-constant A.P. If these are also the three consecutive terms of a G.P., then $\frac{a}{c}$ is equal to:
(1) $\frac{1}{2}$ (2) 4 (3) 2 (4) $\frac{7}{13}$

5. The sum of an infinite geometric series with positive terms is 3 and the sum of the cubes of its terms is $\frac{27}{19}$. Then the common ratio of this series is :
(1) $\frac{4}{9}$ (2) $\frac{2}{9}$ (3) $\frac{2}{3}$ (4) $\frac{1}{3}$

6. Let $a_1, a_2, \ldots\ldots, a_{10}$ be a G.P. If $\frac{a_3}{a_1} = 25$, then $\frac{a_9}{a_5}$ equals :
(1) $2(5^2)$ (2) $4(5^2)$ (3) 5^4 (4) 5^3

7. If 19th term of a non-zero A.P. is zero, then its (49th term) : (29th term) is :-
(1) 3 : 1 (2) 4 : 1
(3) 2 : 1 (4) 1 : 3

8. Let x, y be positive real numbers and m, n positive integers. The maximum value of the expression $\frac{x^m y^n}{(1+x^{2m})(1+y^{2n})}$ is :-
(1) $\frac{1}{2}$ (2) $\frac{1}{4}$ (3) $\frac{m+n}{6mn}$ (4) 1

9. The product of three consecutive terms of a G.P. is 512. If 4 is added to each of the first and the second of these terms, the three terms now from an A.P. Then the sum of the original three terms of the given G.P. is
(1) 36 (2) 24 (3) 32 (4) 28

10. Let $S_k = \frac{1+2+3+\ldots+k}{k}$.
If $S_1^2 + S_2^2 + \ldots + S_{10}^2 = \frac{5}{12}A$, then A is equal to :
(1) 303 (2) 283 (3) 156 (4) 301

11. If $\sin^4\alpha + 4\cos^4\beta + 2 = 4\sqrt{2}\sin\alpha\cos\beta$;
$\alpha, \beta \in [0, \pi]$, then $\cos(\alpha + \beta) - \cos(\alpha - \beta)$ is equal to :
(1) 0 (2) $-\sqrt{2}$ (3) –1 (4) $\sqrt{2}$

12. If the sum of the first 15 tems of the series $\left(\frac{3}{4}\right)^3 + \left(1\frac{1}{2}\right)^3 + \left(2\frac{1}{4}\right)^3 + 3^3 + \left(3\frac{3}{4}\right)^3 + \ldots$ is equal to 225 k, then k is equal to :
(1) 9 (2) 27 (3) 108 (4) 54

13. The sum of all natural numbers 'n' such that 100 < n < 200 and H.C.F. (91, n) > 1 is :
(1) 3221 (2) 3121
(3) 3203 (4) 3303

14. The sum $\sum_{k=1}^{20} k\frac{1}{2^k}$ is equal to-
(1) $2-\frac{3}{2^{17}}$ (2) $2-\frac{11}{2^{19}}$
(3) $1-\frac{11}{2^{20}}$ (4) $2-\frac{21}{2^{20}}$

15. If three distinct numbers a,b,c are in G.P. and the equations $ax^2 + 2bx + c = 0$ and $dx^2 + 2ex + f = 0$ have a common root, then which one of the following statements is correct?

(1) d,e,f are in A.P.

(2) $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in G.P.

(3) $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in A.P.

(4) d,e,f are in G.P.

16. Let the sum of the first n terms of a non-constant A.P., a_1, a_2, a_3, ... be $50n + \frac{n(n-7)}{2}A$, wherre A is a constant. If d is the common difference of this A.P., then the ordered pair (d, a_{50}) is equal to

(1) (A, 50+46A) (2) (A, 50+45A)

(3) (50, 50+46A) (4) (50, 50+45A)

17. If the sum and product of the first three term in an A.P. are 33 and 1155, respectively, then a value of its 11th term is :-

(1) –25 (2) 25

(3) –36 (4) –35

18. The sum of the series 1 + 2 × 3 + 3 × 5+4 × 7 +.... upto 11th term is :-

(1) 915 (2) 946

(3) 945 (4) 916

19. The sum

$$\frac{3\times 1^3}{1^2} + \frac{5\times(1^3+2^3)}{1^2+2^2} + \frac{7\times(1^3+2^3+3^3)}{1^2+2^2+3^2} + ...$$

upto 10th term, is :

(1) 660 (2) 620

(3) 680 (4) 600

20. If a_1, a_2, a_3, ..., a_n are in A.P. and $a_1 + a_4 + a_7 + ... + a_{16} = 114$, then $a_1 + a_6 + a_{11} + a_{16}$ is equal to :

(1) 38 (2) 98

(3) 76 (4) 64

21. The sum $1 + \frac{1^3+2^3}{1+2} + \frac{1^3+2^3+3^3}{1+2+3} + ...$

$$+ \frac{1^3+2^3+3^3+....+15^3}{1+2+3+....+15} - \frac{1}{2}(1+2+3+....+15)]$$

(1) 1240 (2)1860

(3) 660 (4) 620

22. Let a, b and c be in G. P. with common ratio r, where $a \neq 0$ and $0 < r \leq \frac{1}{2}$. If 3a, 7b and 15c are the first three terms of an A. P., then the 4th term of this A. P. is :

(1) $\frac{7}{3}a$ (2) a

(3) $\frac{2}{3}a$ (4) 5a

23. If α and β are the roots of the equation $375x^2 - 25x - 2 = 0$, then

$\lim_{n\to\infty}\sum_{r=1}^{n}\alpha^r + \lim_{n\to\infty}\sum_{r=1}^{n}\beta^r$ is equal to :

(1) $\frac{21}{346}$ (2) $\frac{29}{358}$ (3) $\frac{1}{12}$ (4) $\frac{7}{116}$

24. Let S_n denote the sum of the first n terms of an A.P. If $S_4 = 16$ and $S_6 = -48$, then S_{10} is equal to :

(1) –320 (2) –260 (3) –380 (4) –410

25. If a_1, a_2, a_3,..... are in A.P. such that $a_1 + a_7 + a_{16} = 40$, then the sum of the first 15 terms of this A.P. is :

(1) 200 (2) 280

(3) 120 (4) 150

26. If α,β and γ are three consecutive terms of a non-constant G.P. such that the equations $\alpha x^2 + 2\beta x + \gamma = 0$ and $x^2 + x - 1 = 0$ have a common root, then α(β + γ) is equal to :

(1) βγ (2) 0 (3) αγ (4) αβ

SOLUTION

1. Ans. (4)

$\frac{b}{r}, b, br \rightarrow G.P. \qquad (|r| \neq 1)$

given a + b + c = xb

$\Rightarrow b/r + b + br = xb$

$\Rightarrow b = 0$ (not possible)

or $1 + r + \frac{1}{r} = x \Rightarrow x - 1 = r + \frac{1}{r}$

$\Rightarrow x - 1 > 2$ or $x - 1 < -2$

$\Rightarrow x > 3$ or $x < -1$

So x can't be '2'

2. Ans. (4)

$S = a_1 + a_2 + ... + a_{30}$

$S = \frac{30}{2}[a_1 + a_{30}]$

$S = 15(a_1 + a_{30}) = 15(a_1 + a_1 + 29d)$

$T = a_1 + a_3 + ... + a_{29}$

$= (a_1) + (a_1 + 2d) ..... + (a_1 + 28d)$

$= 15a_1 + 2d(1 + 2 + ... + 14)$

$T = 15a_1 + 210\,d$

Now use $S - 2T = 75$

$\Rightarrow 15(2a_1 + 29d) - 2(15a_1 + 210\,d) = 75$

$\Rightarrow d = 5$

Given $a_5 = 27 = a_1 + 4d \Rightarrow a_1 = 7$

Now $a_{10} = a_1 + 9d = 7 + 9 \times 5 = 52$

3. Ans. (1)

$$T_n = \frac{(3 + (n-1) \times 3)(1^2 + 2^2 + ... + n^2)}{(2n+1)}$$

$$T_n = \frac{3.\frac{n(n+1)(2n+1)}{6}}{2n+1} = \frac{n^2(n+1)}{2}$$

$$S_{15} = \frac{1}{2}\sum_{n=1}^{15}(n^3 + n^2) = \frac{1}{2}\left[\left(\frac{15(15+1)}{2}\right)^2 + \frac{15 \times 16 \times 31}{6}\right]$$

$= 7820$

4. Ans. (2)

$a = A + 6d$

$b = A + 10d$

$c = A + 12d$

a,b,c are in G.P.

$\Rightarrow (A + 10d)^2 = (A + 6d)(a + 12d)$

$\Rightarrow \frac{A}{d} = -14$

$$\frac{a}{c} = \frac{A + 6d}{A + 12d} = \frac{6 + \frac{A}{d}}{12 + \frac{A}{d}} = \frac{6 - 14}{12 - 14} = 4$$

5. Ans. (3)

$\frac{a}{1-r} = 3 \qquad ...(1)$

$$\frac{a^3}{1-r^3} = \frac{27}{19} \Rightarrow \frac{27(1-r)^3}{1-r^3} = \frac{27}{19}$$

$\Rightarrow 6r^2 - 13r + 6 = 0$

$\Rightarrow r = \frac{2}{3}$ as $|r| < 1$

6. Ans. (3)

$a_1, a_2, ..., a_{10}$ are in G.P.,

Let the common ratio be r

$\frac{a_3}{a_1} = 25 \Rightarrow \frac{a_1 r^2}{a_1} = 25 \Rightarrow r^2 = 25$

$\frac{a_9}{a_5} = \frac{a_1 r^8}{a_1 r^4} = r^4 = 5^4$

7. Ans. (1)

$a + 18d = 0 \qquad ...(1)$

$\frac{a + 48d}{a + 28d} = \frac{-18d + 48d}{-18d + 28d} = \frac{3}{1}$

8. Ans. (2)

$$\frac{x^m y^n}{(1 + x^{2m})(1 + y^{2n})} = \frac{1}{\left(x^m + \frac{1}{x^m}\right)\left(y^n + \frac{1}{y^n}\right)} \leq \frac{1}{4}$$

using AM $\geq$ GM

9. Ans. (4)

Let terms are $\frac{a}{r}, a, ar \rightarrow$ G.P

$\therefore a^3 = 512 \Rightarrow a = 8$

$\frac{8}{r} + 4, 12, 8r \rightarrow$ A.P.

$24 = \frac{8}{r} + 4 + 8r$

$r = 2,\ r = \frac{1}{2}$

r = 2 (4, 8, 16)

$r = \frac{1}{2}(16, 8, 4)$

Sum = 28

10. Ans. (1)

$S_K = \frac{K+1}{2}$

$\Sigma S_k^2 = \frac{5}{12}A$

$\sum_{K=1}^{10}\left(\frac{K+1}{2}\right)^2 = \frac{2^2 + 3^2 + - - + 11^2}{4} = \frac{5}{12}A$

$\frac{11 \times 12 \times 23}{6} - 1 = \frac{5}{3}A$

$505 = \frac{5}{3}A, \quad A = 303$

11. Ans (2)

A.M. $\geq$ G.M.

$\frac{\sin^4\alpha + 4\cos^4\beta + 1 + 1}{4} \geq (\sin^4\alpha . 4\cos^4\beta . 1 . 1)^{\frac{1}{4}}$

$\sin^4\alpha + 4\cos^2\beta + 2 \geq 4\sqrt{2}\sin\alpha\cos\beta$

given that $\sin^4\alpha + 4\cos^4\beta + 2$

$= 4\sqrt{2}\ \sin\alpha\cos\beta$

$\Rightarrow$ A.M. = G.M. $\Rightarrow \sin^4\alpha = 1 = 4\cos^4\beta$

$\sin\alpha = 1,\ \cos\beta = \pm\frac{1}{\sqrt{2}}$

$\Rightarrow \sin\beta = \frac{1}{\sqrt{2}}$ as $\beta \in [0, \pi]$

$\cos(\alpha + \beta) - \cos(\alpha - \beta)$

$= -2\sin\alpha\sin\beta$

$= -\sqrt{2}$

12. Ans. (2)

$S = \left(\frac{3}{4}\right)^3 + \left(\frac{6}{4}\right)^3 + \left(\frac{9}{4}\right)^3 + \left(\frac{12}{4}\right)^3 + \ldots\ldots\ldots\ldots 15\text{ term}$

$= \frac{27}{64}\sum_{r=1}^{15} r^3$

$= \frac{27}{64} \cdot \left[\frac{15(15+1)}{2}\right]^2$

= 225 K (Given in question)

K = 27

13. Official Ans. by NTA (2)

Sol. S_A = sum of numbers between 100 & 200 which are divisible by 7.

$\Rightarrow\ S_A = 105 + 112 + \ldots.. + 196$

$S_A = \frac{14}{2}[105 + 196] = 2107$

S_B = Sum of numbers between 100 & 200 which are divisible by 13.

$S_B = 104 + 117 + \ldots. + 195 = \frac{8}{2}[104 + 195] = 1196$

S_C = Sum of numbers between 100 & 200 which are divisible by both 7 & 13.

$S_C = 182$

$\Rightarrow$ H.C.F. $(91, n) > 1 = S_A + S_B - S_C = 3121$

14. Official Ans. by NTA (2)

Sol. $S = \sum_{k=1}^{20}\frac{1}{2^k}$

$S = \frac{1}{2} + \frac{2}{2^2} + \frac{3}{3^2} + \ldots + \frac{20}{2^{20}}$

$S \times \frac{1}{2} = \frac{1}{2^2} + \frac{2}{2^3} + \ldots + \frac{19}{2^{20}} + \frac{20}{2^{21}}$

$\Rightarrow \left(1 - \frac{1}{2}\right)S = \frac{1}{2} + \frac{1}{2^2} + \ldots + \frac{1}{2^{20}} - \frac{20}{2^{21}}$

$\Rightarrow S = 2 - \frac{11}{2^{19}}$

15. Official Ans. by NTA (3)

Sol. a, b, c in G.P.

say a, ar, ar^2

satisfies $ax^2 + 2bx + c = 0 \Rightarrow x = -r$

x = –r is the common root, satisfies second equation $d(-r)^2 + 2e(-r) + f = 0$

$$\Rightarrow d.\frac{c}{a} - \frac{2ce}{b} + f = 0$$

$$\Rightarrow \frac{d}{a} + \frac{f}{c} = \frac{2e}{b}$$

16. Official Ans. by NTA (1)

Sol. $S_n = 50n + \frac{n(n-7)}{2}A$

$T_n = S_n - S_{n-1}$

$$= 50n + \frac{n(n-7)}{2}A - 50(n-1) - \frac{(n-1)(n-8)}{2}A$$

$$= 50 + \frac{A}{2}[n^2 - 7n - n^2 + 9n - 8]$$

$= 50 + A(n - 4)$

$d = T_n - T_{n-1}$

$= 50 + A(n - 4) - 50 - A(n - 5)$

$= A$

$T_{50} = 50 + 46A$

$(d, A_{50}) = (A, 50+46A)$

17. Official Ans. by NTA (1)

Sol. $a - d + a + a + d = 33 \Rightarrow a = 11$

$a(a^2 - d^2) = 1155$

$121 - d^2 = 105$

$d^2 = 16 \Rightarrow d = \pm 4$

If d = 4 then I[st] term = 7

If d = –4 then I[st] term = 15

$T_{11} = 7 + 40 = 47$

OR $T_{11} = 15 - 40 = -25$

18. Official Ans. by NTA (2)

Sol. $T_r = r(2r - 1)$

$S = \Sigma 2r^2 - \Sigma r$

$$S = \frac{2.n(n+1)(2n+1)}{6} - \frac{n(n+1)}{2}$$

$$S_{11} = \frac{2}{6}.(11)(12)(23) - \frac{11(12)}{2} = (44)(23) - 66 = 946$$

19. Official Ans. by NTA (1)

Sol. $$T_n = \frac{(3+(n-1)\times 2)(1^3 + 2^3 + \ldots + n^3)}{(1^2 + 2^2 + \ldots + n^2)}$$

$$= \frac{3}{2}n(n+1) = \frac{n(n+1)(n+2) - (n-1)n(n+1)}{2}$$

$$\Rightarrow S_n = \frac{n(n+1)(n+2)}{2}$$

$\Rightarrow S_{10} = 660$

20. Official Ans. by NTA (3)

Sol. $a_1 + a_4 + a_7 + a_{10} + a_{13} + a_{16} = 114$

$$\Rightarrow \frac{6}{2}(a_1 + a_{16}) = 114$$

$\Rightarrow a_1 + a_{16} = 38$

$$\text{So, } a_1 + a_6 + a_{11} + a_{16} = \frac{4}{2}(a_1 + a_{16})$$

$= 2 \times 38 = 76$

21. Official Ans. by NTA (4)

Sol. $$\text{Sum} = \sum_{n=1}^{15} \frac{1^3 + 2^3 + \ldots n^3}{1 + 2 + \ldots + n} - \frac{1}{2}.\frac{15.16}{2}$$

$$= \sum_{n=1}^{15} \frac{n(n+1)}{2} - 60$$

$$= \sum_{n=1}^{15} \frac{n(n+1)(n+2-(n-1))}{6} - 60$$

$$= \frac{15.16.17}{6} - 60 = 620$$

22. Official Ans. by NTA (2)

Sol. $b = ar$

$c = ar^2$

3a, 7b and 15 c are in A.P.

$\Rightarrow 14b = 3a + 15c$

$\Rightarrow 14(ar) = 3a + 15\,ar^2$

$\Rightarrow 14r = 3 + 15r^2$

$\Rightarrow 15r^2 - 14r + 3 = 0 \;\Rightarrow (3r-1)(5r-3) = 0$

$r = \frac{1}{3}, \frac{3}{5}.$

Only acceptable value is $r = \frac{1}{3}$, because

$r \in \left(0, \frac{1}{2}\right]$

$\therefore$ c. d $= 7b - 3a = 7ar - 3a = \frac{7}{3}a - 3a = -\frac{2}{3}a$

$\therefore$ 4th term $= 15\,c - \frac{2}{3}a \;=\; \frac{15}{9}a - \frac{2}{3}a \;=\; a$

23. Official Ans. by NTA (3)

Sol. $375x^2 - 25x - 2 = 0$

$\alpha + \beta = \frac{25}{375},\; \alpha\beta = \frac{-2}{375}$

$\Rightarrow (\alpha + \alpha^2 + \ldots$ upto infinite terms)

$+ (\beta + \beta^2 + \ldots$ upto infinite terms)

$= \frac{\alpha}{1-\alpha} + \frac{\beta}{1-\beta} = \frac{1}{12}$

24. Official Ans. by NTA (1)

Sol. $2\{2a+3d\} = 16$

$3(2a + 5d) = -48$

$2a + 3d = 8$

$2a + 5d = -16$

$\boxed{d = -12}$

$S_{10} = 5\,\{44 - 9 \times 12\}$

$= -320$

25. Official Ans. by NTA (1)

Sol. $a_1 + a_7 + a_{16} = 40$

$a + a + 6d + a + 15d = 40$

$\Rightarrow 3a + 21d = 40 \quad \Rightarrow \boxed{a + 7d = \frac{40}{3}}$

$S_{15} = \frac{15}{2}(2a + 14d) = 15(a + 7d)$

$S_{15} = 15 \times \frac{40}{3} \Rightarrow 200 \quad \boxed{S_{15} = 200}$

26. Official Ans. by NTA (1)

Sol. $\alpha x^2 + 2\beta x + \gamma = 0$

Let $\beta = \alpha t,\ \gamma = \alpha t^2$

$\therefore\ \alpha x^2 + 2\alpha t x + \alpha t^2 = 0$

$\Rightarrow x^2 + 2tx + t^2 = 0$

$\Rightarrow (x + t)^2 = 0$

$\Rightarrow x = -t$

it must be root of equation $x^2 + x - 1 = 0$

$\therefore\ t^2 - t - 1 = 0 \qquad (1)$

Now

$\alpha(\beta + \gamma) = \alpha^2(t + t^2)$

Option 1 $\beta\gamma = \alpha t \,.\, \alpha t^2 = \alpha^2 t^3 = a^2 (t^2 + t)$

(from equation 1)