METHOD OF DIFFERENTIATION

1. If x = 3 tan t and y = 3 sec t, then the value of $\frac{d^2y}{dx^2}$ at $t=\frac{\pi}{4}$, is:

(1) $\frac{3}{2\sqrt{2}}$ (2) $\frac{1}{3\sqrt{2}}$ (3) $\frac{1}{6}$ (4) $\frac{1}{6\sqrt{2}}$

2. Let f : R→R be a function such that $f(x) = x^3+x^2f'(1) + xf''(2)+f'''(3)$, x∈R. Then f(2) equal :

(1) 8 (2) –2 (3) –4 (4) 30

3. If $x\log_e(\log_e x) - x^2 + y^2 = 4 (y > 0)$, then dy/dx at x = e is equal to :

(1) $\frac{e}{\sqrt{4+e^2}}$ (2) $\frac{(1+2e)}{2\sqrt{4+e^2}}$

(3) $\frac{(2e-1)}{2\sqrt{4+e^2}}$ (4) $\frac{(1+2e)}{\sqrt{4+e^2}}$

4. For x >1, if $(2x)^{2y} = 4e^{2x-2y}$, then $(1+\log_e 2x)^2\frac{dy}{dx}$ is equal to :

(1) $\log_e 2x$ (2) $\frac{x\log_e 2x+\log_e 2}{x}$

(3) $x\log_e 2x$ (4) $\frac{x\log_e 2x-\log_e 2}{x}$

5. Let *f* be a differentiable function such that *f*(1) = 2 and *f*'(x) = *f*(x) for all x∈R. If h(x) = f(f(x)), then h'(1) is equal to :

(1) 4e (2) $4e^2$ (3) 2e (4) $2e^2$

6. If $2y=\left(\cot^{-1}\left(\frac{\sqrt{3}\cos x+\sin x}{\cos x-\sqrt{3}\sin x}\right)\right)^2, x\in\left(0,\frac{\pi}{2}\right)$, then $\frac{dy}{dx}$ is equal to :

(1) $2x-\frac{\pi}{3}$ (2) $\frac{\pi}{3}-x$

(3) $\frac{\pi}{6}-x$ (4) $x-\frac{\pi}{6}$

7. If *f*(1) = 1, *f*'(1) = 3, then the derivative of $f(f(f(x))) + (f(x))^2$ at x = 1 is :

(1) 12 (2) 33 (3) 9 (4) 15

8. Let $f(x) = \log_e(\sin x)$, $(0 < x < \pi)$ and $g(x) = \sin^{-1}(e^{-x})$, $(x \geq 0)$. If α is a positive real number such that a = (fog)'(α) and b = (fog)(α), then :

(1) $a\alpha^2 - b\alpha - a = 0$
(2) $a\alpha^2 + b\alpha - a = -2\alpha^2$
(3) $a\alpha^2 + b\alpha + a = 0$
(4) $a\alpha^2 - b\alpha - a = 1$

9. If $e^y + xy = e$, the ordered pair $\left(\frac{dy}{dx},\frac{d^2y}{dx^2}\right)$ at x = 0 is equal to :

(1) $\left(-\frac{1}{e},\frac{1}{e^2}\right)$ (2) $\left(\frac{1}{e},\frac{1}{e^2}\right)$

(3) $\left(\frac{1}{e},-\frac{1}{e^2}\right)$ (4) $\left(-\frac{1}{e},-\frac{1}{e^2}\right)$

10. The derivative of $\tan^{-1}\left(\frac{\sin x-\cos x}{\sin x+\cos x}\right)$, with respect to $\frac{x}{2}$, where $\left(x\in\left(0,\frac{\pi}{2}\right)\right)$ is :

(1) $\frac{1}{2}$ (2) $\frac{2}{3}$ (3) 1 (4) 2

SOLUTION

1. Ans. (4)

$$\frac{dx}{dt}=3\sec^2 t$$

$$\frac{dy}{dt}=3\sec t\tan t$$

$$\frac{dy}{dx}=\frac{\tan t}{\sec t}=\sin t$$

$$\frac{d^2y}{dx^2}=\cos t\frac{dt}{dx}$$

$$=\frac{\cos t}{3\sec^2 t}=\frac{\cos^3 t}{3}=\frac{1}{3.2\sqrt{2}}=\frac{1}{6\sqrt{2}}$$

2. Ans. (2)

$f(x) = x^3 + x^2f'(1) + xf''(2) + f'''(3)$

$\Rightarrow f'(x) = 3x^2 + 2xf'(1) + f''(x)$ .....(1)

$\Rightarrow f''(x) = 6x + 2f'(1)$ .....(2)

$\Rightarrow f'''(x) = 6$ .....(3)

put x = 1 in equation (1) :

$f'(1) = 3 + 2f'(1) + f''(2)$ .....(4)

put x = 2 in equation (2) :

$f''(2) = 12 + 2f'(1)$ .....(5)

from equation (4) & (5) :

$-3 - f'(1) = 12 + 2f'(1)$

$\Rightarrow 3f'(1) = -15$

$\Rightarrow f'(1) = -5 \Rightarrow f''(2) = 2$ ....(2)

put x = 3 in equation (3) :

$f'''(3) = 6$

$\therefore f(x) = x^3 - 5x^2 + 2x + 6$

$f(2) = 8 - 20 + 4 + 6 = -2$

3. Ans. (3)

Differentiating with respect to x,

$$x.\frac{1}{\ell nx}.\frac{1}{x}+\ell n(\ell nx)-2x+2y.\frac{dy}{dx}=0$$

at x = e we get

$$1-2e+2y\frac{dy}{dx}=0 \Rightarrow \frac{dy}{dx}=\frac{2e-1}{2y}$$

$$\Rightarrow \frac{dy}{dx}=\frac{2e-1}{2\sqrt{4+e^2}} \text{ as } y(e)=\sqrt{4+e^2}$$

4. Ans. (4)

$(2x)^{2y} = 4e^{2x-2y}$

$2y\ell n2x = \ell n4 + 2x - 2y$

$$y=\frac{x+\ell n2}{1+\ell n2x}$$

$$y'=\frac{(1+\ell n2x)-(x+\ell n2)\frac{1}{x}}{(1+\ell n2x)^2}$$

$$y'(1+\ell n2x)^2=\left[\frac{x\ell n2x-\ell n2}{x}\right]$$

5. Ans (1)

$$\frac{f'(x)}{f(x)} = 1 \ \forall\ x \in R$$

Intergrate & use f(1) = 2

$f(x) = 2e^{x-1} \Rightarrow f'(x) = 2e^{x-1}$

$h(x) = f(f(x)) \Rightarrow h'(x) = f'(f(x))\, f'(x)$

$h'(1) = f'(f(1))\, f'(1)$

$= f'(2)\, f'(1)$

$= 2e \cdot 2 = 4e$

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6. Official Ans. by NTA (4)

Sol. Consider $\cot^{-1}\left(\frac{\frac{\sqrt{3}}{2}\cos x+\frac{1}{2}\sin x}{\frac{1}{2}\sin x-\frac{\sqrt{3}}{2}\sin x}\right)$

$$=\cot^{-1}\left(\frac{\sin\left(x+\frac{\pi}{3}\right)}{\cos\left(x+\frac{\pi}{3}\right)}\right)$$

$$=\cot^{-1}\left(\tan\left(x+\frac{\pi}{3}\right)\right)=\frac{\pi}{2}-\tan^{-1}\left(\tan\left(x+\frac{\pi}{3}\right)\right)$$

$$\begin{cases}\frac{\pi}{2}-\left(x+\frac{\pi}{3}\right)=\left(\frac{\pi}{6}-x\right); & 0<x<\frac{\pi}{6}\\ \frac{\pi}{2}-\left(\left(x-\frac{\pi}{3}\right)-\pi\right)=\left(\frac{7\pi}{6}-x\right); & \frac{\pi}{6}<x<\frac{\pi}{2}\end{cases}$$

$$\therefore\quad 2y=\begin{cases}\left(\frac{\pi}{6}-x\right)^2; & 0<x<\frac{\pi}{6}\\ \left(\frac{7\pi}{6}-x\right)^2; & \frac{\pi}{6}<x<\frac{\pi}{2}\end{cases}$$

$$\therefore\quad 2\frac{dy}{dx}=\begin{cases}2\left(\frac{\pi}{6}-x\right).(-1); & 0<x<\frac{\pi}{6}\\ 2\left(\frac{7\pi}{6}-x\right).(-1); & \frac{\pi}{6}<x<\frac{\pi}{2}\end{cases}$$

7. Official Ans. by NTA (2)

Sol. $y=f(f(f(x)))+(f(x))^2$

$$\frac{dy}{dx}=f'(f(f(x)))f'(f(x))f'(x)+2f(x)f'(x)$$

$$=f'(1)f'(1)f'(1)+2f(1)f'(1)$$
$$= 3 \times 5 \times 3 + 2 \times 1 \times 3$$
$$= 27 + 6$$
$$= 33$$

8. Official Ans. by NTA (4)

Sol. $fog(x) = (-x) \Rightarrow (fg(\alpha)) = -\alpha = b$

$(fg(x))' = -1 \Rightarrow (fg(\alpha))' = -1 = a$

9. Official Ans. by NTA (1)

Sol. $e^y = xy = e$

differentiate w.r.t. x

$$e^y\frac{dy}{dx}+x\frac{dy}{dx}+y=0$$

$$\frac{dy}{dx}(x+e^y)=-y\,,\quad \left.\frac{dy}{dx}\right|_{(0,\,1)}=-\frac{1}{e}$$

again differentiate w.r.t. x

$$e^y\cdot\frac{d^2y}{dx^2}+\frac{dy}{dx}\cdot e^y\cdot\frac{dy}{dx}+x\cdot\frac{d^2y}{dx^2}+\frac{dy}{dx}+\frac{dy}{dx}=0$$

$$(x+e^y)\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2\cdot e^y+2\frac{dy}{dx}=0$$

$$e\frac{d^2y}{dx^2}+\frac{1}{e^2}e+2\left(-\frac{1}{e}\right)=0$$

$$\therefore\quad \frac{d^2y}{dx^2}=\frac{1}{e^2}$$

10. Official Ans. by NTA (4)

Sol. $f(x)=\tan^{-1}\left(\frac{\sin x-\cos x}{\sin x+\cos x}\right)$

$$=\tan^{-1}\left(\frac{\tan x-1}{\tan x+1}\right)=\tan^{-1}\left(\tan\left(x-\frac{\pi}{4}\right)\right)$$

$$\because\quad x-\frac{\pi}{4}\in\left(-\frac{\pi}{4},\frac{\pi}{4}\right)$$

$$\therefore\quad f(x)=x-\frac{\pi}{4}$$

$\Rightarrow$ its derivative w.r.t. $\frac{x}{2}$ is $\frac{1}{1/2}=2$