MATRIX

1. If $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$, then the matrix A^{-50} when $\theta = \frac{\pi}{12}$, is equal to :

(1) $\begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$ (2) $\begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$

(3) $\begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$ (4) $\begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$

2. If $A = \begin{bmatrix} e^t & e^{-t}\cos t & e^{-t}\sin t \\ e^t & -e^{-t}\cos t - e^{-t}\sin t & -e^{-t}\sin t + e^{-t}\cos t \\ e^t & 2e^{-t}\sin t & -2e^{-t}\cos t \end{bmatrix}$

Then A is-

(1) Invertible only if $t = \frac{\pi}{2}$

(2) not invertible for any $t \varepsilon R$

(3) invertible for all $t \varepsilon R$

(4) invertible only if $t = \pi$

3. Let $A = \begin{bmatrix} 2 & b & 1 \\ b & b^2+1 & b \\ 1 & b & 2 \end{bmatrix}$ where $b > 0$. Then the minimum value of $\frac{\det(A)}{b}$ is :

(1) $\sqrt{3}$ (2) $-\sqrt{3}$ (3) $-2\sqrt{3}$ (4) $2\sqrt{3}$

4. Let $A = \begin{pmatrix} 0 & 2q & r \\ p & q & -r \\ p & -q & r \end{pmatrix}$. It $AA^T = I_3$, then $|p|$ is :

(1) $\frac{1}{\sqrt{2}}$ (2) $\frac{1}{\sqrt{5}}$ (3) $\frac{1}{\sqrt{6}}$ (4) $\frac{1}{\sqrt{3}}$

5. Let A and B be two invertible matrices of order 3×3. If $\det(ABA^T) = 8$ and $\det(AB^{-1}) = 8$, then $\det(BA^{-1}B^T)$ is equal to :-

(1) 16 (2) $\frac{1}{16}$ (3) $\frac{1}{4}$ (4) 1

6. Let $P = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 9 & 3 & 1 \end{bmatrix}$ and $Q = [q_{ij}]$ be two 3×3 matrices such that $Q - P^5 = I_3$. Then $\frac{q_{21}+q_{31}}{q_{32}}$ is equal to:

(1) 15 (2) 9 (3) 135 (4) 10

7. If $A = \begin{bmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{bmatrix}$; then for all $\theta \in \left(\frac{3\pi}{4}, \frac{5\pi}{4}\right)$, det(A) lies in the interval :

(1) $\left[\frac{5}{2}, 4\right)$ (2) $\left(\frac{3}{2}, 3\right]$

(3) $\left(0, \frac{3}{2}\right]$ (4) $\left(1, \frac{5}{2}\right]$

8. Let the number 2,b,c be in an A.P. and $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & b & c \\ 4 & b^2 & c^2 \end{bmatrix}$. If $\det(A) \in [2,16]$, then c lies in the interval :

(1) [2,3) (2) $(2 + 2^{3/4}, 4)$

(3) $[3, 2 + 2^{3/4}]$ (4) [4,6]

9. Let $A=\begin{pmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{pmatrix},(\alpha\in R)$ such that $A^{32}=\begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}$. Then a value of α is

(1) $\frac{\pi}{16}$ (2) 0 (3) $\frac{\pi}{32}$ (4) $\frac{\pi}{64}$

10. If $\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}.\begin{bmatrix}1 & 2\\ 0 & 1\end{bmatrix}.\begin{bmatrix}1 & 3\\ 0 & 1\end{bmatrix}......\begin{bmatrix}1 & n-1\\ 0 & 1\end{bmatrix}=\begin{bmatrix}1 & 78\\ 0 & 1\end{bmatrix}$, then the inverse of $\begin{bmatrix}1 & n\\ 0 & 1\end{bmatrix}$ is

(1) $\begin{bmatrix}1 & -13\\ 0 & 1\end{bmatrix}$ (2) $\begin{bmatrix}1 & 0\\ 12 & 1\end{bmatrix}$

(3) $\begin{bmatrix}1 & -12\\ 0 & 1\end{bmatrix}$ (4) $\begin{bmatrix}1 & 0\\ 13 & 1\end{bmatrix}$

11. The total number of matrices $A=\begin{pmatrix}0 & 2y & 1\\ 2x & y & -1\\ 2x & -y & 1\end{pmatrix},(x,y\in R, x\neq y)$ for which $A^TA=3I_3$ is :-

(1) 6 (2) 2 (3) 3 (4) 4

12. If $B=\begin{bmatrix}5 & 2\alpha & 1\\ 0 & 2 & 1\\ \alpha & 3 & -1\end{bmatrix}$ is the inverse of a 3 × 3 matrix A, then the sum of all values of α for which det (A) + 1 = 0, is :

(1) 0 (2) 2 (3) 1 (4) –1

13. If a is A symmetric matrix and B is a skew-symmetrix matrix such that $A+B=\begin{bmatrix}2 & 3\\ 5 & -1\end{bmatrix}$, then AB is equal to :

(1) $\begin{bmatrix}-4 & 2\\ 1 & 4\end{bmatrix}$ (2) $\begin{bmatrix}-4 & -2\\ -1 & 4\end{bmatrix}$

(3) $\begin{bmatrix}4 & -2\\ -1 & -4\end{bmatrix}$ (4) $\begin{bmatrix}4 & -2\\ 1 & -4\end{bmatrix}$

SOLUTION

1. Ans. (1)

Here, $AA^T = I$

$$\Rightarrow A^{-1} = A^T = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

Also, $A^{-n} = \begin{bmatrix} \cos(n\theta) & \sin(n\theta) \\ -\sin(n\theta) & \cos(n\theta) \end{bmatrix}$

$$\therefore A^{-50} = \begin{bmatrix} \cos(50)\theta & \sin(50)\theta \\ -\sin(50)\theta & \cos(50)\theta \end{bmatrix}$$

$$= \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$$

2. Ans. (3)

$$|A| = e^{-t}\begin{vmatrix} 1 & \cos t & \sin t \\ 1 & -\cos t - \sin t & -\sin t + \cos t \\ 1 & 2\sin t & -2\cos t \end{vmatrix}$$

$= e^{-t}[5\cos^2 t + 5\sin^2 t]\ \forall\ t \in R$

$= 5e^{-t} \neq 0\ \forall\ t \in R$

3. Ans. (4)

$$A = \begin{bmatrix} 2 & b & 1 \\ b & b^2+1 & b \\ 1 & b & 2 \end{bmatrix} \ (b > 0)$$

$|A| = 2(2b^2 + 2 - b^2) - b(2b - b) + 1\ (b^2 - b^2 - 1)$

$|A| = 2(b^2 + 2) - b^2 - 1$

$|A| = b^2 + 3$

$$\frac{|A|}{b} = b + \frac{3}{b} \Rightarrow \frac{b + \frac{3}{b}}{2} \geq \sqrt{3}$$

$$b + \frac{3}{b} \geq 2\sqrt{3}$$

Option (4)

4. Ans. (1)

A is orthogonal matrix

$$\Rightarrow 0^2 + p^2 + p^2 = 1 \Rightarrow |p| = \frac{1}{\sqrt{2}}$$

5. Ans. (2)

$$|A|^2.|B| = 8 \text{ and } \frac{|A|}{|B|} = 8 \Rightarrow |A| = 4 \text{ and } |B| = \frac{1}{2}$$

$$\therefore \det(BA^{-1}.B^T) = \frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$$

6. Ans. (4)

$$P = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 9 & 3 & 1 \end{bmatrix}$$

$$P^2 = \begin{bmatrix} 1 & 0 & 0 \\ 3+3 & 1 & 0 \\ 9+9+9 & 3+3 & 1 \end{bmatrix}$$

$$P^3 = \begin{bmatrix} 1 & 0 & 0 \\ 3+3+3 & 1 & 0 \\ 6.9 & 3+3+3 & 1 \end{bmatrix}$$

$$P^n = \begin{bmatrix} 1 & 0 & 0 \\ 3n & 1 & 0 \\ \frac{n(n+1)}{2}3^2 & 3n & 1 \end{bmatrix}$$

$$P^5 = \begin{bmatrix} 1 & 0 & 0 \\ 5.3 & 1 & 0 \\ 15.9 & 5.3 & 1 \end{bmatrix}$$

$Q = P^5 + I_3$

$$Q = \begin{bmatrix} 2 & 0 & 0 \\ 15 & 2 & 0 \\ 135 & 15 & 2 \end{bmatrix}$$

$$\frac{q_{21} + q_{31}}{q_{32}} = \frac{15 + 135}{15} = 10$$

Aliter

$$P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 3 & 0 & 0 \\ 9 & 3 & 0 \end{pmatrix}$$

$P = I + X$

$$X = \begin{pmatrix} 0 & 0 & 0 \\ 3 & 0 & 0 \\ 9 & 3 & 0 \end{pmatrix}$$

$$X^2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 9 & 0 & 0 \end{pmatrix}$$

$X^3 = 0$

$P^5 = I + 5X + 10X^2$

$Q = P^5 + I = 2I + 5X + 10X^2$

$$Q = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 15 & 0 & 0 \\ 15 & 15 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 90 & 0 & 0 \end{pmatrix}$$

$$\Rightarrow Q = \begin{pmatrix} 2 & 0 & 0 \\ 15 & 2 & 0 \\ 135 & 15 & 2 \end{pmatrix}$$

7. Ans (2)

$$|A| = \begin{vmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{vmatrix}$$

$= 2(1 + \sin^2\theta)$

$$\theta \in \left(\frac{3\pi}{4}, \frac{5\pi}{4}\right) \Rightarrow \frac{1}{\sqrt{2}} < \sin\theta < \frac{1}{\sqrt{2}}$$

$$\Rightarrow 0 \le \sin^2\theta < \frac{1}{2}$$

$\therefore |A| \in [2, 3)$

8. Official Ans. by NTA (4)

Sol. put $b = \frac{2+c}{2}$ in determinant of A

$$|A| = \frac{c^3 - 6c^2 + 12c - 8}{4} \in [2,16]$$

$\Rightarrow (c-2)^3 \in [8, 64]$

$\Rightarrow c \in [4, 6]$

9. Official Ans. by NTA (4)

Sol. $A=\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}$

$A^2=\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}$

$=\begin{bmatrix}\cos2\alpha & -\sin2\alpha\\ \sin2\alpha & \cos2\alpha\end{bmatrix}$

$A^3=\begin{bmatrix}\cos2\alpha & -\sin2\alpha\\ \sin2\alpha & \cos2\alpha\end{bmatrix}\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}$

$=\begin{bmatrix}\cos3\alpha & -\sin3\alpha\\ \sin3\alpha & \cos3\alpha\end{bmatrix}$

Similarly

$A^{32}=\begin{bmatrix}\cos32\alpha & -\sin32\alpha\\ \sin32\alpha & \cos32\alpha\end{bmatrix}=\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}$

$\Rightarrow$ $\cos32\alpha = 0$ & $\sin32\alpha = 1$

$\Rightarrow$ $32\alpha=(4n+1)\frac{\pi}{2}, n\in I$

$\alpha=(4n+1)\frac{\pi}{64}, n\in I$

$\alpha=\frac{\pi}{64}$ for $n=0$

10. Official Ans. by NTA (1)

Sol. $\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}\begin{bmatrix}1 & 2\\ 0 & 1\end{bmatrix}\begin{bmatrix}1 & 2\\ 0 & 1\end{bmatrix}\begin{bmatrix}1 & 3\\ 0 & 1\end{bmatrix}.....\begin{bmatrix}1 & n-1\\ 0 & 1\end{bmatrix}=\begin{bmatrix}1 & 78\\ 0 & 1\end{bmatrix}$

$\Rightarrow\begin{bmatrix}1 & 1+2+3+.....+n-1\\ 0 & 1\end{bmatrix}=\begin{bmatrix}1 & 78\\ 0 & 1\end{bmatrix}$

$\Rightarrow \frac{n(n-2)}{2}=78 \Rightarrow$ n = 13, –12(reject)

$\therefore$ We have to find inverse of $\begin{bmatrix}1 & 13\\ 0 & 1\end{bmatrix}$

$\therefore \begin{bmatrix}1 & -13\\ 0 & 1\end{bmatrix}$

11. Official Ans. by NTA (4)

Sol. $A^TA = 3I_3$

$\begin{pmatrix}0 & 2x & 2x\\ 2y & y & -y\\ 1 & -1 & 1\end{pmatrix}\begin{pmatrix}0 & 2y & 1\\ 2x & y & -1\\ 2x & -y & 1\end{pmatrix}=\begin{pmatrix}3 & 0 & 0\\ 0 & 3 & 0\\ 0 & 0 & 3\end{pmatrix}$

$\begin{pmatrix}8x^2 & 0 & 0\\ 0 & 6y^2 & 0\\ 0 & 0 & 3\end{pmatrix}=\begin{bmatrix}3 & 0 & 0\\ 0 & 3 & 0\\ 0 & 0 & 3\end{bmatrix}$

$8x^2 = 3$

$6y^2 = 3$

$x^2 = 3/8$

$y^2 = 1/2$

$x=\pm\sqrt{\frac{3}{8}}; y=\pm\sqrt{\frac{1}{2}}$

12. Official Ans. by NTA (3)

Sol. $|B| = 5(-5) - 2\alpha(-\alpha) - 2\alpha$

$= 2\alpha^2 - 2\alpha - 25$

$1 + |A| = 0$

$\alpha^2 - \alpha - 12 = 0$

Sum = 1

13. Official Ans. by NTA (3)

Sol. A = A', B = –B'

$A + B = \begin{bmatrix}2 & 3\\ 5 & -1\end{bmatrix}$ ...(1)

$A' + B' = \begin{bmatrix}2 & 5\\ 3 & -1\end{bmatrix}$

$A - B = \begin{bmatrix}2 & 5\\ 3 & -1\end{bmatrix}$ ...(2)

After adding Eq. (1) & (2)

$A=\begin{bmatrix}2 & 4\\ 4 & -1\end{bmatrix}, B=\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}$

$AB = \begin{bmatrix}4 & -2\\ -1 & -4\end{bmatrix}$