QUADRATIC EQUATION

1. Let α and β be two roots of the equation $x^2 + 2x + 2 = 0$, then $\alpha^{15} + \beta^{15}$ is equal to :

(1) 512 (2) –512 (3) –256 (4) 256

2. If both the roots of the quadratic equation $x^2 - mx + 4 = 0$ are real and distinct and they lie in the interval [1,5], then m lies in the interval:

(1) (4,5) (2) (3,4)

(3) (5,6) (4) (–5,–4)

3. The number of all possible positive integral values of α for which the roots of the quadratic equation, $6x^2-11x+\alpha = 0$ are rational numbers is :

(1) 2 (2) 5 (3) 3 (4) 4

4. Consider the quadratic equation $(c-5)x^2-2cx + (c-4) = 0, c\neq5$. Let S be the set of all integral values of c for which one root of the equation lies in the interval (0,2) and its other root lies in the interval (2,3). Then the number of elements in S is :

(1) 11 (2) 18 (3) 10 (4) 12

5. The values of λ such that sum of the squares of the roots of the quadratic equation,

$x^2 + (3 - \lambda) x + 2 = \lambda$ has the least value is :

(1) 2 (2) $\frac{4}{9}$

(3) $\frac{15}{8}$ (4) 1

6. If one real root of the quadratic equation $81x^2 + kx + 256 = 0$ is cube of the other root, then a value of k is

(1) –81 (2) 100 (3) –300 (4) 144

7. Let α and β be the roots of the quadratic equation

$x^2 \sin\theta - x(\sin\theta\cos\theta + 1) + \cos\theta = 0$

$(0 < \theta < 45°)$, and $\alpha < \beta$. Then $\sum_{n=0}^{\infty}\left(\alpha^n + \frac{(-1)^n}{\beta^n}\right)$ is equal to :-

(1) $\frac{1}{1-\cos\theta} + \frac{1}{1+\sin\theta}$

(2) $\frac{1}{1+\cos\theta} + \frac{1}{1-\sin\theta}$

(3) $\frac{1}{1-\cos\theta} - \frac{1}{1+\sin\theta}$

(4) $\frac{1}{1+\cos\theta} - \frac{1}{1-\sin\theta}$

8. If λ be the ratio of the roots of the quadratic equation in x, $3m^2x^2+m(m-4)x+2 = 0$, then the least value of m for which $\lambda + \frac{1}{\lambda} = 1$, is :

(1) $2-\sqrt{3}$ (2) $4-3\sqrt{2}$

(3) $-2+\sqrt{2}$ (4) $4-2\sqrt{3}$

9. The number of integral values of m for which the quadratic expression. $(1 + 2m)x^2 - 2(1 + 3m)x + 4(1 + m)$, $x\in R$, is always positive, is :

(1) 8 (2) 7 (3) 6 (4) 3

10. If α and β be the roots of the equation $x^2 - 2x + 2 = 0$, then the least value of n for which $\left(\frac{\alpha}{\beta}\right)^n = 1$ is :

(1) 2 (2) 3

(3) 4 (4) 5

11. The sum of the solutions of the equation $\left|\sqrt{x}-2\right|+\sqrt{x}\left(\sqrt{x}-4\right)+2=0, (x>0)$ is equal to :

(1) 4 (2) 9

(3) 10 (4) 12

12. The number of integral values of m for which the equation

$(1 + m^2)x^2 - 2(1 + 3m)x + (1 + 8m) = 0$

has no real root is :

(1) infinitely many (2) 2

(3) 3 (4) 1

13. Let p, q $\in$ R. If $2-\sqrt{3}$ is a root of the quadratic equation, $x^2 + px + q = 0$, then :

(1) $q^2 + 4p + 14 = 0$ (2) $p^2 - 4q - 12 = 0$

(3) $q^2 - 4p - 16 = 0$ (4) $p^2 - 4q + 12 = 0$

14. If m is chosen in the quadratic equation $(m^2 + 1)x^2 - 3x + (m^2 + 1)^2 = 0$ such that the sum of its roots is greatest, then the absolute difference of the cubes of its roots is :-

(1) $8\sqrt{3}$ (2) $4\sqrt{3}$

(3) $10\sqrt{5}$ (4) $8\sqrt{5}$

15. If α and β are the roots of the quadratic equation, $x^2 + x\sin\theta - 2\sin\theta = 0$, $\theta \in \left(0,\frac{\pi}{2}\right)$,

then $\frac{\alpha^{12}+\beta^{12}}{\left(\alpha^{-12}+\beta^{-12}\right)(\alpha-\beta)^{24}}$ is equal to :

(1) $\frac{2^6}{(\sin\theta+8)^{12}}$ (2) $\frac{2^{12}}{(\sin\theta-8)^{6}}$

(3) $\frac{2^{12}}{(\sin\theta-4)^{12}}$ (4) $\frac{2^{12}}{(\sin\theta+8)^{12}}$

SOLUTION

1. Ans. (3)

We have

$(x + 1)^2 + 1 = 0$

$\Rightarrow (x + 1)^2 - (i)^2 = 0$

$\Rightarrow (x + 1 + i)\,(x + 1 - i) = 0$

$\therefore\ x = \underbrace{-(1+i)}_{\alpha(\text{let})}\ \underbrace{-(1-i)}_{\beta(\text{let})}$

So, $\alpha^{15} + \beta^{15} = (\alpha^2)^7\,\alpha + (\beta^2)^7\,\beta$

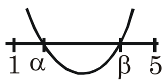
$= -128\,(-i + 1 + i + 1)$

$= -256$

2. Ans. (1)

$x^2 - mx + 4 = 0$

$\alpha,\beta \in [1,5]$



(1) $D > 0 \Rightarrow m^2 - 16 > 0$

$\Rightarrow m \in (-\infty,-4) \cup (4,\infty)$

(2) $f(1) \geq 0 \Rightarrow 5 - m \geq 0 \Rightarrow m \in (-\infty, 5]$

(3) $f(5) \geq 0 \Rightarrow 29 - 5m \geq 0 \Rightarrow m \in \left(-\infty, \frac{29}{5}\right]$

(4) $1 < \frac{-b}{2a} < 5 \Rightarrow 1 < \frac{m}{2} < 5 \Rightarrow m \in (2,10)$

$\Rightarrow m \in (4,5)$

No option correct : Bonus

* If we consider $\alpha,\beta \in (1,5)$ then option (1) is correct.

3. Ans. (3)

$6x^2 - 11x + \alpha = 0$

given roots are rational

$\Rightarrow$ D must be perfect square

$\Rightarrow 121 - 24\alpha = \lambda^2$

$\Rightarrow$ maximum value of α is 5

$\alpha = 1 \Rightarrow \lambda \notin I$

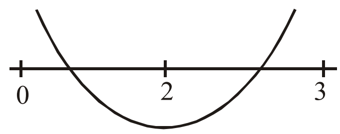
$\alpha = 2 \Rightarrow \lambda \notin I$

$\alpha = 3 \Rightarrow \lambda \in I \quad \Rightarrow$ 3 integral values

$\alpha = 4 \Rightarrow \lambda \in I$

$\alpha = 5 \Rightarrow \lambda \in I$

4. Ans. (1)



Let $f(x) = (c - 5)x^2 - 2cx + c - 4$

$\therefore\ f(0)f(2) < 0$ .....(1)

& $f(2)f(3) < 0$ .....(2)

from (1) & (2)

$(c - 4)(c - 24) < 0$

& $(c - 24)(4c - 49) < 0$

$\Rightarrow \frac{49}{4} < c < 24$

$\therefore\ s = \{13, 14, 15, ... 23\}$

Number of elements in set S = 11

5. Ans. (1)

$\alpha + \beta = \lambda - 3$

$\alpha\beta = 2 - \lambda$

$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = (\lambda - 3)^2 - 2(2 - \lambda)$

$= \lambda^2 + 9 - 6\lambda - 4 + 2\lambda$

$= \lambda^2 - 4\lambda + 5$

$= (\lambda - 2)^2 + 1$

$\therefore\ \lambda = 2$

Option (1)

6. Ans. (3)

$81x^2 + kx + 256 = 0\ ;\ x = \alpha, \alpha^3$

$\Rightarrow \alpha^4 = \frac{256}{81} \Rightarrow \alpha = \pm\frac{4}{3}$

Now $-\frac{k}{81} = \alpha + \alpha^3 = \pm\frac{100}{27} \Rightarrow k = \pm 300$

7. Ans. (1)

$D = (1 + \sin\theta\cos\theta)^2 - 4\sin\theta\cos\theta$

$= (1 - \sin\theta\cos\theta)^2$

$\Rightarrow$ roots are $\beta = \operatorname{cosec}\theta$ and $\alpha = \cos\theta$

$\Rightarrow \sum_{n=0}^{\infty}\left(\alpha^n + \left(-\frac{1}{\beta}\right)^n\right) = \sum_{n=0}^{\infty}(\cos\theta)^n + \sum_{n=0}^{n}(-\sin\theta)^n$

$= \frac{1}{1-\cos\theta} + \frac{1}{1+\sin\theta}$

8. Ans. (2)

$3m^2x^2 + m(m-4)x + 2 = 0$

$\lambda + \frac{1}{\lambda} = 1$, $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = 1$, $\alpha^2 + \beta^2 = \alpha\beta$

$(\alpha + \beta)^2 = 3\alpha\beta$

$$\left(-\frac{m(m-4)}{3m^2}\right)^2 = \frac{3(2)}{3m^2}, \frac{(m-4)^2}{9m^2} = \frac{6}{3m}$$

$(m-4)^2 = 18$, $m = 4 \pm \sqrt{18}, 4 \pm 3\sqrt{2}$

9. Ans. (2)

Exprsssion is always positve it

$2m + 1 > 0 \Rightarrow m > -\frac{1}{2}$ & $D < 0$

$\Rightarrow m^2 - 6m - 3 < 0$

$3 - \sqrt{12} < m < 3 + \sqrt{12}$ .... (iii)

$\therefore$ Common interval is

$3 - \sqrt{12} < m < 3 + \sqrt{12}$

$\therefore$ Intgral value of m $\{0,1,2,3,4,5,6\}$

10. Official Ans. by NTA (3)

Sol. $(x-1)^2 + 1 = 0 \Rightarrow x = 1 + i, 1 - i$

$$\therefore \left(\frac{\alpha}{\beta}\right)^n = 1 \Rightarrow (\pm i)^n = 1$$

$\therefore$ n (least natural number) = 4

11. Official Ans. by NTA (3)

Sol. $\left|\sqrt{x} - 2\right| + \sqrt{x}\left(\sqrt{x} - 4\right) + 2 = 0$

$\left|\sqrt{x} - 2\right| + \left(\sqrt{x}\right)^2 - 4\sqrt{x} + 2 = 0$

$\left|\sqrt{x} - 2\right|^2 + \left|\sqrt{x} - 2\right| - 2 = 0$

$\left|\sqrt{x} - 2\right| = -2$ (not possible) or $\left|\sqrt{x} - 2\right| = 1$

$\sqrt{x} - 2 = 1, -1$

$\sqrt{x} = 3, 1$

$x = 9, 1$

Sum = 10

12. Official Ans. by NTA (1)

Sol. $D < 0$

$4(1 + 3m)^2 - 4(1 + m^2)(1 + 8m) < 0$

$\Rightarrow m(2m-1)^2 > 0 \Rightarrow m > 0$

13. Official Ans. by NTA (2)

ALLEN Ans. (2) or (Bonus)

Sol. In given question p, q $\in$ R. If we take other root as any real number α,

then quadratic equation will be

$x^2 - (\alpha + 2 - \sqrt{3})x + \alpha.\left(2 - \sqrt{3}\right) = 0$

Now, we can have none or any of the options can be correct depending upon 'α'

Instead of p, q $\in$ R it should be p, q $\in$ Q then other root will be $2 + \sqrt{3}$

$\Rightarrow p = -\left(2 + \sqrt{3} - 2 - \sqrt{3}\right) = -4$

and $q = \left(2 + \sqrt{3}\right)\left(2 - \sqrt{3}\right) = 1$

$\Rightarrow p^2 - 4q - 12 = (-4)^2 - 4 - 12$

$= 16 - 16 = 0$

Option (2) is correct

14. Official Ans. by NTA (4)

Sol. $SOR = \frac{3}{m^2 + 1} \Rightarrow (S.O.R)_{max} = 3$

when m = 0

$x^2 - 3x + 1 = 0$ → α, β

$\alpha + \beta = 3$

$\alpha\beta = 1$

$|\alpha^3 - \beta^3| = ||\alpha - \beta|(\alpha^2 + \beta^2 + \alpha\beta)|$

$= \left|\sqrt{(\alpha - \beta)^2 - \alpha\beta}\ \ ((\alpha + \beta)^2 - \alpha\beta)\right|$

$= \left|\sqrt{9 - 4}\ (9 - 1)\right|$

$= \sqrt{5} \times 8$

15. Official Ans. by NTA (4)

Sol. $$\frac{\alpha^{12}+\beta^{12}}{\left(\frac{1}{\alpha^{12}}+\frac{1}{\beta^{12}}\right)(\alpha-\beta)^{24}}=\frac{(\alpha\beta)^{12}}{(\alpha-\beta)^{24}}$$

$$=\frac{(\alpha\beta)^{12}}{\left[(\alpha+\beta)^2-4\alpha\beta\right]^{12}}=\left[\frac{\alpha\beta}{(\alpha+\beta)^2-4\alpha\beta}\right]^{12}$$

$$=\left(\frac{-2\sin\theta}{\sin^2\theta+8\sin\theta}\right)^{12}=\frac{2^{12}}{(\sin\theta+8)^{12}}$$