MAXIMA & MINIMA

1. The maximum volume (in cu. m) of the right circular cone having slant height 3m is :

(1) $3\sqrt{3}\ \pi$ (2) $6\ \pi$ (3) $2\sqrt{3}\ \pi$ (4) $\frac{4}{3}\ \pi$

2. The shortest distance between the point $\left(\frac{3}{2},0\right)$ and the curve $y=\sqrt{x},(x>0)$ is :

(1) $\frac{\sqrt{5}}{2}$ (2) $\frac{5}{4}$

(3) $\frac{3}{2}$ (4) $\frac{\sqrt{3}}{2}$

3. The maximum value of the function $f(x) = 3x^3 - 18x^2 + 27x - 40$ on the set $S=\{x \in R : x^2 + 30 \le 11x\}$ is :

(1) 122 (2) –222

(3) –122 (4) 222

4. The maximum area (in sq. units) of a rectangle having its base on the x-axis and its other two vertices on the parabola, $y = 12-x^2$ such that the rectangle lies inside the parabola, is :-

(1) $20\sqrt{2}$ (2) $18\sqrt{3}$

(3) 32 (4) 36

5. The shortest distance between the line $y = x$ and the curve $y^2 = x - 2$ is :

(1) $\frac{7}{4\sqrt{2}}$ (2) $\frac{7}{8}$

(3) $\frac{11}{4\sqrt{2}}$ (4) 2

6. If S_1 and S_2 are respectively the sets of local minimum and local maximum points of the function, $f(x) = 9x^4 + 12x^3 - 36x^2 + 25$, $x \in R$, then :

(1) $S_1 = \{-2, 1\}$; $S_2 = \{0\}$

(2) $S_1 = \{-2, 0\}$; $S_2 = \{1\}$

(3) $S_1 = \{-2\}$; $S_2 = \{0, 1\}$

(4) $S_1 = \{-1\}$; $S_2 = \{0, 2\}$

7. The height of a right circular cylinder of maximum volume inscribed in a sphere of radius 3 is

(1) $2\sqrt{3}$ (2) $\sqrt{3}$

(3) $\sqrt{6}$ (4) $\frac{2}{3}\sqrt{3}$

8. If $f(x)$ is a non-zero polynomial of degree four, having local extreme points at $x = -1, 0, 1$; then the set $S = \{x \in R : f(x) = f(0)\}$ Contains exactly :

(1) four irrational numbers.

(2) two irrational and one rational number.

(3) four rational numbers.

(4) two irrational and two rational numbes.

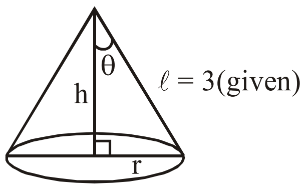
9. Let a_1, a_2, a_3,......be an A. P. with $a_6 = 2$. Then the common difference of this A. P., which maximises the produce $a_1a_4a_5$, is :

(1) $\frac{6}{5}$ (2) $\frac{8}{5}$

(3) $\frac{2}{3}$ (4) $\frac{3}{2}$

SOLUTION

1. Ans. (3)



$\therefore$ h = 3 cos θ

r = 3 sin θ

Now,

$$V=\frac{1}{3}\pi r^2h=\frac{\pi}{3}\left(9\sin^2\theta\right).\left(3\cos\theta\right)$$

$$\therefore\ \frac{dV}{d\theta}=0\ \Rightarrow\ \sin\theta=\sqrt{\frac{2}{3}}$$

Also, $\left.\frac{d^2V}{d\theta^2}\right]_{\sin\theta=\sqrt{\frac{2}{3}}}$ = negative

$\Rightarrow$ Volume is maximum,

when $\sin\theta=\sqrt{\frac{2}{3}}$

$$\therefore\ V_{max}\left(\sin\theta=\sqrt{\frac{2}{3}}\right)=2\sqrt{3}\pi\ \text{(in cu. m)}$$

2. Ans. (1)

Let points $\left(\frac{3}{2},0\right)$, (t^2, t), $t > 0$

$$\text{Distance}=\sqrt{t^2+\left(t^2-\frac{3}{2}\right)^2}$$

$$=\sqrt{t^4-2t^2+\frac{9}{4}}=\sqrt{(t^2-1)^2+\frac{5}{4}}$$

So minimum distance is $\sqrt{\frac{5}{4}}=\frac{\sqrt{5}}{2}$

3. Ans. (1)

S = {x ∈ R, $x^2 + 30 - 11x \le 0$}

= {x ∈ R, $5 \le x \le 6$}

Now $f(x) = 3x^3 - 18x^2 + 27x - 40$

$\Rightarrow f'(x) = 9(x - 1)(x - 3)$,

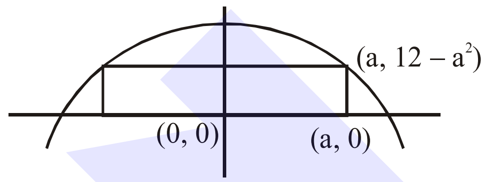
which is positive in [5, 6]

$\Rightarrow f(x)$ increasing in [5, 6]

Hence maximum value = $f(6) = 122$

4. Ans. (3)

$f(a) = 2a(12 - a)^2$



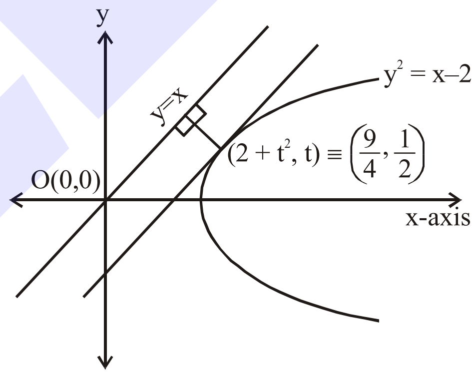
$f'(a) = 2(12 - 3a^2)$

maximum at a = 2

maximum area = f(2) = 32

5. Official Ans. by NTA (1)

Sol.



we have, $2y.\frac{dy}{dx}=1\Rightarrow\left.\frac{dy}{dx}\right]_{P(2+t^2,t)}=\frac{1}{2t}=1$

$\Rightarrow\quad t=\frac{1}{2}$

$\therefore\quad P\left(\frac{9}{4},\frac{1}{2}\right)$

So, shortest distance

$$=\frac{\left|\frac{9}{4}-\frac{2}{4}\right|}{\sqrt{2}}=\frac{7}{4\sqrt{2}}$$

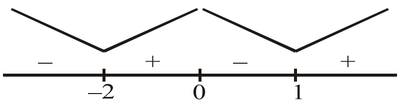
6. Official Ans. by NTA (1)

Sol. $f(x) = 9x^4 + 12x^3 - 36x^2 + 25$

$f'(x) = 36x^3 + 36x^2 - 72x$

$= 36x(x^2 + x - 2)$

$= 36x(x - 1)(x + 2)$

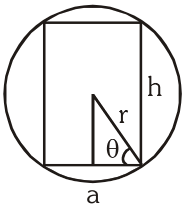


Points of minima = $\{-2, 1\} = S_1$

Point of maxima = $\{0\} = S_2$

7. Official Ans. by NTA (1)

Sol.



$h = 2r\sin\theta$

$a = 2r\cos\theta$

$v = \pi(r\cos\theta)^2(2r\sin\theta)$

$v = 2\pi r^3\cos^2\theta\sin\theta$

$$\frac{dv}{d\theta} = \pi r^3\left(-2\cos\theta\sin^2\theta + \cos^3\theta\right) = 0$$

$$\text{or } \tan\theta = \frac{1}{\sqrt{2}}$$

$$\because\ h = 2\times 3\times\frac{1}{\sqrt{3}} = 2\sqrt{3}$$

8. Official Ans. by NTA (2)

Sol. $f'(x) = \lambda(x + 1)(x - 0)(x - 1) = \lambda(x^3 - x)$

$$\Rightarrow f(x) = \lambda\left(\frac{x^4}{4} - \frac{x^2}{2}\right) + \mu$$

Now $f(x) = f(0)$

$$\Rightarrow \lambda\left(\frac{x^4}{4} - \frac{x^2}{2}\right) + \mu = \mu$$

$\Rightarrow x = 0, 0, \pm\sqrt{2}$

Two irrational and one rational number

9. Official Ans. by NTA (2)

Sol. Let a is first term and d is common difference

then, $a + 5d = 2$ (given) ...(1)

$f(d) = (2 - 5d)(2 - 2d)(2 - d)$

$f'(d) = 0 \quad \Rightarrow d = \frac{2}{3}, \frac{8}{5}$

$f''(d) < 0$ at $d = 8/5$

$$\Rightarrow d = \frac{8}{5}$$