PROBABILITY

1. Two cards are drawn successively with replacement from a well-shuffled deck of 52 cards. Let X denote the random variable of number of aces obtained in the two drawn cards. Then P(X = 1) + P(X = 2) equals :

(1) 52/169 (2) 25/169
(3) 49/169 (4) 24/169

2. An urn contains 5 red and 2 green balls. A ball is drawn at random from the urn. If the drawn ball is green, then a red ball is added to the urn and if the drawn ball is red, then a green ball is added to the urn; the original ball is not returned to the urn. Now, a second ball is drawn at random from it. The probability that the second ball is red, is :

(1) $\frac{26}{49}$ (2) $\frac{32}{49}$ (3) $\frac{27}{49}$ (4) $\frac{21}{49}$

3. An unbiased coin is tossed. If the outcome is a head then a pair of unbiased dice is rolled and the sum of the numbers obtained on them is noted. If the toss of the coin results in tail then a card from a well-shuffled pack of nine cards numbered 1,2,3,...,9 is randomly picked and the number on the card is noted. The probability that the noted number is either 7 or 8 is :

(1) $\frac{13}{36}$ (2) $\frac{19}{36}$ (3) $\frac{19}{72}$ (4) $\frac{15}{72}$

4. If the probability of hitting a target by a shooter, in any shot, is 1/3, then the minimum number of independent shots at the target required by him so that the probability of hitting the target at least once is greater than $\frac{5}{6}$, is :

(1) 6 (2) 5 (3) 4 (4) 3

5. Two integers are selected at random from the set {1, 2,...., 11}. Given that the sum of selected numbers is even, the conditional probability that both the numbers are even is :

(1) $\frac{2}{5}$ (2) $\frac{1}{2}$ (3) $\frac{3}{5}$ (4) $\frac{7}{10}$

6. Let S = {1, 2, ... , 20}. A subset B of S is said to be "nice", if the sum of the elements of B is 203. Then the probability that a randomly chosen subset of S is "nice" is :-

(1) $\frac{6}{2^{20}}$ (2) $\frac{5}{2^{20}}$
(3) $\frac{4}{2^{20}}$ (4) $\frac{7}{2^{20}}$

7. A bag contains 30 white balls and 10 red balls. 16 balls are drawn one by one randomly from the bag with replacement. If X be the number of white balls drawn, the $\left(\frac{\text{mean of } X}{\text{standard deviation of } X}\right)$ is equal to :-

(1) 4 (2) $\frac{4\sqrt{3}}{3}$ (3) $4\sqrt{3}$ (4) $3\sqrt{2}$

8. In a random experiment, a fair die is rolled until two fours are obtained in succession. The probability that the experiment will end in the fifth throw of the die is equal to :

(1) $\frac{150}{6^5}$ (2) $\frac{175}{6^5}$ (3) $\frac{200}{6^5}$ (4) $\frac{225}{6^5}$

9. Consider three boxes, each containing 10 balls labelled 1,2,....,10. Suppose one ball is randomly drawn from each of the boxes. Denote by n_i, the label of the ball drawn from the i^{th} box, (i = 1, 2, 3). Then, the number of ways in which the balls can be chosen such that $n_1 < n_2 < n_3$ is :

(1) 82 (2) 240 (3) 164 (4) 120

10. In a game, a man wins Rs. 100 if he gets 5 of 6 on a throw of a fair die and loses Rs. 50 for getting any other number on the die. If he decides to throw the die either till he gets a five or a six or to a maximum of three throws, then his expected gain/loss (in rupees) is :

(1) $\frac{400}{3}$ gain (2) $\frac{400}{3}$ loss

(3) 0 (4) $\frac{400}{9}$ loss

11. In a class of 60 students, 40 opted for NCC, 30 opted for NSS and 20 opted for both NCC and NSS. If one of these students is selected at random, then the probability that the student selected has opted neither for NCC nor for NSS is :

(1) $\frac{2}{3}$ (2) $\frac{1}{6}$

(3) $\frac{1}{3}$ (4) $\frac{5}{6}$

12. Let A and B be two non-null events such that $A \subset B$. Then, which of the following statements is always correct ?

(1) P(A|B) = 1

(2) P(A|B) = P(B) – P(A)

(3) P(A|B) ≤ P(A)

(4) P(A|B) ≥ P(A)

13. The minimum number of times one has to toss a fair coin so that the probability of observing at least one head is at least 90% is :

(1) 5 (2) 3 (3) 2 (4) 4

14. Four persons can hit a target correctly with probabilities $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ and $\frac{1}{8}$ respectively. if all hit at the target independently, then the probability that the target would be hit, is

(1) $\frac{25}{192}$ (2) $\frac{1}{192}$

(3) $\frac{25}{32}$ (4) $\frac{7}{32}$

15. Assume that each born child is equally likely to be a boy or a girl. If two families have two children each, then the conditional probability that all children are girls given that at least two are girls is :

(1) $\frac{1}{11}$ (2) $\frac{1}{17}$

(3) $\frac{1}{10}$ (4) $\frac{1}{12}$

16. Minimum number of times a fair coin must be tossed so that the probability of getting at least one head is more than 99% is :

(1) 5 (2) 6

(3) 7 (4) 8

17. If three of the six vertices of a regular hexagon are chosen at random, then the probability that the triangle formed with these chosen vertices is equilateral is :

(1) $\frac{3}{10}$ (2) $\frac{1}{10}$

(3) $\frac{3}{20}$ (4) $\frac{1}{5}$

18. Let a random variable X have a binomial distribution with mean 8 and variance 4. If $P(x \le 2) = \frac{k}{2^{16}}$, then k is equal to :

(1) 17 (2) 1

(3) 121 (4) 137

19. For and initial screening of an admission test, a candidate is given fifty problems to solve. If the probability that the candidate can solve any problem is $\frac{4}{5}$, then the probability that he is unable to solve less than two problems is :

(1) $\frac{316}{25}\left(\frac{4}{5}\right)^{48}$ (2) $\frac{54}{5}\left(\frac{4}{5}\right)^{49}$

(3) $\frac{164}{25}\left(\frac{1}{5}\right)^{48}$ (4) $\frac{201}{5}\left(\frac{1}{5}\right)^{49}$

20. A person throws two fair dice. He wins Rs. 15 for throwing a doublet (same numbers on the two dice), wins Rs.12 when the throw results in the sum of 9, and loses Rs. 6 for any other outcome on the throw. Then the expected gain/loss (in Rs.) of the person is :

(1) 2 gain (2) $\frac{1}{2}$ loss (3) $\frac{1}{4}$ loss (4) $\frac{1}{2}$ gain

SOLUTION

1. Ans. (2)

Two cards are drawn successively with replacement

4 Aces 48 Non Aces

$$P(x=1)=\frac{{}^{4}C_1}{{}^{52}C_1}\times\frac{48C_1}{52C_1}+\frac{48C_1}{52C_1}\times\frac{4C_1}{52C_1}=\frac{24}{169}$$

$$P(x=2)=\frac{{}^{4}C_1}{{}^{52}C_1}\times\frac{{}^{4}C_1}{{}^{52}C_1}=\frac{1}{169}$$

$$P(x=1)+P(x=2)=\frac{25}{169}$$

2. Ans. (2)

E_1 : Event of drawing a Red ball and placing a green ball in the bag

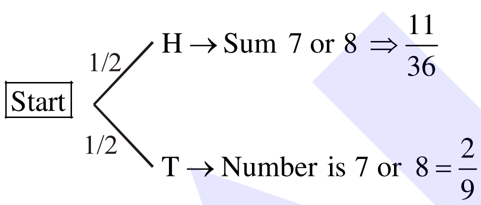
E_2 : Event of drawing a green ball and placing a red ball in the bag

E : Event of drawing a red ball in second draw

$$P(E)=P(E_1)\times P\left(\frac{E}{E_1}\right)+P(E_2)\times P\left(\frac{E}{E_2}\right)$$

$$=\frac{5}{7}\times\frac{4}{7}+\frac{2}{7}\times\frac{6}{7}=\frac{32}{49}$$

3. Ans. (3)



$$P(A)=\frac{1}{2}\times\frac{11}{36}+\frac{1}{2}\times\frac{2}{9}=\frac{19}{72}$$

4. Ans. (2)

$$1-{}^{n}C_0\left(\frac{1}{3}\right)^0\left(\frac{2}{3}\right)^n>\frac{5}{6}$$

$$\frac{1}{6}>\left(\frac{2}{3}\right)^n \Rightarrow 0.1666>\left(\frac{2}{3}\right)^n$$

$n_{min}=5 \Rightarrow$ Option (2)

5. Ans. (1)

Since sum of two numbers is even so either both are odd or both are even. Hence number of elements in reduced samples space

$= {}^{5}C_2 + {}^{6}C_2$

so required probability $=\dfrac{{}^{5}C_2}{{}^{5}C_2+{}^{6}C_2}$

6. Ans. (2)

7,

1,6

$$\boxed{P=\frac{5}{2^{20}}}$$

2,5

3,4

1,2,4

7. Ans. (3)

p (probability of getting white ball) $=\dfrac{30}{40}$

$q=\dfrac{1}{4}$ and n = 16

mean =np $=16.\dfrac{3}{4}=12$

and standard diviation

$$=\sqrt{npq}=\sqrt{16.\frac{3}{4}.\frac{1}{4}}=\sqrt{3}$$

8. Ans. (2)

_ _ _ 4 4

$$\frac{1}{6^2}\left(\frac{5^3}{6^3}+\frac{2C_1.5^2}{6^3}\right)=\frac{175}{6^5}$$

9. Ans. (4)

No. of ways = $10C_3 = 120$

10. Ans. (3)

Expected Gain/ Loss

= w × 100 + Lw (–50 + 100) + L^2w (–50 –50 + 100) + L^3 (–150)

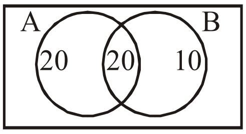
$$=\frac{1}{3}\times 100+\frac{2}{3}.\frac{1}{3}(50)+\left(\frac{2}{3}\right)^2\left(\frac{1}{3}\right)(0)$$

$$+\left(\frac{2}{3}\right)^3(-150)=0$$

here w denotes probability that outcome 5 or 6 ($w=\frac{2}{6}=\frac{1}{3}$)

here L denotes probability that outcome 1,2,3,4 ($L=\frac{4}{6}=\frac{2}{3}$)

11. Ans. (2)



A → opted NCC

B → opted NSS

$\therefore$ P (neither A nor B) = $\frac{10}{60}=\frac{1}{6}$

12. Official Ans. by NTA (4)

Sol. $P(A|B)=\frac{P(A\cap B)}{P(B)}=\frac{P(A)}{P(B)}$

$(\text{as } A\subset B\Rightarrow P(A\cap B)=P(A))$

$\Rightarrow \quad P(A|B)\ge P(A)$

13. Official Ans. by NTA (4)

Sol. Probability of observing at least one head out of n tosses

$$=1-\left(\frac{1}{2}\right)^n\ge 0.9$$

$$\Rightarrow\left(\frac{1}{2}\right)^n\le 0.1$$

$\Rightarrow n\ge 4$

$\Rightarrow$ minimum number of tosses = 4

14. Official Ans. by NTA (3)

Sol. Let persons be A,B,C,D

P(Hit) = 1 – P(none of them hits)

$$=1-P\left(\bar{A}\cap\bar{B}\cap\bar{C}\cap\bar{D}\right)$$

$$=1-P\left(\bar{A}\right).P\left(\bar{B}\right).P\left(\bar{C}\right).P\left(\bar{D}\right)$$

$$=1-\frac{1}{2}.\frac{2}{3}.\frac{3}{4}.\frac{7}{8}$$

$$=\frac{25}{32}$$

15. Official Ans. by NTA (1)

Sol. P(B) = P(G) = 1/2

Required Proballity =

$$\frac{\text{all 4 girls}}{(\text{all 4girls})+(\text{exactly 3 girls}+1\text{boy})+(\text{exactly 2girls}+2\text{boys})}$$

$$=\frac{\left(\frac{1}{2}\right)^4}{\left(\frac{1}{2}\right)^4+{}^4C_3\left(\frac{1}{2}\right)^4+{}^4C_2\left(\frac{1}{2}\right)^4}=\frac{1}{11}$$

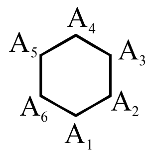
16. Official Ans. by NTA (3)

Sol. $1-\left(\frac{1}{2}\right)^n>\frac{99}{100}$

$$\Rightarrow\left(\frac{1}{2}\right)^n<\frac{1}{100}\Rightarrow n=7.$$

17. Official Ans. by NTA (2)

Sol.



Only two equilateral tringles are possible $A_1 A_3 A_5$ and $A_2A_5A_6$

$$\frac{2}{6_{C_3}}=\frac{2}{20}=\frac{1}{10}$$

18. Official Ans. by NTA (4)

Sol. np = 8

npq = 4

$q = \frac{1}{2} \Rightarrow p = \frac{1}{2}$

n = 16

$p\,(x = r) = {}^{16}C_r\left(\frac{1}{2}\right)^{16}$

$p\,(x \le 2) = \frac{{}^{16}C_0 + {}^{16}C_1 + {}^{16}C_2}{2^{16}}$

$= \frac{137}{2^{16}}$

19. Official Ans. by NTA (2)

Sol. Let X be random varibale which denotes number of problems that candidate is unbale to solve

$\because\ p = \frac{1}{5}$ and $X < 2$

$\Rightarrow P(X < 2) = P(X = 0) + P(X = 1)$

$= \left(\frac{4}{5}\right)^{50} + {}^{50}C_1 \cdot \left(\frac{1}{5}\right) \cdot \left(\frac{4}{5}\right)^{49}$

20. Official Ans. by NTA (2)

Sol. win Rs.15 $\rightarrow$ number of cases = 6

win Rs.12 $\rightarrow$ number of cases = 4

loss Rs.6 $\rightarrow$ number of cases = 26

p(expected gain/loss) $= 15 \times \frac{6}{36} + 12 \times \frac{4}{36} -$

$6 \times \frac{26}{36} = -\frac{1}{2}$