INVERSE TRIGONOMETRIC FUNCTION

1. If $\cos^{-1}\left(\frac{2}{3x}\right)+\cos^{-1}\left(\frac{3}{4x}\right)=\frac{\pi}{2}\left(x>\frac{3}{4}\right)$ then x is equal to :

(1) $\frac{\sqrt{145}}{12}$ (2) $\frac{\sqrt{145}}{10}$

(3) $\frac{\sqrt{146}}{12}$ (4) $\frac{\sqrt{145}}{11}$

2. If $x = \sin^{-1}(\sin 10)$ and $y=\cos^{-1}(\cos 10)$, then y–x is equal to:

(1) π (2) 7π (3) 0 (4) 10

3. The value of $\cot\left(\sum_{n=1}^{19}\cot^{-1}\left(1+\sum_{p=1}^{n}2p\right)\right)$ is :

(1) $\frac{22}{23}$ (2) $\frac{23}{22}$

(3) $\frac{21}{19}$ (4) $\frac{19}{21}$

4. All x satisfying the inequality $(\cot^{-1} x)^2 - 7(\cot^{-1} x) + 10 > 0$, lie in the interval:-

(1) $(-\infty, \cot 5) \cup (\cot 4, \cot 2)$

(2) (cot 5, cot 4)

(3) (cot 2, ∞)

(4) $(-\infty, \cot 5) \cup (\cot 2, \infty)$

5. Considering only the principal values of inverse functions, the set $A=\left\{x\geq 0:\tan^{-1}(2x)+\tan^{-1}(3x)=\frac{\pi}{4}\right\}$

(1) is an empty set

(2) Contains more than two elements

(3) Contains two elements

(4) is a singleton

6. If $\alpha=\cos^{-1}\left(\frac{3}{5}\right), \beta=\tan^{-1}\left(\frac{1}{3}\right)$,

where $0<\alpha,\beta<\frac{\pi}{2}$, then $\alpha - \beta$ is equal to :

(1) $\sin^{-1}\left(\frac{9}{5\sqrt{10}}\right)$ (2) $\tan^{-1}\left(\frac{9}{14}\right)$

(3) $\cos^{-1}\left(\frac{9}{5\sqrt{10}}\right)$ (4) $\tan^{-1}\left(\frac{9}{5\sqrt{10}}\right)$

7. If $\cos^{-1}x - \cos^{-1}\frac{y}{2} = \alpha$,

where $-1 \le x \le 1, -2 \le y \le 2, x \le \frac{y}{2}$, then for all x, y, $4x^2 - 4xy\cos\alpha + y^2$ is equal to

(1) $4\sin^2\alpha - 2x^2y^2$ (2) $4\cos^2\alpha + 2x^2y^2$

(3) $4\sin^2\alpha$ (4) $2\sin^2\alpha$

8. The value of $\sin^{-1}\left(\frac{12}{13}\right)-\sin^{-1}\left(\frac{3}{5}\right)$ is equal to:

(1) $\pi-\sin^{-1}\left(\frac{63}{65}\right)$ (2) $\pi-\cos^{-1}\left(\frac{33}{65}\right)$

(3) $\frac{\pi}{2}-\sin^{-1}\left(\frac{56}{65}\right)$ (4) $\frac{\pi}{2}-\cos^{-1}\left(\frac{9}{65}\right)$

SOLUTION

1. Ans. (1)

$$\cos^{-1}\left(\frac{2}{3x}\right) + \cos^{-1}\left(\frac{3}{4x}\right) = \frac{\pi}{2} \quad \left(x > \frac{3}{4}\right)$$

$$\cos^{-1}\left(\frac{3}{4x}\right) = \frac{\pi}{2} - \cos^{-1}\left(\frac{2}{3x}\right)$$

$$\cos^{-1}\left(\frac{3}{4x}\right) = \sin^{-1}\left(\frac{2}{3x}\right)$$

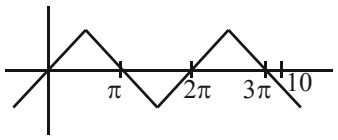
$$\cos\left(\cos^{-1}\left(\frac{3}{4x}\right)\right) = \cos\left(\sin^{-1}\frac{2}{3x}\right)$$

$$\frac{3}{4x} = \frac{\sqrt{9x^2-4}}{3x}$$

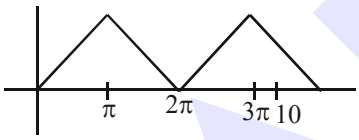
$$\frac{81}{16} + 4 = 9x^2$$

$$x^2 = \frac{145}{16\times 9} \quad \Rightarrow \quad x = \frac{\sqrt{145}}{12}$$

2. Ans. (1)



$x = \sin^{-1}(\sin 10) = 3\pi - 10$



$y = \cos^{-1}(\cos 10) = 4\pi - 10$

$y - x = \pi$

3. Ans. (3)

$$\cot\left(\sum_{n=1}^{19}\cot^{-1}(1+n(n+1)\right)$$

$$\cot\left(\sum_{n=1}^{19}\cot^{-1}(n^2+n+1)\right) = \cot\left(\sum_{n=1}^{19}\tan^{-1}\frac{1}{1+n(n+1)}\right)$$

$$\sum_{n=1}^{19}(\tan^{-1}(n+1) - \tan^{-1}n)$$

$$\cot(\tan^{-1}20 - \tan^{-1}1) = \frac{\cot A\cot\beta + 1}{\cot\beta - \cot A}$$

(Where tanA=20, tanB=1) $\quad \dfrac{1\left(\frac{1}{20}\right)+1}{1-\frac{1}{20}} = \frac{21}{19}$

$\therefore$ Option (3)

4. Ans. (3)

$\cot^{-1}x > 5$ (reject), $\cot^{-1}x < 2$

$\therefore\ x > \cot 2$

$\therefore\ x \in (\cot 2, \infty)$

5. Ans. (4)

$\tan^{-1}(2x) + \tan^{-1}(3x) = \pi/4$

$$\Rightarrow \frac{5x}{1-6x^2} = 1$$

$\Rightarrow 6x^2 + 5x - 1 = 0$

$$x = -1 \text{ or } x = \frac{1}{6}$$

$$x = \frac{1}{6} \quad \because x > 0$$

6. Official Ans. by NTA (1)

Sol. $\cos\alpha=\frac{3}{5},\tan\beta=\frac{1}{3}$

$\Rightarrow \quad \tan\alpha=\frac{4}{3}$

$\Rightarrow \quad \tan(\alpha-\beta)=\frac{\frac{4}{3}-\frac{1}{3}}{1+\frac{4}{3}.\frac{1}{3}}=\frac{9}{13}$

$\Rightarrow \quad \sin(\alpha-\beta)=\frac{9}{5\sqrt{10}}$

$\Rightarrow \quad \alpha-\beta=\sin^{-1}\left(\frac{9}{5\sqrt{10}}\right)$

7. Official Ans. by NTA (3)

Sol. $\cos^{-1}x-\cos^{-1}\frac{y}{2}=\alpha$

$\cos(\cos^{-1}x-\cos^{-1}\frac{y}{2})=\cos\alpha$

$\Rightarrow x\times\frac{y}{2}+\sqrt{1-x^2}\sqrt{1-\frac{y^2}{4}}=\cos\alpha$

$\Rightarrow \left(\cos\alpha-\frac{xy}{2}\right)^2=\left(1-x^2\right)\left(1-\frac{y^2}{4}\right)$

$x^2+\frac{y^2}{4}-xy\cos\alpha=1-\cos^2\alpha=\sin^2\alpha$

8. Official Ans. by NTA (3)

Sol. $\sin^{-1}\left(\frac{12}{13}\right)-\sin^{-1}\left(\frac{3}{5}\right)$

$\sin^{-1}\left(x\sqrt{1-y^2}-y\sqrt{1-x^2}\right)$

$=\sin^{1}\left(\frac{33}{65}\right)=\cos^{-1}\left(\frac{56}{65}\right)=\frac{\pi}{2}-\sin^{1}\left(\frac{56}{65}\right)$