FUNCTION

1. For $x \in R-\{0, 1\}$, let $f_1(x) = \frac{1}{x}$, $f_2(x) = 1-x$ and $f_3(x) = \frac{1}{1-x}$ be three given functions. If a function, J(x) satisfies $(f_2oJof_1)(x) = f_3(x)$ then J(x) is equal to :-

(1) $f_3(x)$

(2) $f_1(x)$

(3) $f_2(x)$

(4) $\frac{1}{x}f_3(x)$

2. Let A={xεR:x is not a positive integer}. Define a function f :A→R as $f(x)=\frac{2x}{x-1}$ then f is

(1) injective but not surjective

(2) not injective

(3) surjective but not injective

(4) neither injective nor surjective

3. Let N be the set of natural numbers and two functions f and g be defined as f,g : N→N such that : $f(n)=\begin{cases}\frac{n+1}{2} & \text{if n is odd}\\ \frac{n}{2} & \text{if n is even}\end{cases}$ and $g(n) = n-(-1)^n$. The fog is :

(1) Both one-one and onto

(2) One-one but not onto

(3) Neither one-one nor onto

(4) onto but not one-one

4. Let f : R → R be defined by $f(x) = \frac{x}{1+x^2}$, $x \in R$. Then the range of f is :

(1) $(-1, 1) - \{0\}$ (2) $\left[-\frac{1}{2}, \frac{1}{2}\right]$

(3) $R-\left[-\frac{1}{2}, \frac{1}{2}\right]$ (4) $R - [-1, 1]$

5. Let a function f : (0, ∞) → (0, ∞) be defined by $f(x) = \left|1-\frac{1}{x}\right|$. Then f is :-

(1) Injective only

(2) Not injective but it is surjective

(3) Both injective as well as surjective

(4) Neither injective nor surjective

6. The number of functions f from {1, 2, 3, ..., 20} onto {1, 2, 3, ..., 20} such that f(k) is a multiple of 3, whenever k is a multiple of 4, is :-

(1) (15)! × 6! (2) $5^6 \times 15$

(3) 5! × 6! (4) $6^5 \times (15)!$

7. If $f(x)=\log_e\left(\frac{1-x}{1+x}\right), |x|<1$, then $f\left(\frac{2x}{1+x^2}\right)$ is equal to :

(1) $2f(x)$ (2) $2f(x^2)$

(3) $(f(x))^2$ (4) $-2f(x)$

8. Let $f(x) = a^x$ (a > 0) be written as $f(x) = f_1(x) + f_2(x)$, where $f_1(x)$ is an even function of $f_2(x)$ is an odd function. Then $f_1(x + y) + f_1(x - y)$ equals

(1) $2f_1(x)f_1(y)$

(2) $2f_1(x)f_2(y)$

(3) $2f_1(x+y)f_2(x-y)$

(4) $2f_1(x+y)f_1(x-y)$

9. Let $\sum_{k=1}^{10} f(a+k)=16(2^{10}-1)$, where the function f satisfies $f(x + y) = f(x)f(y)$ for all natural numbers x, y and $f(1) = 2$. then the natural number 'a' is

(1) 4 (2) 3 (3) 16 (4) 2

10. If the function f : R – {1, –1} → A defined by $f(x)=\frac{x^2}{1-x^2}$, is surjective, then A is equal to

(1) R – [–1, 0)

(2) R – (–1, 0)

(3) R – {–1}

(4) [0, ∞)

11. The domain of the definition of the function $f(x)=\frac{1}{4-x^2}+\log_{10}(x^3-x)$ is :-

(1) $(1, 2) \cup (2, \infty)$

(2) $(-1, 0) \cup (1, 2) \cup (3, \infty)$

(3) $(-1, 0) \cup (1, 2) \cup (2, \infty)$

(4) $(-2, -1) \cup (-1, 0) \cup (2, \infty)$

12. Le $f(x) = x^2$, $x \in R$. For any $A \subseteq R$, define $g(A) = \{x \in R, f(x) \in A\}$. If $S = [0, 4]$, then which one of the following statements is not true ?

(1) $f(g(S)) \neq f(S)$ (2) $f(g(S)) = S$

(3) $g(f(S)) = g(S)$ (4) $g(f(S)) \neq S$

13. The number of real roots of the equation $5 + |2^x - 1| = 2^x (2^x - 2)$ is :

(1) 2 (2) 3 (3) 4 (4) 1

14. For $x \in \left(0, \frac{3}{2}\right)$, let $f(x) = \sqrt{x}$, $g(x) = \tan x$ and $h(x)=\frac{1-x^2}{1+x^2}$. If $\phi(x) = ((hof)og)(x)$, then $\phi\left(\frac{\pi}{3}\right)$ is equal to :

(1) $\tan\frac{\pi}{12}$ (2) $\tan\frac{7\pi}{12}$

(3) $\tan\frac{11\pi}{12}$ (4) $\tan\frac{5\pi}{12}$

15. For $x \in R$, let $[x]$ denote the greatest integer $\leq$ x, then the sum of the series

$$\left[-\frac{1}{3}\right]+\left[-\frac{1}{3}-\frac{1}{100}\right]+\left[-\frac{1}{3}-\frac{2}{100}\right]+.....+\left[-\frac{1}{3}-\frac{99}{100}\right]$$

is

(1) –153 (2) –133

(3) –131 (4) –135

SOLUTION

1. Ans. (1)

Given $f_1(x)=\frac{1}{x}, f_2(x)=1-x$ and $f_3(x)=\frac{1}{1-x}$

$(f_2 \circ J \circ f_1)(x)=f_3(x)$

$f_2 \circ (J(f_1(x))) = f_3(x)$

$f_2 \circ \left(J\left(\frac{1}{x}\right)\right)=\frac{1}{1-x}$

$1-J\left(\frac{1}{x}\right)=\frac{1}{1-x}$

$J\left(\frac{1}{x}\right)=1-\frac{1}{1-x}=\frac{-x}{1-x}=\frac{x}{x-1}$

Now $x \to \frac{1}{x}$, $J(x)=\frac{\frac{1}{x}}{\frac{1}{x}-1}=\frac{1}{1-x}=f_3(x)$

2. Ans. (1)

$f(x)=2\left(1+\frac{1}{x-1}\right)$

$f'(x)=-\frac{2}{(x-1)^2}$

$\Rightarrow f$ is one-one but not onto

3. Ans. (4)

$$f(x)=\begin{cases}\frac{n+1}{2} & n \text{ is odd}\\ n/2 & n \text{ is even}\end{cases}$$

$$g(x)=n-(-1)^n\begin{cases}n+1\ ; & n \text{ is odd}\\ n-1\ ; & n \text{ is even}\end{cases}$$

$$f(g(n))=\begin{cases}\frac{n}{2}; & n \text{ is even}\\ \frac{n+1}{2}; & n \text{ is odd}\end{cases}$$

$\therefore$ many one but onto

Option (4)

4. Ans. (2)

$f(0)=0$ & $f(x)$ is odd.

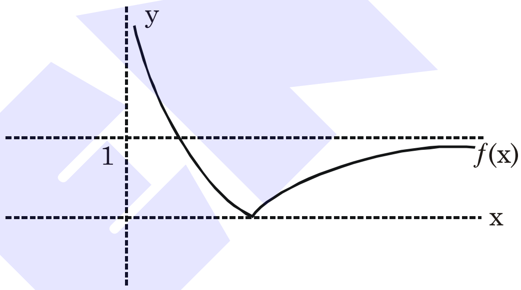
Further, if $x>0$ then

$f(x)=\frac{1}{x+\frac{1}{x}}\in\left(0,\frac{1}{2}\right]$

Hence, $f(x)\in\left[-\frac{1}{2},\frac{1}{2}\right]$

5. Ans. (Bonus)

$$f(x)=\left|1-\frac{1}{x}\right|=\frac{|x-1|}{x}=\begin{cases}\frac{1-x}{x} & 0<x\le 1\\ \frac{x-1}{x} & x\ge 1\end{cases}$$



$\Rightarrow f(x)$ is not injective

but range of function is $[0,\infty)$

Remark : If co-domain is $[0,\infty)$, then $f(x)$ will be surjective

6. Ans. (1)

$f(k) = 3m$ (3,6,9,12,15,18)

for k = 4,8,12,16,20 6.5.4.3.2 ways

For rest numbers 15! ways

Total ways = 6!(15!)

7. Official Ans. by NTA (1)

Sol. $f(x)=\log_e\left(\frac{1-x}{1+x}\right), |x|<1$

$$f\left(\frac{2x}{1+x^2}\right)=\ell n\left(\frac{1-\frac{2x}{1+2x^2}}{1+\frac{2x}{1+x^2}}\right)$$

$$=\ell n\left(\frac{(x-1)^2}{(x+1)^2}\right)=2\ell n\left|\frac{1-x}{1+x}\right|=2f(x)$$

8. Official Ans. by NTA (1)

Sol. $f(x) = a^x, a > 0$

$$f(x) = \frac{a^x + a^{-x} + a^x - a^{-x}}{2}$$

$$\Rightarrow f_1(x) = \frac{a^x + a^{-x}}{2}$$

$$f_2(x) = \frac{a^x - a^{-x}}{2}$$

$$\Rightarrow f_1(x+y) + f_1(x-y)$$

$$= \frac{a^{x+y} + a^{-x-y}}{2} + \frac{a^{x-y} + a^{-x+y}}{2}$$

$$= \frac{(a^x + a^{-x})}{2}(a^y + a^{-y})$$

$$= f_1(x) \times 2f_1(y)$$

$$= 2f_1(x)\, f_1(y)$$

9. Official Ans. by NTA (2)

Sol. From the given functional equation :

$f(x) = 2^x \quad \forall\, x \in N$

$2^{a+1} + 2^{a+2} + ... + 2^{a+10} = 16(2^{10} - 1)$

$2^a (2 + 2^2 + ... + 2^{10}) = 16(2^{10} - 1)$

$$2^a . \frac{2.(2^{10} - 1)}{1} = 16(2^{10} - 1)$$

$2^{a+1} = 16 = 2^4$

$a = 3$

10. Official Ans. by NTA (1)

Sol. $y = \frac{x^2}{1 - x^2}$

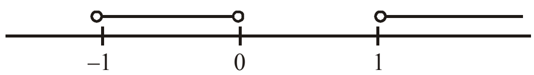
Range of y : $R - [-1, 0)$

for surjective funciton, A must be same as above range.

11. Official Ans. by NTA (3)

Sol. $4 - x^2 \neq 0$; $x^3 - x > 0$

$x = \pm 2 \qquad x(x-1)(x+1) > 0$



$\therefore D_f \in (-1, 0) \cup (1, 2) \cup (2, \infty)$

12. Official Ans. by NTA (3)

Sol. $g(S) = [-2, 2]$

So, $f(g(S)) = [0, 4] = S$

And $f(S) = [0, 16] \Rightarrow f(g(S)) \neq f(S)$

Also, $g(f(S)) = [-4, 4] \neq g(S)$

So, $g(f(S)) \neq S$

13. Official Ans. by NTA (4)

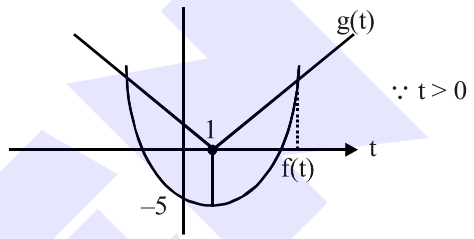
Sol. Let $2^x = t$

$5 + |t - 1| = t^2 - 2t$

$\Rightarrow |t - 1| = (t^2 - 2t - 5)$

g(t) f(t)

From the graph



So, number of real root is 1.

14. Official Ans. by NTA (3)

Sol. $f(x) = \sqrt{x}$, $g(x) = \tan x$, $h(x) = \frac{1 - x^2}{1 + x^2}$

$fog(x) = \sqrt{\tan x}$

$$hofog(x) = h(\sqrt{\tan x}) = \frac{1 - \tan x}{1 + \tan x}$$

$$= -\tan\left(\frac{\pi}{4} - x\right)$$

$$\phi(x) = \tan\left(\frac{\pi}{4} - x\right)$$

$$\phi\left(\frac{\pi}{3}\right) = \tan\left(\frac{\pi}{4} - \frac{\pi}{3}\right) = \tan\left(-\frac{\pi}{12}\right) = -\tan\frac{\pi}{12}$$

$$= \tan\left(\pi - \frac{\pi}{12}\right) = \tan\frac{11\pi}{12}$$

15. Official Ans. by NTA (2)

Sol.
$$\underbrace{\left[-\frac{1}{3}\right] + \left[-\frac{1}{3} - \frac{1}{100}\right] + ... + \left[-\frac{1}{3} - \frac{66}{100}\right]}_{(-1)67}$$

$$+ \underbrace{\left[-\frac{1}{3} - \frac{67}{100}\right] + ... + \left[-\frac{1}{3} - \frac{99}{100}\right]}_{-2(33)} = -133$$