HYPERBOLA

1. Let $0 < \theta < \frac{\pi}{2}$. If the eccentricity of the hyperbola $\frac{x^2}{\cos^2\theta} - \frac{y^2}{\sin^2\theta} = 1$ is greater than 2, then the length of its latus rectum lies in the interval :

(1) (2, 3] (2) $(3, \infty)$

(3) (3/2, 2] (4) (1, 3/2]

2. A hyperbola has its centre at the origin, passes through the point (4,2) and has transverse axis of length 4 along the x-axis. Then the eccentricity of the hyperbola is :

(1) $\frac{2}{\sqrt{3}}$ (2) $\frac{3}{2}$

(3) $\sqrt{3}$ (4) 2

3. The equation of a tangent to the hyperbola $4x^2-5y^2 = 20$ parallel to the line $x-y = 2$ is :

(1) $x-y+9 = 0$

(2) $x-y+7 = 0$

(3) $x-y+1 = 0$

(4) $x-y-3 = 0$

4. Let $S = \left\{(x,y) \in R^2 : \frac{y^2}{1+r} - \frac{x^2}{1-r} = 1\right\}$, where $r \neq \pm 1$. Then S represents :

(1) A hyperbola whose eccentricity is $\frac{2}{\sqrt{r+1}}$, where $0 < r < 1$.

(2) An ellipse whose eccentricity is $\frac{1}{\sqrt{r+1}}$, where $r > 1$

(3) A hyperbola whose eccentricity is $\frac{2}{\sqrt{1-r}}$, when $0 < r < 1$.

(4) An ellipse whose eccentricity is $\sqrt{\frac{2}{r+1}}$, when $r > 1$

5. Equation of a common tangent to the parabola $y^2 = 4x$ and the hyperbole $xy = 2$ is :

(1) $x + 2y + 4 = 0$

(2) $x - 2y + 4 = 0$

(3) $x + y + 1 = 0$

(4) $4x + 2y + 1 = 0$

6. If a hyperbola has length of its conjugate axis equal to 5 and the distance between its foci is 13, then the eccentricity of the hyperbola is :-

(1) 2 (2) $\frac{13}{6}$ (3) $\frac{13}{8}$ (4) $\frac{13}{12}$

7. If the vertices of a hyperbola be at (–2, 0) and (2, 0) and one of its foci be at (–3, 0), then which one of the following points does not lie on this hyperbola?

(1) $(4, \sqrt{15})$ (2) $(-6, 2\sqrt{10})$

(3) $(6, 5\sqrt{2})$ (4) $(2\sqrt{6}, 5)$

8. If the eccentricity of the standard hyperbola passing through the point (4,6) is 2, then the equation of the tangent to the hyperbola at (4,6) is-

(1) $2x - y - 2 = 0$

(2) $3x - 2y = 0$

(3) $2x - 3y + 10 = 0$

(4) $x - 2y + 8 = 0$

9. If the line $y = mx + 7\sqrt{3}$ is normal to the hyperbola $\frac{x^2}{24} - \frac{y^2}{18} = 1$, then a value of m is

(1) $\frac{\sqrt{5}}{2}$ (2) $\frac{3}{\sqrt{5}}$

(3) $\frac{2}{\sqrt{5}}$ (4) $\frac{\sqrt{15}}{2}$

10. If a directrix of a hyperbola centred at the origin and passing through the point $\left(4,-2\sqrt{3}\right)$ is $5x = 4\sqrt{5}$ and its eccentricity is e, then :

(1) $4e^4 - 24e^2 + 35 = 0$

(2) $4e^4 + 8e^2 - 35 = 0$

(3) $4e^4 - 12e^2 - 27 = 0$

(4) $4e^4 - 24e^2 + 27 = 0$

11. If $5x + 9 = 0$ is the directrix of the hyperbola $16x^2 - 9y^2 = 144$, then its corresponding focus is :

(1) $\left(-\frac{5}{3},0\right)$ (2) (5, 0)

(3) (–5, 0) (4) $\left(\frac{5}{3},0\right)$

12. Let P be the point of intersection of the common tangents to the parabola $y^2 = 12x$ and the hyperbola $8x^2 - y^2 = 8$. If S and S' denote the foci of the hyperbola where S lies on the positive x-axis then P divides SS' in a ratio:

(1) 5:4 (2) 14:13 (3) 2:1 (4) 13:11

SOLUTION

1. Ans. (2)

$e=\sqrt{1+\tan^2\theta}=\sec\theta$

As, $\sec\theta > 2 \Rightarrow \cos\theta < \frac{1}{2}$

$\Rightarrow \theta \in (60^\circ, 90^\circ)$

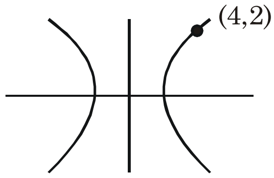
Now, $\ell(L{\cdot}R) = \frac{2b^2}{a}=2\frac{(1-\cos^2\theta)}{\cos\theta}$

$=2(\sec\theta - \cos\theta)$

Which is strictly increasing, so

ℓ (L.R) $\in(3,\infty)$.

2. Ans. (1)



$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

$2a = 4 \Rightarrow a = 2$

$\frac{x^2}{4}-\frac{y^2}{b^2}=1$

Passes through (4,2)

$4-\frac{4}{b^2}=1\Rightarrow b^2=\frac{4}{3}\Rightarrow e=\frac{2}{\sqrt{3}}$

3. Ans. (3)

Hyperbola $\frac{x^2}{5}-\frac{y^2}{4}=1$

slope of tangent = 1

equation of tangent $y=x\pm\sqrt{5-4}$

$\Rightarrow y = x \pm 1$

$\Rightarrow y = x + 1$ or $y = x - 1$

4. Ans. (4)

$\frac{y^2}{1+r}-\frac{x^2}{1-r}=1$

for $r > 1$, $\frac{y^2}{1+r}+\frac{x^2}{r-1}=1$

$e=\sqrt{1-\left(\frac{r-1}{r+1}\right)}$

$=\sqrt{\frac{(r+1)-(r-1)}{(r+1)}}$

$=\sqrt{\frac{2}{r+1}}=\sqrt{\frac{2}{r+1}}$

Option (4)

5. Ans. (1)

Let the equation of tangent to parabola

$y^2 = 4x$ be $y=mx+\frac{1}{m}$

It is also a tangent to hyperbola $xy = 2$

$\Rightarrow x\left(mx+\frac{1}{m}\right)=2$

$\Rightarrow x^2m+\frac{x}{m}-2=0$

$D = 0 \Rightarrow m=-\frac{1}{2}$

So tangent is $2y + x + 4 = 0$

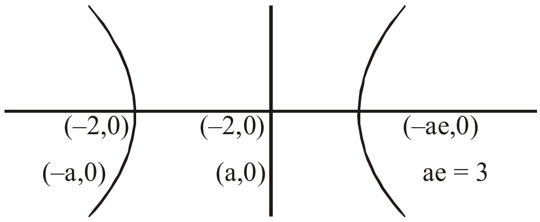
6. Ans. (4)

$2b = 5$ and $2ae = 13$

$b^2 = a^2(e^2 - 1) \Rightarrow \frac{25}{4}=\frac{169}{4}-a^2$

$\Rightarrow a=6 \Rightarrow e=\frac{13}{12}$

7. Ans. (3)



$ae = 3,\ e = \frac{3}{2},\ b^2 = 4\left(\frac{9}{4} - 1\right),\ b^2 = 5$

$\frac{x^2}{4} - \frac{y^2}{5} = 1$

8. Official Ans. by NTA (1)

Sol. Let us Suppose equation of hyperbola is

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$e = 2 \Rightarrow b^2 = 3a^2$

passing through (4,6) $\Rightarrow a^2 = 4,\ b^2 = 12$

$\Rightarrow$ equaiton of tangent

$x - \frac{y}{2} = 1$

$\Rightarrow 2x - y - 2 = 0$

9. Official Ans. by NTA (3)

Sol. $\frac{x^2}{24} - \frac{y^2}{18} = 1 \Rightarrow a = \sqrt{24}; b = \sqrt{18}$

Parametric normal :

$\sqrt{24}\cos\theta.x + \sqrt{18}.y\cot\theta = 42$

At x = 0 : $y = \frac{42}{\sqrt{18}}\tan\theta = 7\sqrt{3}$ (from given equation)

$\Rightarrow \tan\theta = \sqrt{\frac{3}{2}} \Rightarrow \sin\theta = \pm\sqrt{\frac{3}{5}}$

slope of parametric normal $= \frac{-\sqrt{24}\cos\theta}{\sqrt{18}\cot\theta} = m$

$\Rightarrow m = -\sqrt{\frac{4}{3}}\sin\theta = -\frac{2}{\sqrt{5}}$ or $\frac{2}{\sqrt{5}}$

10. Official Ans. by NTA (1)

Sol. Hyperbola is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$\frac{a}{e} = \frac{4}{\sqrt{5}}$ and $\frac{16}{a^2} - \frac{12}{b^2} = 1$

$a^2 = \frac{16}{5}e^2$ ....(1) and $\frac{16}{a^2} - \frac{12}{a^2(e^2-1)} = 1$ ....(2)

From (1) & (2)

$16\left(\frac{5}{16e^2}\right) - \frac{12}{(e^2-1)}\left(\frac{5}{16e^2}\right) = 1$

$\Rightarrow 4e^4 - 24e^2 + 35 = 0$

11. Official Ans. by NTA (3)

Sol. $\frac{x^2}{9} - \frac{y^2}{16} = 1$

$a = 3,\ b = 4\ \&\ e = \sqrt{1 + \frac{16}{9}} = \frac{5}{3}$

corresponding focus will be (–ae, 0) i.e., (–5, 0).

12. Official Ans. by NTA (1)

Sol. Equation of tangents

$y^2 = 12x \quad \Rightarrow \quad y = 2x + \frac{3}{m}$

$\frac{x^2}{1} - \frac{y^2}{8} = 1 \Rightarrow y = mx \pm \sqrt{m^2 - 8}$

Since they are common tangent

$\therefore \frac{3}{m} = \pm\sqrt{m^2 - 8}$ | $\frac{x^2}{1} - \frac{y^2}{8} = 1$

$m^4 - 8m^2 - 9 = 0$ | $e = 3$

$m = \pm 3$ | $ae = 3$

$\therefore$ $y = 3x + 1$, $y = -3x - 1$ > $P\left(-\frac{1}{3}, 0\right)$, $S = (3, 0)$, $S' = (-3, 0)$