INDEFINITE INTEGRATION

1. For $x^2 \neq n\pi + 1$, $n \in N$ (the set of natural numbers), the integral

$$\int x\sqrt{\frac{2\sin(x^2-1)-\sin 2(x^2-1)}{2\sin(x^2-1)+\sin 2(x^2-1)}}dx$$

is equal to :
(where c is a constant of integration)

(1) $\log_e\left|\sec\left(\frac{x^2-1}{2}\right)\right|+c$

(2) $\log_e\left|\frac{1}{2}\sec^2(x^2-1)\right|+c$

(3) $\frac{1}{2}\log_e\left|\sec^2\left(\frac{x^2-1}{2}\right)\right|+c$

(4) $\frac{1}{2}\log_e\left|\sec(x^2-1)\right|+c$

2. If $f(x)=\int\frac{5x^8+7x^6}{(x^2+1+2x^7)^2}dx, (x \geq 0)$ and $f(0) = 0$, then the value of $f(1)$ is :

(1) $-\frac{1}{2}$ (2) $\frac{1}{2}$ (3) $-\frac{1}{4}$ (4) $\frac{1}{4}$

3. Let $n \geq 2$ be a natural number and $0<\theta<\pi/2$.

Then $\int\frac{(\sin^n\theta-\sin\theta)^{\frac{1}{n}}\cos\theta}{\sin^{n+1}\theta}d\theta$ is equal to :

(Where C is a constant of integration)

(1) $\frac{n}{n^2-1}\left(1-\frac{1}{\sin^{n+1}\theta}\right)^{\frac{n+1}{n}}+C$

(2) $\frac{n}{n^2+1}\left(1-\frac{1}{\sin^{n-1}\theta}\right)^{\frac{n+1}{n}}+C$

(3) $\frac{n}{n^2-1}\left(1-\frac{1}{\sin^{n-1}\theta}\right)^{\frac{n+1}{n}}+C$

(4) $\frac{n}{n^2-1}\left(1+\frac{1}{\sin^{n-1}\theta}\right)^{\frac{n+1}{n}}+C$

4. If $\int x^5 e^{-4x^3}dx=\frac{1}{48}e^{-4x^3}f(x)+C$, where C is a constant of integration, then f(x) is equal to :

(1) $-4x^3 - 1$ (2) $4x^3 + 1$

(3) $-2x^3 - 1$ (4) $-2x^3 + 1$

5. If $\int\frac{\sqrt{1-x^2}}{x^4}dx = A(x)\left(\sqrt{1-x^2}\right)^m + C$, for a suitable chosen integer m and a function A(x), where C is a constant of integration then $(A(x))^m$ equals :

(1) $\frac{-1}{3x^3}$ (2) $\frac{-1}{27x^9}$ (3) $\frac{1}{9x^4}$ (4) $\frac{1}{27x^6}$

6. If $\int\frac{x+1}{\sqrt{2x-1}}dx = f(x)\sqrt{2x-1}+C$, where C is a constant of integration, then f(x) is equal to :-

(1) $\frac{1}{3}(x+4)$ (2) $\frac{1}{3}(x+1)$

(3) $\frac{2}{3}(x+2)$ (4) $\frac{2}{3}(x-4)$

7. The integral $\int\cos(\log_e x)dx$ is equal to :

(where C is a constant of integration)

(1) $\frac{x}{2}[\sin(\log_e x)-\cos(\log_e x)]+C$

(2) $\frac{x}{2}[\cos(\log_e x)+\sin(\log_e x)]+C$

(3) $x[\cos(\log_e x)+\sin(\log_e x)]+C$

(4) $x[\cos(\log_e x)-\sin(\log_e x)]+C$

8. The integral $\int \frac{3x^{13}+2x^{11}}{\left(2x^4+3x^2+1\right)^4}dx$ is equal to :

(where C is a constant of integration)

(1) $\frac{x^4}{\left(2x^4+3x^2+1\right)^3}+C$

(2) $\frac{x^{12}}{6\left(2x^4+3x^2+1\right)^3}+C$

(3) $\frac{x^4}{6\left(2x^4+3x^2+1\right)^3}+C$

(4) $\frac{x^{12}}{\left(2x^4+3x^2+1\right)^3}+C$

9. $\int \frac{\sin\frac{5x}{2}}{\sin\frac{x}{2}}dx$ is equal to :

(where c is a constant of integration)

(1) $2x + \sin x + 2\sin 2x + c$

(2) $x + 2\sin x + 2\sin 2x + c$

(3) $x + 2\sin x + \sin 2x + c$

(4) $2x + \sin x + \sin 2x + c$

10. If $\int \frac{dx}{x^3\left(1+x^6\right)^{2/3}} = xf(x)\left(1+x^6\right)^{\frac{1}{3}}+C$

where C is a constant of integration, then the function $f(x)$ is equal to-

(1) $-\frac{1}{6x^3}$ (2) $\frac{3}{x^2}$

(3) $-\frac{1}{2x^2}$ (4) $-\frac{1}{2x^3}$

11. The integral $\int \sec^{2/3}x \operatorname{cosec}^{4/3}x\, dx$ is equal to

(Hence C is a constant of integration)

(1) $3\tan^{-1/3}x + C$ (2) $-\frac{3}{4}\tan^{-4/3}x + C$

(3) $-3\cot^{-1/3}x + C$ (4) $-3\tan^{-1/3}x + C$

12. If $\int \frac{dx}{\left(x^2-2x+10\right)^2}$

$= A\left(\tan^{-1}\left(\frac{x-1}{3}\right)+\frac{f(x)}{x^2-2x+10}\right)+C$

where C is a constant of integration, then :

(1) $A = \frac{1}{27}$ and $f(x) = 9(x-1)$

(2) $A = \frac{1}{81}$ and $f(x) = 3(x-1)$

(3) $A = \frac{1}{54}$ and $f(x) = 9(x-1)^2$

(4) $A = \frac{1}{54}$ and $f(x) = 3(x-1)$

13. If $\int x^5 e^{-x^2}dx = g(x)e^{-x^2}+c$, where c is a constant of integration, then $g(-1)$ is equal to :

(1) $-\frac{5}{2}$ (2) 1

(3) $-\frac{1}{2}$ (4) –1

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14. The integral $\int \frac{2x^3-1}{x^4+x}dx$ is equal to :

(Here C is a constant of integration)

(1) $\log_e\left|\frac{x^3+1}{x}\right|+C$

(2) $\frac{1}{2}\log_e\frac{(x^3+1)^2}{|x^3|}+C$

(3) $\frac{1}{2}\log_e\frac{|x^3+1|}{x^2}+C$

(4) $\log_e\frac{|x^3+1|}{x^2}+C$

15. Let $\alpha \in (0, \pi/2)$ be fixed. If the integral

$\int \frac{\tan x+\tan\alpha}{\tan x-\tan\alpha}dx =$

A(x) cos2α + B(x) sin2α + C, where C is a constant of integration, then the functions A(x) and B(x) are respectively :

(1) $x - \alpha$ and $\log_e|\cos(x - \alpha)|$

(2) $x + \alpha$ and $\log_e|\sin(x - \alpha)|$

(3) $x - \alpha$ and $\log_e|\sin(x - \alpha)|$

(4) $x + \alpha$ and $\log_e|\sin(x + \alpha)|$

16. If $\int e^{\sec x}(\sec x\tan x f(x)+(\sec x\tan x+\sec^2 x))dx =$ $e^{\sec x}f(x) + C$, then a possible choice of f(x) is :-

(1) $\sec x-\tan x-\frac{1}{2}$

(2) $x\sec x+\tan x+\frac{1}{2}$

(3) $\sec x+x\tan x-\frac{1}{2}$

(4) $\sec x+\tan x+\frac{1}{2}$

SOLUTION

1. **(1 or 3)**

Put $(x^2 - 1) = 1$

$\Rightarrow 2xdx = dt$

$$\therefore \ I = \frac{1}{2}\int \sqrt{\frac{1-\cos t}{1+\cos t}}\,dt$$

$$= \frac{1}{2}\int \tan\left(\frac{t}{2}\right)dt$$

$$= \ln\left|\sec\frac{t}{2}\right| + c$$

$$I = \ln\left|\sec\left(\frac{x^2-1}{2}\right)\right| + c$$

2. **Ans. (4)**

$$\int \frac{5x^8 + 7x^6}{\left(x^2+1+2x^7\right)^2}dx$$

$$= \int \frac{5x^{-6}+7x^{-8}}{\left(\frac{1}{x^7}+\frac{1}{x^5}+2\right)^2}dx = \frac{1}{2+\frac{1}{x^5}+\frac{1}{x^7}} + C$$

As $f(0) = 0$, $f(x) = \frac{x^7}{2x^7+x^2+1}$

$$f(1) = \frac{1}{4}$$

3. **Ans. (3)**

$$\int \frac{(\sin^n\theta - \sin\theta)^{1/n}\cos\theta}{\sin^{n+1}\theta}d\theta$$

$$= \int \frac{\sin\theta\left(1-\frac{1}{\sin^{n-1}\theta}\right)^{1/n}}{\sin^{n+1}\theta}d\theta$$

Put $1 - \frac{1}{\sin^{n-1}\theta} = t$

So $\frac{(n-1)}{\sin^n\theta}\cos\theta d\theta = dt$

Now $\frac{1}{n-1}\int (t)^{1/n}dt$

$$= \frac{1}{(n-1)}\frac{(t)^{\frac{1}{n}+1}}{\frac{1}{n}+1} + C$$

$$= \frac{1}{(n-1)}\left(1-\frac{1}{\sin^{n-1}\theta}\right)^{\frac{1}{n}+1} + C$$

4. **Ans. (1)**

$$\int x^5 \cdot e^{-4x^3}dx = \frac{1}{48}e^{-4x^3}f(x) + c$$

Put $x^3 = t$

$3x^2\,dx = dt$

$$\int x^3 \cdot e^{-4x^3}\cdot x^2dx$$

$$\frac{1}{3}\int t\cdot e^{-4t}\,dt$$

$$\frac{1}{3}\left[t\cdot\frac{e^{-4t}}{-4} - \int\frac{e^{-4t}}{-4}dt\right]$$

$$-\frac{e^{-4t}}{48}[4t+1]+c$$

$$\frac{-e^{-4x^3}}{48}[4x^3+1]+c$$

$\therefore f(x) = -1 - 4x^3$

Option (1)

(From the given options (1) is most suitable)

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5. Ans. (2)

$$\int \frac{\sqrt{1-x^2}}{x^4}dx = A(x)\left(\sqrt{1-x^2}\right)^m + C$$

$$\int \frac{|x|\sqrt{\frac{1}{x^2}-1}}{x^4}dx,$$

Put $\frac{1}{x^2}-1=t \Rightarrow \frac{dt}{dx}=\frac{-2}{x^3}$

Case-1 $x \geq 0$

$$-\frac{1}{2}\int\sqrt{t}\,dt \Rightarrow -\frac{t^{3/2}}{3}+C$$

$$\Rightarrow -\frac{1}{3}\left(\frac{1}{x^2}-1\right)^{3/2} \Rightarrow \frac{\left(\sqrt{1-x^2}\right)^3}{-3x^2}+C$$

$A(x) = -\frac{1}{3x^3}$ and m = 3

$$(A(x))^m = \left(-\frac{1}{3x^3}\right)^3 = -\frac{1}{27x^9}$$

Case-II $x \leq 0$

We get $\frac{\left(\sqrt{1-x^2}\right)^3}{-3x^3}+C$

$A(x) = \frac{1}{-3x^3}$, m = 3

$$(A(x))^m = \frac{-1}{27x^9}$$

6. Ans. (1)

$\sqrt{2x-1}=t \Rightarrow 2x-1=t^2 \Rightarrow 2dx = 2t.dt$

$$\int\frac{x+1}{\sqrt{2x-1}}dx = \int\frac{\frac{t^2+1}{2}+1}{t}t\,dt = \int\frac{t^2+3}{2}dt$$

$$=\frac{1}{2}\left(\frac{t^3}{3}+3t\right)=\frac{t}{6}\left(t^2+9\right)+c$$

$$=\sqrt{2x-1}\left(\frac{2x-1+9}{6}\right)+c=\sqrt{2x-1}\left(\frac{x+4}{3}\right)+c$$

$$\Rightarrow f(x)=\frac{x+4}{3}$$

7. Ans. (2)

$$I=\int\cos(\ell n\,x)dx$$

$$I=\cos(\ln x).x+\int\sin(\ell n\,x)\,dx$$

$$\cos(\ell n\,x)x+[\sin(\ell n\,x).x-\int\cos(\ell n\,x)dx]$$

$$I=\frac{x}{2}[\sin(\ell n\,x)+\cos(\ell n\,x)]+C$$

8. Ans (2)

$$\int\frac{3x^{13}+2x^{11}}{(2x^4+3x^2+1)^4}dx$$

$$\int\frac{\left(\frac{3}{x^3}+\frac{2}{x^5}\right)dx}{\left(2+\frac{3}{x^2}+\frac{1}{x^4}\right)^4}$$

Let $\left(2+\frac{3}{x^2}+\frac{1}{x^4}\right)=t$

$$-\frac{1}{2}\int\frac{dt}{t^4}=\frac{1}{6t^3}+C \Rightarrow \frac{x^{12}}{6(2x^4+3x^2+1)^3}+C$$

9. Official Ans. by NTA (3)

Sol. $\int\frac{\sin\frac{5x}{2}}{\sin\frac{x}{2}}dx = \int\frac{2\sin\frac{5x}{2}\cos\frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}}dx$

$$=\int\frac{\sin 3x+\sin 2x}{\sin x}dx$$

$$=\int\frac{3\sin x-4\sin^3 x-2\sin x\cos x}{\sin x}dx$$

$$=\int(3-4\sin^2 x+2\cos x)dx$$

$$=\int\left(3-2\left(1-\cos 2x\right)+2\cos x\right)dx$$

$$=\int\left(1+2\cos 2x+2\cos x\right)dx$$

= x + sin2x + 2sinx + c

10. Official Ans. by NTA (4)

Sol. $\int \frac{dx}{x^3\left(1+x^6\right)^{2/3}} = x f(x)(1+x^6)^{1/3} + c$

$\int \frac{dx}{x^7\left(\frac{1}{x^6}+1\right)^{2/3}} = x f(x)(1+x^6)^{1/3} + c$

Let $t = \frac{1}{x^6}+1$

$dt = \frac{-6}{x^7}dx$

$-\frac{1}{6}\int \frac{dt}{t^{2/3}} = -\frac{1}{2}t^{1/3}$

$= -\frac{1}{2}\left(\frac{1}{x^6}+1\right)^{1/3} = -\frac{1}{2}\frac{\left(1+x^6\right)^{1/3}}{x^2}$

$\therefore\ f(x) = -\frac{1}{2x^3}$

11. Official Ans. by NTA (4)

Sol. $I = \int \frac{dx}{(\sin x)^{4/3}.(\cos x)^{2/3}}$

$I = \int \frac{dx}{\left(\frac{\sin x}{\cos x}\right)^{4/3}.\cos^2 x}$

$\Rightarrow\ I = \int \frac{\sec^2 x}{(\tan x)^{4/3}}dx$

put $\tan x = t \Rightarrow \sec^2 x dx = dt$

$\therefore\ I = \int \frac{dt}{t^{4/3}} \Rightarrow I = \frac{-3}{t^{1/3}} + c$

$\Rightarrow\ I = \frac{-3}{(\tan x)^{1/3}} + c$

12. Official Ans. by NTA (4)

Sol. $\int \frac{dx}{\left((x-1)^2+9\right)^2} = \frac{1}{27}\int \cos^2\theta\, d\theta$

(Put $x-1 = 3\tan\theta$)

$= \frac{1}{54}\int (1+\cos 2\theta)d\theta = \frac{1}{54}\left(\theta + \frac{\sin 2\theta}{2}\right) + C$

$= \frac{1}{54}\left(\tan^{-1}\left(\frac{x-1}{3}\right) + \frac{3(x-1)}{x^2-2x+10}\right) + C$

13. Official Ans. by NTA (1)

Sol. Let $x^2 = t$ $\quad$ $2xdx = dt$

$\Rightarrow \frac{1}{2}\int t^2.e^{-t}dt = \frac{1}{2}\left[-t^2.e^{-t} + \int 2t.e^{-t}.dt\right]$

$= \frac{1}{2}\left(-t^2.e^{-t}\right) + \left(-t.e^{-t} + \int 1.e^{-t}.dt\right)$

$= -\frac{t^2e^{-t}}{2} - te^{-t} - e^{-t} = \left(-\frac{t^2}{2} - t - 1\right)e^{-t}$

$= \left(-\frac{x^4}{2} - x^2 - 1\right)e^{-x^2} + C$

$g(x) = -1 - x^2 - \frac{x^4}{2} + ke^{x^2}$

for $k = 0$

$g(-1) = -1 - 1 - \frac{1}{2} = -\frac{5}{2}$

14. Official Ans. by NTA (1)

Sol. $\int \frac{2x^3-1}{x^4+x}dx$

$\int \frac{2x-\frac{1}{x^2}}{x^2+\frac{1}{x}}dx$

$x^2+\frac{1}{x}=t$

$\left(2x-\frac{1}{x^2}\right)dx=dt$

$\int \frac{dt}{t}=\ell n(t)+C$

$=\ell n\left(x^2+\frac{1}{x}\right)+C$

15. Official Ans. by NTA (3)

Sol. $\int \frac{\tan x+\tan\alpha}{\tan x-\tan\alpha}dx=\int \frac{\sin(x+\alpha)}{\sin(x-\alpha)}dx$

Let $x - \alpha = t$

$\Rightarrow \int \frac{\sin(t+2\alpha)}{\sin t}dt=\int \cos 2\alpha dt+\int \cot(t)\sin 2\alpha dt$

$=t.\cos 2\alpha+\ell n|\sin t|.\sin 2\alpha+C$

$= (x - \alpha) \cos 2\alpha + \ln|\sin(x - \alpha)|.\sin 2\alpha + C$

16. Official Ans. by NTA (4)

Sol. $\int e^{\sec x}(\sec x \tan x\, f(x)+(\sec x \tan x+\sec^2 x)\, dx$

$= e^{\sec x} f(x) + C$

Diff. both sides w.r.t. 'x'

$e^{\sec x}(\sec x \tan x\, f(x)+(\sec x \tan x+\sec^2 x))$

$= e^{\sec x}\cdot \sec x \tan x\, f(x) + e^{\sec x} f'(x)$

$f'(x) = \sec^2 x + \tan x \sec x$

$\Rightarrow f(x) = \tan x + \sec x + c$