ELLIPSE

1. If tangents are drawn to the ellipse $x^2 + 2y^2 = 2$ at all points on the ellipse other than its four vertices then the mid points of the tangents intercepted betwen the coordinate axes lie on the curve :

(1) $\frac{x^2}{2}+\frac{y^2}{4}=1$ (2) $\frac{x^2}{4}+\frac{y^2}{2}=1$

(3) $\frac{1}{2x^2}+\frac{1}{4y^2}=1$ (4) $\frac{1}{4x^2}+\frac{1}{2y^2}=1$

2. Let the length of the latus rectum of an ellipse with its major axis along x-axis and centre at the origin, be 8. If the distance between the foci of this ellipse is equal to the length of its minor axis, then which one of the following points lies on it ?

(1) $(4\sqrt{3}, 2\sqrt{3})$

(2) $(4\sqrt{3}, 2\sqrt{2})$

(3) $(4\sqrt{2}, 2\sqrt{2})$

(4) $(4\sqrt{2}, 2\sqrt{3})$

3. Let S and S' be the foci of the ellipse and B be any one of the extremities of its minor axis. If $\Delta S'BS$ is a right angled triangle with right angle at B and area $(\Delta S'BS) = 8$ sq. units, then the length of a latus rectum of the ellipse is :

(1) $2\sqrt{2}$ (2) 2

(3) 4 (4) $4\sqrt{2}$

4. If the tangents on the ellipse $4x^2 + y^2 = 8$ at the points (1, 2) and (a, b) are perpendicular to each other, then a^2 is equal to :

(1) $\frac{64}{17}$ (2) $\frac{2}{17}$

(3) $\frac{128}{17}$ (4) $\frac{4}{17}$

5. In an ellipse, with centre at the origin, if the difference of the lengths of major axis and minor axis is 10 and one of the foci is at $(0, 5\sqrt{3})$, then the length of its latus rectum is:

(1) 10 (2) 8 (3) 5 (4) 6

6. If the line $x - 2y = 12$ is tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at the point $\left(3, \frac{-9}{2}\right)$, then the length of the latus recturm of the ellipse is :

(1) 9 (2) $8\sqrt{3}$ (3) $12\sqrt{2}$ (4) 5

7. The tangent and normal to the ellipse $3x^2 + 5y^2 = 32$ at the point P(2, 2) meet the x-axis at Q and R, respectively. Then the area (in sq. units) of the triangle PQR is :

(1) $\frac{14}{3}$ (2) $\frac{16}{3}$ (3) $\frac{68}{15}$ (4) $\frac{34}{15}$

8. If the normal to the ellipse $3x^2 + 4y^2 = 12$ at a point P on it is parallel to the line, $2x + y = 4$ and the tangent to the ellipse at P passes through Q(4, 4) then PQ is equal to :

(1) $\frac{\sqrt{221}}{2}$ (2) $\frac{\sqrt{157}}{2}$ (3) $\frac{\sqrt{61}}{2}$ (4) $\frac{5\sqrt{5}}{2}$

9. An ellipse, with foci at (0, 2) and (0, –2) and minor axis of length 4, passes through which of the following points ?

(1) $(1, 2\sqrt{2})$

(2) $(2, \sqrt{2})$

(3) $(2, 2\sqrt{2})$

(4) $(\sqrt{2}, 2)$

SOLUTION

1. Ans. (3)

Equation of general tangent on ellipse

$$\frac{x}{a\sec\theta}+\frac{y}{b\operatorname{cosec}\theta}=1$$

$a=\sqrt{2}$, b = 1

$$\Rightarrow \frac{x}{\sqrt{2}\sec\theta}+\frac{y}{\operatorname{cosec}\theta}=1$$

Let the midpoint be (h, k)

$$h=\frac{\sqrt{2}\sec\theta}{2} \Rightarrow \cos\theta=\frac{1}{\sqrt{2}h}$$

and $k=\frac{\operatorname{cosec}\theta}{2} \Rightarrow \sin\theta=\frac{1}{2k}$

$\because \sin^2\theta+\cos^2\theta=1$

$$\Rightarrow \frac{1}{2h^2}+\frac{1}{4k^2}=1$$

$$\Rightarrow \frac{1}{2x^2}+\frac{1}{4y^2}=1$$

2. Ans. (2)

$\frac{2b^2}{a}=8$ and $2ae=2b$

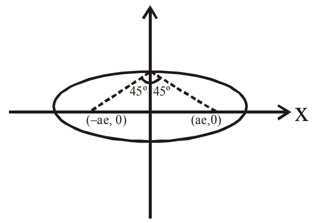
$\Rightarrow \frac{b}{a}=e$ and $1-e^2=e^2 \Rightarrow e=\frac{1}{\sqrt{2}}$

$\Rightarrow b=4\sqrt{2}$ and a = 8

so equation of ellipse is $\frac{x^2}{64}+\frac{y^2}{32}=1$

3. Ans. (3)

$m_{SB}\,.\,m_{S'B}=-1$



$b^2=a^2e^2$ .... (i)

$\frac{1}{2}S'B\,.\,SB=8$

S'B. SB = 16

$a^2e^2+b^2=16$ ..... (ii)

$b^2=a^2(1-e^2)$ ..... (iii)

using (i),(ii), (iii) a = 4

$b=2\sqrt{2}$

$e=\frac{1}{\sqrt{2}}$

$\therefore \ell$ (L.R) $=\frac{2b^2}{a}=4$ Ans.3

4. Official Ans. by NTA (2)

Sol. $4a^2+b^2=8$ ....(1)

also $\left.\frac{dy}{dx}\right)_{(1,2)}=-\frac{4x}{y}=-2$

$\Rightarrow -\frac{4a}{b}=\frac{1}{2}$

$b=-8a$

$\Rightarrow b^2=64a^2$

$68a^2=8$

$a^2=\frac{2}{17}$

5. Official Ans. by NTA (3)

Sol. Let equation of ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

$2a - 2b = 10$ ...(1)

$ae = 5\sqrt{3}$ ...(2)

$\frac{2b^2}{a} = ?$

$b^2 = a^2(1 - e^2)$

$b^2 = a^2 - a^2e^2$

$b^2 = a^2 - 25 \times 3$

$\Rightarrow b = 5$ and $a = 10$

$\therefore$ length of L.R. = $\frac{2(25)}{10} = 5$

6. Official Ans. by NTA (1)

Sol. Tangent at $\left(3, -\frac{9}{2}\right)$

$\frac{3x}{a^2} - \frac{9y}{2b^2} = 1$

Comparing this with $x - 2y = 12$

$\frac{3}{a^2} = \frac{9}{4b^2} = \frac{1}{12}$

we get $a = 6$ and $b = 3\sqrt{3}$

$L(LR) = \frac{2b^2}{a} = 9$

7. Official Ans. by NTA (3)

Sol. $3x^2 + 5y^2 = 32$

$\left.\frac{dy}{dx}\right|_{(2,2)} = -\frac{3}{5}$

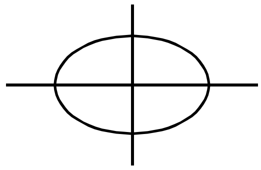
Tangent : $y - 2 = -\frac{3}{5}(x - 2) \Rightarrow Q\left(\frac{16}{3}, 0\right)$

Normal : $y - 2 = \frac{5}{3}(x - 2) \Rightarrow R\left(\frac{4}{5}, 0\right)$

Area is = $\frac{1}{2}(QR) \times 2$ = QR = $\frac{68}{15}$.

8. Official Ans. by NTA (4)

Sol. $3x^2 + 4y^2 = 12$



$\frac{x^2}{4}+\frac{y^2}{3}=1$

$x = 2\cos\theta,\ y = \sqrt{3}\sin\theta$

Let $P(2\cos\theta, \sqrt{3}\sin\theta)$

Equation of normal is $\frac{a^2x}{x_1} - \frac{b^2y}{y_1} = a^2 - b^2$

$2x\sin\theta - \sqrt{3}\cos\theta y = \sin\theta\cos\theta$

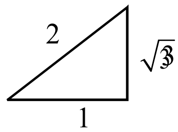
Slope $\frac{2}{\sqrt{3}}\tan\theta = -2 \quad \therefore \tan\theta = -\sqrt{3}$

Equation of tangent is

it passes through (4, 4)

$3x\cos\theta + 2\sqrt{3}\sin\theta\, y = 6$

$12\cos\theta + 8\sqrt{3}\sin\theta = 6$



$\cos\theta = -\frac{1}{2},\ \sin\theta = \frac{\sqrt{3}}{2} \quad \therefore \theta = 120°$

Hence point P is $(2\cos 120°, \sqrt{3}\sin 120°)$

$P\left(-1, \frac{3}{2}\right)$, Q (4, 4)

$PQ = \frac{5\sqrt{5}}{2}$

9. Official Ans. by NTA (4)

Sol. given that $be = 2$ and $a = 2$

(here $a < b$)

$\because a^2 = b^2(1 - e^2)$

$\therefore b^2 = 8$

$\therefore$ equation of ellipse $\frac{x^2}{4}+\frac{y^2}{8}=1$