PARABOLA

1. Equation of a common tangent to the circle, $x^2 + y^2 - 6x = 0$ and the parabola, $y^2 = 4x$, is:

(1) $2\sqrt{3}\,y = 12x + 1$ (2) $2\sqrt{3}\,y = -x - 12$

(3) $\sqrt{3}\,y = x + 3$ (4) $\sqrt{3}\,y = 3x + 1$

2. Axis of a parabola lies along x-axis. If its vertex and focus are at distances 2 and 4 respectively from the origin, on the positive x-axis then which of the following points does not lie on it ?

(1) (4, –4) (2) $(5, 2\sqrt{6})$

(3) (8, 6) (4) $6, 4\sqrt{2}$

3. Let A(4,–4) and B(9,6) be points on the parabola, $y^2 = 4x$. Let C be chosen on the arc AOB of the parabola, where O is the origin, such that the area of ΔACB is maximum. Then, the area (in sq. units) of ΔACB, is:

(1) $31\frac{3}{4}$ (2) 32 (3) $30\frac{1}{2}$ (4) $31\frac{1}{4}$

4. If the parabolas $y^2=4b(x-c)$ and $y^2=8ax$ have a common normal, then which one of the following is a valid choice for the ordered triad (a,b,c)

(1) (1, 1, 0) (2) $\left(\frac{1}{2},2,3\right)$

(3) $\left(\frac{1}{2},2,0\right)$ (4) (1, 1, 3)

5. The length of the chord of the parabola $x^2 = 4y$ having equation $x - \sqrt{2}y + 4\sqrt{2} = 0$ is :

(1) $2\sqrt{11}$ (2) $3\sqrt{2}$

(3) $6\sqrt{3}$ (4) $8\sqrt{2}$

6. If the area of the triangle whose one vertex is at the vertex of the parabola, $y^2 + 4(x - a^2) = 0$ and the other two vertices are the points of intersection of the parabola and y-axis, is 250 sq. units, then a value of 'a' is :-

(1) $5\sqrt{5}$ (2) $(10)^{2/3}$ (3) $5(2^{1/3})$ (4) 5

7. The area (in sq. units) in the first quadrant bounded by the parabola, $y = x^2 + 1$, the tangent to it at the point (2, 5) and the coordinate axes is :-

(1) $\frac{14}{3}$ (2) $\frac{187}{24}$

(3) $\frac{37}{24}$ (4) $\frac{8}{3}$

8. Let P(4, –4) and Q(9, 6) be two points on the parabola, $y^2 = 4x$ and let X be any point on the arc POQ of this parabola, where O is the vertex of this parabola, such that the area of ΔPXQ is maximum. Then this maximum area (in sq. units) is :

(1) $\frac{125}{4}$ (2) $\frac{125}{2}$

(3) $\frac{625}{4}$ (4) $\frac{75}{2}$

9. The equation of a tangent to the parabola, $x^2 = 8y$, which makes an angle θ with the positive direction of x-axis, is :

(1) $x = y\cot\theta + 2\tan\theta$

(2) $x = y\cot\theta - 2\tan\theta$

(3) $y = x\tan\theta - 2\cot\theta$

(4) $y = x\tan\theta + 2\cot\theta$

10. The tangent to the parabola $y^2 = 4x$ at the point where it intersects the circle $x^2 + y^2 = 5$ in the first quadrant, passes through the point :

(1) $\left(-\frac{1}{3}, \frac{4}{3}\right)$ (2) $\left(-\frac{1}{4}, \frac{1}{2}\right)$

(3) $\left(\frac{3}{4}, \frac{7}{4}\right)$ (4) $\left(\frac{1}{4}, \frac{3}{4}\right)$

11. If one end of a focal chord of the parabola, $y^2 = 16x$ is at $(1, 4)$, then the length of this focal chord is

(1) 25 (2) 24 (3) 20 (4) 22

12. If the tangent to the parabola $y^2 = x$ at a point (α, β), $(\beta > 0)$ is also a tangent to the ellipse, $x^2 + 2y^2 = 1$, then α is equal to :

(1) $2\sqrt{2}+1$ (2) $\sqrt{2}-1$

(3) $\sqrt{2}+1$ (4) $2\sqrt{2}-1$

13. The area (in sq. units) of the smaller of the two circles that touch the parabola, $y^2 = 4x$ at the point $(1, 2)$ and the x-axis is :-

(1) $4\pi(2-\sqrt{2})$ (2) $8\pi(3-2\sqrt{2})$

(3) $4\pi(3+\sqrt{2})$ (4) $8\pi(2-\sqrt{2})$

14. If the line $ax + y = c$, touches both the curves $x^2 + y^2 = 1$ and $y^2 = 4\sqrt{2}x$, then $|c|$ is equal to:

(1) 1/2 (2) 2

(3) $\sqrt{2}$ (4) $\frac{1}{\sqrt{2}}$

15. The tangents to the curve $y = (x - 2)^2 - 1$ at its points of intersection with the line $x - y = 3$, intersect at the point :

(1) $\left(-\frac{5}{2}, -1\right)$ (2) $\left(-\frac{5}{2}, 1\right)$

(3) $\left(\frac{5}{2}, -1\right)$ (4) $\left(\frac{5}{2}, 1\right)$

16. The equation of a common tangent to the curves, $y^2 = 16x$ and $xy = -4$ is :

(1) $x + y + 4 = 0$ (2) $x - 2y + 16 = 0$

(3) $2x - y + 2 = 0$ (4) $x - y + 4 = 0$

SOLUTION

1. Ans. (3)

Let equation of tangent to the parabola $y^2 = 4x$ is

$y = mx + \frac{1}{m}$,

$\Rightarrow m^2x - ym + 1 = 0$ is tangent to $x^2 + y^2 - 6x = 0$

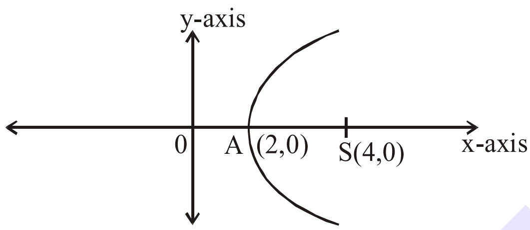
$\Rightarrow \frac{|3m^2 + 1|}{\sqrt{m^4 + m^2}} = 3$

$m = \pm\frac{1}{\sqrt{3}}$

$\Rightarrow$ tangent are $x + \sqrt{3}y + 3 = 0$

and $x - \sqrt{3}y + 3 = 0$

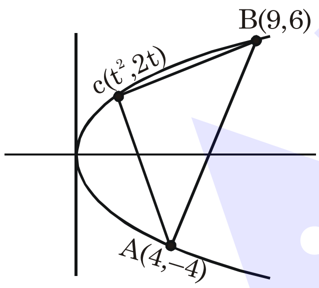
2. Ans. (3)



equation of parabola is $y^2 = 8(x - 2)$

(8, 6) does not lie on parabola.

3. Ans. (4)



Area = $5|t^2 - t - 6| = 5\left|\left(t - \frac{1}{2}\right)^2 - \frac{25}{4}\right|$

is maximum if $t = \frac{1}{2}$

4. Ans. (1,2,3,4)

Normal to these two curves are

$y = m(x - c) - 2bm - bm^3$,

$y = mx - 4am - 2am^3$

If they have a common normal

$(c + 2b)\,m + bm^3 = 4am + 2am^3$

Now $(4a - c - 2b)\,m = (b - 2a)m^3$

We get all options are correct for m = 0

(common normal x-axis)

Ans. (1), (2), (3), (4)

Remark :

If we consider question as

If the parabolas $y^2 = 4b(x - c)$ and $y^2 = 8ax$ have a common normal other than x-axis, then which one of the following is a valid choice for the ordered triad (a, b, c) ?

When $m \neq 0$: $(4a - c - 2b) = (b - 2a)m^2$

$m^2 = \frac{c}{2a - b} - 2 > 0 \Rightarrow \frac{c}{2a - b} > 2$

Now according to options, option 4 is correct

5. Ans. (3)

$x^2 = 4y$

$x - \sqrt{2}y + 4\sqrt{2} = 0$

Solving together we get

$x^2 = 4\left(\frac{x+4\sqrt{2}}{\sqrt{2}}\right)$

$\sqrt{2}x^2 + 4x + 16\sqrt{2}$

$\sqrt{2}x^2 - 4x - 16\sqrt{2} = 0$

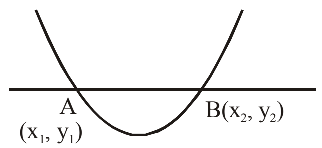
$x_1 + x_2 = 2\sqrt{2}$; $x_1x_2 = \frac{-16\sqrt{2}}{\sqrt{2}} = -16$

Similarly,

$\left(\sqrt{2}y - 4\sqrt{2}\right)^2 = 4y$

$2y^2 + 32 - 16y = 4y$

$2y^2 - 20y + 32 = 0$ → $y_1 + y_2 = 10$, $y_1 y_2 = 16$



$\ell_{AB} = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

$= \sqrt{\left(2\sqrt{2}\right)^2 + 64 + (10)^2 - 4(16)}$

$= \sqrt{8+64+100-64}$

$= \sqrt{108} = 6\sqrt{3}$

Option (3)

6. Ans. (4)

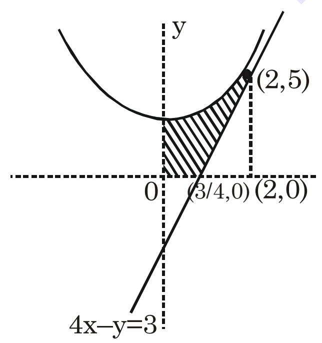
Vertex is $(a^2,0)$

$y^2 = -(x - a^2)$ and $x = 0 \Rightarrow (0, \pm 2a)$

Area of triangle is $= \frac{1}{2}.4a.\left(a^2\right) = 250$

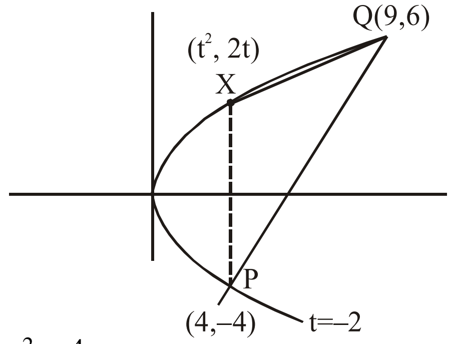
$\Rightarrow a^3 = 125$ or $a = 5$

7. Ans. (3)



Area $= \int_0^2 \left(x^2+1\right)dx - \frac{1}{2}\left(\frac{5}{4}\right)(5) = \frac{37}{24}$

8. Ans. (1)



$y^2 = 4x$

$2yy' = 4$

$y' = \frac{1}{t} = 2$, $t = \frac{1}{2}$

Area $= \frac{1}{2}\begin{vmatrix} \frac{1}{4} & 1 & 1 \\ 9 & 6 & 1 \\ 4 & -4 & 1 \end{vmatrix} = \frac{125}{4}$

9. Ans (1)

$x^2 = 8y$

$\Rightarrow \frac{dy}{dx} = \frac{x}{4} = \tan\theta$

$\therefore x_1 = 4\tan\theta$

$y_1 = 2\tan^2\theta$

Equation of tangent :-

$y - 2\tan^2\theta = \tan\theta\,(x - 4\tan\theta)$

$\Rightarrow x = y\cot\theta + 2\tan\theta$

10. Official Ans. by NTA (3)

Sol. Given $y^2 = 4x$ ...(1)

and $x^2 + y^2 = 5$ ...(2)

by (1) and (2)

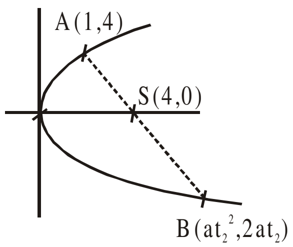
$\Rightarrow x = 1$ and $y = 2$

equation of tangent at (1,2) to $y^2 = 4x$

is $y = x + 1$

11. Official Ans. by NTA (1)

Sol.



$y^2 = 4ax = 16x \Rightarrow a = 4$

$A(1,4) \Rightarrow 2.4.t_1 = 4 \Rightarrow t_1 = \frac{1}{2}$

$\therefore$ length of focal chord $= a\left(t + \frac{1}{t}\right)^2$

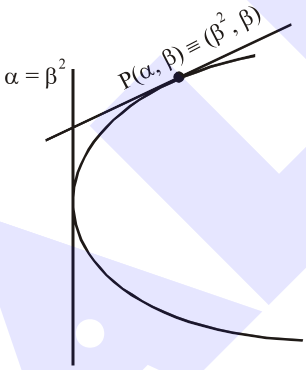
$= 4\left(\frac{1}{2} + 2\right)^2 = 4.\frac{25}{4} = 25$

12. Official Ans. by NTA (3)

Sol. $T : y(\beta) = \frac{1}{2}(x + \beta^2)$

$2y\beta = x + \beta^2$

$y = \left(\frac{1}{2\beta}\right)x + \frac{\beta}{2}$



$m = \frac{1}{2\beta}; C = \frac{\beta}{2}$

$\frac{\beta}{2} = \pm\sqrt{\frac{1}{4\beta^2} + \frac{1}{2}}$

$\frac{\beta^2}{4} = \frac{1}{4\beta^2} + \frac{1}{2}$

$\frac{\beta^2}{4} = \frac{1 + 2\beta^2}{4\beta^2}$

$\Rightarrow \beta^4 - 2\beta^2 - 1 = 0$

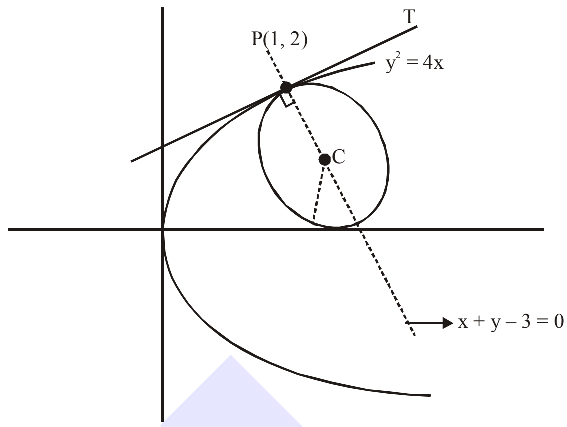
$(\beta^2 - 1)^2 = 2$

$\beta^2 - 1 = \sqrt{2}$

$\beta^2 = \sqrt{2} + 1$

13. Official Ans. by NTA (2)

Sol.



Equation of circle is

$(x - 1)^2 + (y - 2)^2 + \lambda(x - y + 1) = 0$

$\Rightarrow x^2 + y^2 + x(\lambda - 2) + y(-4 - \lambda) + (5 + \lambda) = 0$

As cirlce touches x axis then $g^2 - c = 0$

$\frac{(\lambda - 2)^2}{4} = (5 + \lambda)$

$\lambda^2 + 4 - 4\lambda = 20 + 4\lambda$

$\lambda^2 - 8\lambda - 16 = 0$

$\lambda = \frac{8 \pm \sqrt{128}}{2}$

$\lambda = 4 \pm 4\sqrt{2}$

Radius = $\left|\frac{(-4 - \lambda)}{2}\right|$

Put λ and get least radius.

14. Official Ans. by NTA (3)

Sol. Tangent to $y^2 = 4\sqrt{2}\,x$ is $y = mx + \frac{\sqrt{2}}{m}$

it is also tangent to $x^2 + y^2 = 1$

$\Rightarrow \left|\frac{\sqrt{2}/m}{\sqrt{1 + m^2}}\right| = 1 \quad \Rightarrow m = \pm 1$

$\Rightarrow$ Tagent will be $y = x + \sqrt{2}$ or $y = -x - \sqrt{2}$

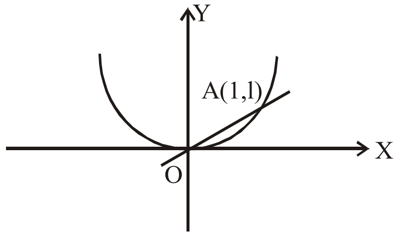
compare with $y = -ax + C$

$\Rightarrow a = \pm 1$ & $C = \pm\sqrt{2}$

15. Official Ans. by NTA (3)

Sol. Put $x - 2 = X$ & $y + 1 = Y$

$\therefore$ given curve becomes $Y = X^2$ and $Y = X$



tangent at origin is X-axis

and tangent at A(1,1) is $Y + 1 = 2X$

$\therefore$ there intersection is $\left(\frac{1}{2}, 0\right)$

$\therefore \quad x - 2 = \frac{1}{2}$ & $y + 1 = 0$

therefore $x = \frac{5}{2}, y = -1$

16. Official Ans. by NTA (4)

Sol. tangent to the parabola $y^2 = 16x$ is $y = mx + \frac{4}{m}$

solve it by curve $xy = -4$

i.e. $mx^2 + \frac{4}{m}x + 4 = 0$

condition of common tangent is $D = 0$

$\therefore \ m^3 = 1$

$\Rightarrow m = 1$

$\therefore$ equation of common tangent is $y = x + 4$