

PERMUTATION & COMBINATION

1. Consider a class of 5 girls and 7 boys. The number of different teams consisting of 2 girls and 3 boys that can be formed from this class, if there are two specific boys A and B, who refuse to be the members of the same team, is :
(1) 200 (2) 300 (3) 500 (4) 350
2. The number of natural numbers less than 7,000 which can be formed by using the digits 0,1,3,7,9 (repetition of digits allowed) is equal to :
(1) 250 (2) 374 (3) 372 (4) 375
3. The sum of all two digit positive numbers which when divided by 7 yield 2 or 5 as remainder is :
(1) 1365 (2) 1256
(3) 1465 (4) 1356
4. Let $S = \{1,2,3, \dots, 100\}$. The number of non-empty subsets A of S such that the product of elements in A is even is :-
(1) $2^{50}(2^{50}-1)$ (2) $2^{100}-1$
(3) $2^{50}-1$ (4) $2^{50}+1$
5. There are m men and two women participating in a chess tournament. Each participant plays two games with every other participant. If the number of games played by the men between themselves exceeds the number of games played between the men and the women by 84, then the value of m is :
(1) 9 (2) 11 (3) 12 (4) 7
6. If ${}^nC_4, {}^nC_5$ and nC_6 are in A.P., then n can be:
(1) 14 (2) 11 (3) 9 (4) 12
7. All possible numbers are formed using the digits 1, 1, 2, 2, 2, 2, 3, 4, 4 taken all at a time. The number of such numbers in which the odd digits occupy even places is :
(1) 175 (2) 162
(3) 160 (4) 180
8. The number of four-digit numbers strictly greater than 4321 that can be formed using the digits 0,1,2,3,4,5 (repetition of digits is allowed) is :
(1) 288 (2) 306 (3) 360 (4) 310
9. A committee of 11 members is to be formed from 8 males and 5 females. If m is the number of ways the committee is formed with at least 6 males and n is the number of ways the committee is formed with at least 3 females, then :
(1) $m = n = 78$ (2) $n = m - 8$
(3) $m + n = 68$ (4) $m = n = 68$
10. Some identical balls are arranged in rows to form an equilateral triangle. The first row consists of one ball, the second row consists of two balls and so on. If 99 more identical balls are added to the total number of balls used in forming the equilateral triangle, then all these balls can be arranged in a square whose each side contains exactly 2 balls less than the number of balls each side of the triangle contains. Then the number of balls used to form the equilateral triangle is :-
(1) 190 (2) 262 (3) 225 (4) 157
11. The number of 6 digit numbers that can be formed using the digits 0, 1, 2, 5, 7 and 9 which are divisible by 11 and no digit is repeated, is :
(1) 36 (2) 60 (3) 48 (4) 72
12. Suppose that 20 pillars of the same height have been erected along the boundary of a circular stadium. If the top of each pillar has been connected by beams with the top of all its non-adjacent pillars, then the total number beams is :
(1) 210 (2) 190
(3) 170 (4) 180

13. A group of students comprises of 5 boys and n girls. If the number of ways, in which a team of 3 students can randomly be selected from this group such that there is at least one boy and at least one girl in each team, is 1750, then n is equal to :
- (1) 25 (2) 28 (3) 27 (4) 24
14. The number of ways of choosing 10 objects out of 31 objects of which 10 are identical and the remaining 21 are distinct, is :
- (1) 2^{20} (2) $2^{20} - 1$
(3) $2^{20} + 1$ (4) 2^{21}

SOLUTION

- Ans. (2)**
 Required number of ways
 = Total number of ways – When A and B are always included.
 $= {}^5C_2 \cdot {}^7C_3 - {}^5C_1 \cdot {}^5C_2 = 300$
- Ans. (2)**

a_1	a_2	a_3
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Number of numbers = $5^3 - 1$

a_4	a_1	a_2	a_3
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2 ways for a_4
 Number of numbers = 2×5^3
 Required number = $5^3 + 2 \times 5^3 - 1 = 374$
- Ans. (4)**

$$\sum_{r=2}^{13} (7r+2) = 7 \cdot \frac{2+13}{2} \times 6 + 2 \times 12$$

$$= 7 \times 90 + 24 = 654$$

$$\sum_{r=1}^{13} (7r+5) = 7 \left(\frac{1+13}{2} \right) \times 13 + 5 \times 13 = 702$$

Total = $654 + 702 = 1356$
- Ans. (1)**
 $S = \{1, 2, 3, \dots, 100\}$
 = Total non empty subsets-subsets with product of element is odd
 $= 2^{100} - 1 - [(2^{50} - 1)]$
 $= 2^{100} - 2^{50}$
 $= 2^{50}(2^{50} - 1)$
- Ans (3)**
 Let m-men, 2-women
 ${}^mC_2 \times 2 = {}^mC_1 \cdot {}^2C_1 \cdot 2 + 84$
 $m^2 - 5m - 84 = 0 \Rightarrow (m - 12)(m + 7) = 0$
 $m = 12$

- Ans. (1)**
 $2 \cdot {}^nC_5 = {}^nC_4 + {}^nC_6$

$$2 \cdot \frac{|n}{|5|n-5} = \frac{|n}{|4|n-4} + \frac{|n}{|6|n-6}$$

$$\frac{2}{5} \cdot \frac{1}{n-5} = \frac{1}{(n-4)(n-5)} + \frac{1}{30}$$

$n = 14$ satisfying equation.
- Official Ans. by NTA (4)**

Sol.

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\downarrow \downarrow \downarrow \downarrow
 2nd place 4th place 6th place 8th place (even places)

Number of such numbers
 $= {}^4C_3 \times \frac{3!}{2!} \times \frac{6!}{2!4!} = 180$
- Official Ans. by NTA (4)**

Sol. (1) The number of four-digit numbers Starting with 5 is equal to $6^3 = 216$
 (2) Starting with 44 and 55 is equal to $36 \times 2 = 72$
 (3) Starting with 433, 434 and 435 is equal to $6 \times 3 = 18$
 (3) Remaining numbers are 4322, 4323, 4324, 4325 is equal to 4
 so total numbers are $216 + 72 + 18 + 4 = 310$
- Official Ans. by NTA (1)**

Sol. Since there are 8 males and 5 females. Out of these 13, if we select 11 persons, then there will be at least 6 males and atleast 3 females in the selection.

$$m = n = \binom{13}{11} = \binom{13}{2} = \frac{13 \times 12}{2} = 78$$
- Official Ans. by NTA (1)**

Sol. $\frac{n(n+1)}{2} + 99 = (n-2)^2$

$$n^2 + n + 198 = 2(n^2 + 4 - 4n)$$

$$n^2 - 9n - 190 = 0$$

$$n^2 - 19n + 10 - 190 = 0$$

$$n(n - 19) + 10(n - 19) = 0$$

$$n = 19$$

11. Official Ans. by NTA (2)

Sol. Sum of given digits 0, 1, 2, 5, 7, 9 is 24.

Let the six digit number be abcdef and to be divisible by 11

so $|a + c + e - (b + d + f)|$ is multiple of 11.

Hence only possibility is $a + c + e = 12 = b + d + f$

Case-I $\{a, c, e\} = \{9, 2, 1\}$ & $\{b, d, f\} = \{7, 5, 0\}$

So, Number of numbers = $3! \times 3! = 36$

Case-II $\{a, c, e\} = \{7, 5, 0\}$ and $\{b, d, f\} = \{9, 2, 1\}$

So, Number of numbers $2 \times 2! \times 3! = 24$

Total = 60

12. Official Ans. by NTA (3)

Sol. Total cases = number of diagonals
 $= {}^{20}C_2 - 20 = 170$

13. Official Ans. by NTA (1)

Sol. ${}^5C_1 \cdot {}^nC_2 + {}^5C_2 \cdot {}^nC_1 = 1750$
 $n^2 + 3n = 700$
 $\therefore n = 25$

14. Official Ans. by NTA (1)

10 Identical	21 Distinct	10 Object
0	10	${}^{21}C_{10} \times 1$
1	9	${}^{21}C_9 \times 1$
⋮	⋮	⋮
10	0	${}^{21}C_0 \times 1$
${}^{21}C_0 + \dots + {}^{21}C_{10} + {}^{21}C_1 + \dots + {}^{21}C_0 = 2^{21}$		
$({}^{21}C_0 + \dots + {}^{21}C_{10}) = 2^{20}$		