MONOTONICITY

1. Let $f(x)=\frac{x}{\sqrt{a^2+x^2}}-\frac{d-x}{\sqrt{b^2+(d-x)^2}}$, $x\in R$, where a, b and d are non-zero real constants. Then :-

(1) f is a decreasing function of x

(2) f is neither increasing nor decreasing function of x

(3) f' is not a continuous function of x

(4) f is an increasing function of x

2. If the function f given by

$f(x) = x^3 - 3(a-2)x^2 + 3ax + 7$,

for some $a\in R$ is increasing in (0, 1] and decreasing in [1, 5), then a root of the equation, $\frac{f(x)-14}{(x-1)^2}=0 (x\neq 1)$ is :

(1) 6 (2) 5 (3) 7 (4) –7

3. Let $f : [0, 2] \to R$ be a twice differentiable function such that $f''(x) > 0$, for all $x \in (0, 2)$. If $\phi(x) = f(x) + f(2-x)$, then ϕ is :

(1) decreasing on (0, 2)

(2) decreasing on (0, 1) and increasing on (1, 2)

(3) increasing on (0, 2)

(4) increasing on (0, 1) and decreasing on (1, 2)

4. Let $f(x) = e^x - x$ and $g(x) = x^2 - x$, $\forall x \in R$. Then the set of all $x \in R$, where the function $h(x) = (fog)(x)$ is increasing, is :

(1) $\left[-1,\frac{-1}{2}\right]\cup\left[\frac{1}{2},\infty\right)$ (2) $\left[0,\frac{1}{2}\right]\cup[1,\infty)$

(3) $\left[\frac{-1}{2},0\right]\cup[1,\infty)$ (4) $[0,\infty)$

5. If m is the minimum value of k for which the function $f(x)=x\sqrt{kx-x^2}$ is increasing in the interval [0, 3] and M is the maximum value of f in [0, 3] when k = m, then the ordered pair (m, M) is equal to :

(1) $(4, 3\sqrt{2})$ (2) $(4, 3\sqrt{3})$

(3) $(3, 3\sqrt{3})$ (4) $(5, 3\sqrt{6})$

SOLUTION

1. Ans. (4)

$$f(x)=\frac{x}{\sqrt{a^2+x^2}}-\frac{d-x}{\sqrt{b^2+(d-x)^2}}$$

$$f'(x)=\frac{a^2}{\left(a^2+x^2\right)^{3/2}}+\frac{b^2}{\left(b^2+(d-x)^2\right)^{3/2}}>0\ \forall x\in R$$

$f(x)$ is an increasing function.

2. Ans (3)

$f'(x) = 3x^2 - 6(a-2)x + 3a$

$f'(x) \geq 0\ \forall\ x \in (0, 1]$

$f'(x) \leq 0\ \forall\ x \in [1, 5)$

$\Rightarrow f'(x) = 0$ at $x = 1 \Rightarrow a = 5$

$f(x) - 14 = (x-1)^2 (x-7)$

$$\frac{f(x)-14}{(x-1)^2} = x - 7$$

3. Official Ans. by NTA (2)

Sol. $\phi(x) = f(x) + f(2-x)$

$\phi'(x) = f'(x) - f'(2-x)$ ......(1)

Since $f''(x) > 0$

$\Rightarrow$ $f'(x)$ is increasing $\forall\ x \in (0, 2)$

Case-I : When $x > 2 - x \quad \Rightarrow \quad x > 1$

$\Rightarrow \quad \phi'(x) > 0\ \forall\ x \in (1, 2)$

$\therefore \quad \phi(x)$ is increasing on $(1, 2)$

Case-II : When $x < 2 - x \quad \Rightarrow \quad x < 1$

$\Rightarrow \quad \phi'(x) < 0\ \forall\ x \in (0,1)$

$\therefore \quad \phi(x)$ is decreasing on $(0, 1)$

4. Official Ans. by NTA (2)

Sol. $h(x) = f(g(x))$

$\Rightarrow h'(x) = f'(g(x)).\ g'(x)$ and $f'(x) = e^x - 1$

$\Rightarrow h'(x) = (e^{g(x)} - 1)\ g'(x)$

$\Rightarrow h'(x) = \left(e^{x^2-x}-1\right)(2x-1) \geq 0$

Case-I $e^{x^2-x} \geq 1$ and $2x - 1 \geq 0$

$\Rightarrow x \in [1, \infty)$ ......(1)

Case-II $e^{x^2-x} \leq 1$ and $2x - 1 \leq 0$

$\Rightarrow x \in \left[0, \frac{1}{2}\right]$.....(2)

Hence, $x \in \left[0, \frac{1}{2}\right] \cup [1, \infty)$

5. Official Ans. by NTA (2)

Sol. $f(x) = x\sqrt{kx-x^2}$

$$f'(x) = \frac{3kx-4x^2}{2\sqrt{kx-x^2}}$$

For $\uparrow$ $f'(x) \geq 0$	$3kx-4x^2 \geq 0$
$kx - x^2 \geq 0$	$4x^2-3kx \leq 0$
$x^2 - kx \leq 0$	$4x(x-\frac{3k}{4}) \leq 0$
$x(x-k) \leq 0$ so $x \in [0, 3]$	$3-\frac{3k}{4} \leq 0$
+ve $\boxed{x \geq 3}$	$\boxed{k \geq 4}$

minimum value of k is $\boxed{m = 4}$

$f(x) = x\sqrt{kx-x^2}$

$= 3\sqrt{4\times 3-3^2} = 3\sqrt{3},\ M = 3\sqrt{3}$