LIMIT

1. $\lim_{y\to 0}\dfrac{\sqrt{1+\sqrt{1+y^4}}-\sqrt{2}}{y^4}$

(1) exists and equals $\dfrac{1}{4\sqrt{2}}$

(2) does not exist

(3) exists and equals $\dfrac{1}{2\sqrt{2}}$

(4) exists and equals $\dfrac{1}{2\sqrt{2}\left(\sqrt{2}+1\right)}$

2. For each xεR, let [x] be the greatest integer less than or equal to x. Then $\lim_{x\to 0^-}\dfrac{x\left([x]+|x|\right)\sin[x]}{|x|}$ is equal to

(1) – sin1 (2) 0
(3) 1 (4) sin1

3. For each t∈R, let [t] be the greatest integer less than or equal to t. Then,

$$\lim_{x\to 1+}\frac{(1-|x|+\sin|1-x|)\sin\left(\frac{\pi}{2}[1-x]\right)}{|1-x|[1-x]}$$

(1) equals –1
(2) equals 1
(3) does not exist
(4) equals 0

4. Let [x] denote the greatest integer less than or equal to x. Then :-

$$\lim_{x\to 0}\frac{\tan(\pi\sin^2 x)+\left(|x|-\sin\left(x[x]\right)\right)^2}{x^2}$$

(1) equals π (2) equals 0
(3) equals π + 1 (4) does not exist

5. $\lim_{x\to 0}\dfrac{x\cot(4x)}{\sin^2 x\cot^2(2x)}$ is equal to :-

(1) 2 (2) 0
(3) 4 (4) 1

6. $\lim_{x\to \pi/4}\dfrac{\cot^3 x-\tan x}{\cos\left(x+\pi/4\right)}$ is :

(1) 4 (2) $8\sqrt{2}$
(3) 8 (4) $4\sqrt{2}$

7. $\lim_{x\to 1-}\dfrac{\sqrt{\pi}-\sqrt{2\sin^{-1}x}}{\sqrt{1-x}}$ equal to :

(1) $\dfrac{1}{\sqrt{2\pi}}$ (2) $\sqrt{\dfrac{\pi}{2}}$

(3) $\sqrt{\dfrac{2}{\pi}}$ (4) $\sqrt{\pi}$

8. $\lim_{x\to 0}\dfrac{\sin^2 x}{\sqrt{2}-\sqrt{1+\cos x}}$ equals :

(1) $2\sqrt{2}$ (2) $4\sqrt{2}$
(3) $\sqrt{2}$ (4) 4

9. Let f : R → R be a differentiable function satisfying $f'(3) + f'(2) = 0$.

Then $\lim_{x\to 0}\left(\dfrac{1+f(3+x)-f(3)}{1+f(2-x)-f(2)}\right)^{\frac{1}{x}}$ is equal to

(1) e^2 (2) e (3) e^{-1} (4) 1

10. If $f(x)=[x]-\left[\dfrac{x}{4}\right], x\in R$, where [x] denotes the greatest integer function, then :

(1) Both $\lim_{x\to 4-}f(x)$ and $\lim_{x\to 4+}f(x)$ exist but are not equal

(2) $\lim_{x\to 4-}f(x)$ exists but $\lim_{x\to 4+}f(x)$ does not exist

(3) $\lim_{x\to 4+}f(x)$ exists but $\lim_{x\to 4-}f(x)$ does not exist

(4) f is continuous at x = 4

11. If $\lim_{x\to 1}\frac{x^4-1}{x-1}=\lim_{x\to k}\frac{x^3-k^3}{x^2-k^2}$, then k is :

(1) $\frac{3}{8}$ (2) $\frac{3}{2}$ (3) $\frac{4}{3}$ (4) $\frac{8}{3}$

12. If $\lim_{x\to 1}\frac{x^2-ax+b}{x-1}=5$, then a + b is equal to :-

(1) –7 (2) – 4 (3) 5 (4) 1

13. $\lim_{x\to 0}\frac{x+2\sin x}{\sqrt{x^2+2\sin x+1}-\sqrt{\sin^2 x-x+1}}$ is :

(1) 3 (2) 2 (3) 6 (4) 1

14. Let f(x) = 5– |x–2| and g(x) = |x + 1|, $x \in R$. If f(x) attains maximum value at α and g(x) attains minimum value at β, then

$\lim_{x\to -\alpha\beta}\frac{(x-1)(x^2-5x+6)}{x^2-6x+8}$ is equal to :

(1) 1/2 (2) –3/2 (3) 3/2 (4) –1/2

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SOLUTION

1. Ans. (1)

$$\lim_{y\to 0}\frac{\sqrt{1+\sqrt{1+y^4}}-\sqrt{2}}{y^4}$$

$$=\lim_{y\to 0}\frac{1+\sqrt{1+y^4}-2}{y^4\left(\sqrt{1+\sqrt{1+y^4}}+\sqrt{2}\right)}$$

$$=\lim_{y\to 0}\frac{\left(\sqrt{1+y^4}-1\right)\left(\sqrt{1+y^4}+1\right)}{y^4\left(\sqrt{1+\sqrt{1+y^4}}+\sqrt{2}\right)\left(\sqrt{1+y^4}+1\right)}$$

$$=\lim_{y\to 0}\frac{1+y^4-1}{y^4\left(\sqrt{1+\sqrt{1+y^4}}+\sqrt{2}\right)\left(\sqrt{1+y^4}+1\right)}$$

$$=\lim_{y\to 0}\frac{1}{\left(\sqrt{1+\sqrt{1+y^4}}+\sqrt{2}\right)\left(\sqrt{1+y^4}+1\right)}=\frac{1}{4\sqrt{2}}$$

2. Ans. (1)

$$\lim_{x\to 0^-}\frac{x\left([x]+|x|\right)\sin[x]}{|x|}$$

$x\to 0^-$

$[x]=-1\Rightarrow \lim_{x\to 0^-}\frac{x(-x-1)\sin(-1)}{-x}=-\sin 1$

$|x|=-x$

3. Ans. (4)

$$\lim_{x\to 1^+}\frac{(1-|x|+\sin|1-x|)\sin\left(\frac{\pi}{2}[1-x]\right)}{|1-x|[1-x]}$$

$$=\lim_{x\to 1^+}\frac{(1-x)+\sin(x-1)}{(x-1)(-1)}\sin\left(\frac{\pi}{2}(-1)\right)$$

$$=\lim_{x\to 1^+}\left(1-\frac{\sin(x-1)}{(x-1)}\right)(-1)=(1-1)(-1)=0$$

4. Ans. (4)

$$\text{R.H.L.}=\lim_{x\to 0^+}\frac{\tan(\pi\sin^2 x)+\left(|x|-\sin(x[x])\right)^2}{x^2}$$

(as $x\to 0^+ \Rightarrow [x]=0$)

$$=\lim_{x\to 0^+}\frac{\tan(\pi\sin^2 x)+x^2}{x^2}$$

$$=\lim_{x\to 0^+}\frac{\tan(\pi\sin^2 x)}{(\pi\sin^2 x)}+1=\pi+1$$

$$\text{L.H.L.}=\lim_{x\to 0^-}\frac{\tan(\pi\sin^2 x)+(-x+\sin x)^2}{x^2}$$

(as $x\to 0^- \Rightarrow [x]=-1$)

$$\lim_{x\to 0+}\frac{\tan(\pi\sin^2 x)}{\pi\sin^2 x}\cdot\frac{\pi\sin^2 x}{x^2}+\left(-1+\frac{\sin x}{x}\right)^2\Rightarrow \pi$$

R.H.L. $\neq$ L.H.L.

5. Ans. (4)

$$\lim_{x\to 0}\frac{x\tan^2 2x}{\tan 4x\sin^2 x}=\lim_{x\to 0}\frac{x\left(\frac{\tan^2 2x}{4x^2}\right)4x^2}{\left(\frac{\tan 4x}{4x}\right)4x\left(\frac{\sin^2 x}{x^2}\right)x^2}=1$$

6. Ans. (3)

$$\lim_{x\to\pi/4}\frac{\cot^3 x-\tan x}{\cos\left(x+\frac{\pi}{4}\right)}$$

$$\lim_{x\to\pi/4}\frac{(1-\tan^4 x)}{\cos(x+\pi/4)}$$

$$2\lim_{x\to\pi/4}\frac{(1-\tan^2 x)}{\cos(x+\pi/4)}$$

$$R\lim_{x\to\pi/4}\frac{\cos^2 x-\sin^2 x}{\frac{\cos x-\sin x}{\sqrt{2}}}\,\frac{1}{\cos^2 x}$$

$$4\sqrt{2}\lim_{x\to\pi/4}(\cos x+\sin x)=8$$

7. Ans. (3)

$$\lim_{x\to 1^-}\frac{\sqrt{\pi}-\sqrt{2\sin^{-1}x}}{\sqrt{1-x}}\times\frac{\sqrt{\pi}+\sqrt{2\sin^{-1}x}}{\sqrt{\pi}+\sqrt{2\sin^{-1}x}}$$

$$\lim_{x\to 1^-}\frac{2\left(\frac{\pi}{2}-\sin^{-1}x\right)}{\sqrt{1-x}\cdot\left(\sqrt{\pi}+\sqrt{2\sin^{-1}x}\right)}$$

$$\lim_{x\to 1^-}\frac{2\cos^{-1}x}{\sqrt{1-x}}\cdot\frac{1}{2\sqrt{\pi}}$$

Put $x=\cos\theta$

$$\lim_{\theta\to 0^+}\frac{2\theta}{\sqrt{2}\sin\left(\frac{\theta}{2}\right)}\cdot\frac{1}{2\sqrt{\pi}}=\sqrt{\frac{2}{\pi}}$$

8. Official Ans. by NTA (2)

Sol. $$\lim_{x\to 0}\frac{\left(\frac{\sin^2 x}{x^2}\right)\left(\sqrt{2}+\sqrt{1+\cos x}\right)}{\left(\frac{1-\cos x}{x^2}\right)}$$

$$=\frac{(1)^2.\left(2\sqrt{2}\right)}{\frac{1}{2}}=4\sqrt{2}$$

9. Official Ans. by NTA (4)

Sol. $$\lim_{x\to 0}\left(\frac{1+f(3+x)-f(3)}{1+f(2-x)-f(2)}\right)^{\frac{1}{x}}\quad (1^{\infty}\text{ form})$$

$$\Rightarrow e^{\lim\limits_{x\to 0}\frac{f(3+x)-f(2-x)-f(3)+f(2)}{x(1+f(2-x)-f(2))}}$$

using L'Hopital

$$\Rightarrow e^{\lim\limits_{x\to 0}\frac{f'(3+x)+f'(2-x)}{-xf'(2-x)+(1+f(2-x)-f(2))}}$$

$$\Rightarrow e^{\frac{f'(3)+f'(2)}{1}}=1$$

10. Official Ans. by NTA (4)

Sol. $f(x)=[x]-\left[\frac{x}{4}\right]$

$$\lim_{x\to 4+}f(x)=\lim_{x\to 4+}\left(\left([x]-\left[\frac{x}{4}\right]\right)\right)=4-1=3$$

$$\lim_{x\to 4-}f(x)=\lim_{x\to 4-}\left([x]-\frac{x}{4}\right)=3-0=3$$

f(x) = 3

$\therefore$ continuous at x = 4

11. Official Ans. by NTA (4)

Sol. $$\lim_{x\to 1}\frac{x^4-1}{x-1}=\lim_{x\to k}\frac{x^3-k^3}{x^2-k^2}$$

$$\Rightarrow\lim_{x\to 1}(x+1)(x^2+1)=\frac{k^2+k^2+k^2}{2k}$$

$\Rightarrow$ k = 8/3

12. Official Ans. by NTA (1)

Sol. $$\lim_{x\to 1}\frac{x^2-ax+b}{x-1}=5$$

1 – a + b = 0 ...(i)

2 – a = 5 ...(ii)

$\Rightarrow$ a + b = – 7.

13. Official Ans. by NTA (2)

Sol. Rationalize

$$\lim_{x\to 0}\frac{(x+2\sin x)\left(\sqrt{x^2+2\sin x+1}+\sqrt{\sin^2 x-x+1}\right)}{x^2+2\sin x+1-\sin^2 x+x-1}$$

$$\lim_{x\to 0}\frac{(x+2\sin x)(2)}{x^2+2\sin x-\sin^2 x+x}$$

$\frac{0}{0}$ form using L' hospital

$$\Rightarrow\lim_{x\to 0}\frac{(1+2\cos x)\times 2}{2x+2\cos x-2\sin x\cos x+1}\Rightarrow\frac{2\times 3}{(2+1)}=2$$

14. Official Ans. by NTA (1)

Sol. Maxima of f(x) occured at x = 2 i.e. α = 2

Minima of g(x) occured at x = –1 i.e. β = –1

$$\therefore\lim_{x\to 2}\frac{(x-1)(x-2)(x-3)}{(x-2)(x-4)}=\frac{1}{2}$$

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