DIFFERENTIAL EQUATION

1. If y = y(x) is the solution of the differential equation, $x\frac{dy}{dx}+2y=x^2$ satisfying y(1) = 1, then $y\left(\frac{1}{2}\right)$ is equal to :

(1) $\frac{7}{64}$ (2) $\frac{13}{16}$

(3) $\frac{49}{16}$ (4) $\frac{1}{4}$

2. Let f:[0,1]→R be such that $f(xy) = f(x).f(y)$ for all x,y,ε[0,1], and $f(0)\neq 0$. If y = y(x) satisfies the differential equation, $\frac{dy}{dx}=f(x)$ with y(0) = 1, then $y\left(\frac{1}{4}\right)+y\left(\frac{3}{4}\right)$ is equal to

(1) 4 (2) 3

(3) 5 (4) 2

3. If $\frac{dy}{dx}+\frac{3}{\cos^2 x}y=\frac{1}{\cos^2 x}, x\in\left(\frac{-\pi}{3},\frac{\pi}{3}\right)$, and $y\left(\frac{\pi}{4}\right)=\frac{4}{3}$, then $y\left(-\frac{\pi}{4}\right)$ equals :

(1) $\frac{1}{3}+e^6$ (2) $\frac{1}{3}$

(3) $-\frac{4}{3}$ (4) $\frac{1}{3}+e^3$

4. Let f be a differentiable function such that $f'(x)=7-\frac{3}{4}\frac{f(x)}{x},(x>0)$ and f(1) ≠ 4. Then $\lim_{x\to 0^+} xf\left(\frac{1}{x}\right)$:

(1) Exists and equals 4

(2) Does not exist

(3) Exist and equals 0

(4) Exists and equals $\frac{4}{7}$

5. The curve amongst the family of curves, represented by the differential equation, $(x^2-y^2)dx+2xy\,dy=0$ which passes through (1,1) is :

(1) A circle with centre on the y-axis

(2) A circle with centre on the x-axis

(3) An ellipse with major axis along the y-axis

(4) A hyperbola with transverse axis along the x-axis

6. If y(x) is the solution of the differential equation $\frac{dy}{dx}+\left(\frac{2x+1}{x}\right)y=e^{-2x}$, x > 0, where $y(1)=\frac{1}{2}e^{-2}$, then :

(1) y(x) is decreasing in (0, 1)

(2) y(x) is decreasing in $\left(\frac{1}{2},1\right)$

(3) $y(\log_e 2)=\frac{\log_e 2}{4}$

(4) $y(\log_e 2)=\log_e 4$

7. The solution of the differential equation, $\frac{dy}{dx}=(x-y)^2$, when y(1) = 1, is :-

(1) $\log_e\left|\frac{2-y}{2-x}\right|=2(y-1)$

(2) $\log_e\left|\frac{2-x}{2-y}\right|=x-y$

(3) $-\log_e\left|\frac{1+x-y}{1-x+y}\right|=x+y-2$

(4) $-\log_e\left|\frac{1-x+y}{1+x-y}\right|=2(x-1)$

8. Lety = y(x) be the solution of the differential equation, $x\frac{dy}{dx}+y = x\ \log_e x,(x > 1)$. If $2y(2) = \log_e 4-1$, then y(e) is equal to :-

(1) $\frac{e^2}{4}$ (2) $\frac{e}{4}$

(3) $-\frac{e}{2}$ (4) $-\frac{e^2}{2}$

9. If a cuver passes through the point (1, –2) and has slope of the tangent at any point (x, y) on it as $\frac{x^2-2y}{x}$, then the curve also passes through the point :

(1) $\left(-\sqrt{2},1\right)$ (2) $\left(\sqrt{3},0\right)$

(3) (–1, 2) (4) (3, 0)

10. Let y = y(x) be the solution of the differential equation, $\left(x^2+1\right)^2\frac{dy}{dx}+2x\left(x^2+1\right)y=1$ such that $y(0) = 0$. If $\sqrt{a}y(1)=\frac{\pi}{32}$, then the value of 'a' is :

(1) $\frac{1}{2}$ (2) $\frac{1}{16}$

(3) $\frac{1}{4}$ (4) 1

11. Given that the slope of the tangent to a curve $y = y(x)$ at any point (x,y) is $\frac{2y}{x^2}$. If the curve passes through the centre of the circle $x^2 + y^2 - 2x - 2y = 0$, then its equation is :

(1) $x \log_e|y| = 2(x - 1)$

(2) $x \log_e|y| = x - 1$

(3) $x^2 \log_e|y| = -2(x - 1)$

(4) $x \log_e|y| = -2(x - 1)$

12. The solution of the differential equation $x\frac{dy}{dx}+2y=x^2$ $(x \neq 0)$ with y(1) = 1, is

(1) $y=\frac{x^3}{5}+\frac{1}{5x^2}$

(2) $y=\frac{4}{5}x^3+\frac{1}{5x^2}$

(3) $y=\frac{3}{4}x^2+\frac{1}{4x^2}$

(4) $y=\frac{x^2}{4}+\frac{3}{4x^2}$

13. If $\cos x\frac{dy}{dx}-y\sin x=6x,(0<x<\frac{\pi}{2})$ and $y\left(\frac{\pi}{3}\right)=0$, then $y\left(\frac{\pi}{6}\right)$ is equal to :-

(1) $-\frac{\pi^2}{4\sqrt{3}}$ (2) $-\frac{\pi^2}{2}$

(3) $-\frac{\pi^2}{2\sqrt{3}}$ (4) $\frac{\pi^2}{2\sqrt{3}}$

14. If y = y (x) is the solution of the differential equation $\frac{dy}{dx}$ = (tanx –y) $\sec^2 x$, $x\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$, such that y(0) = 0, then $y\left(-\frac{\pi}{4}\right)$ is equal to :

(1) $2+\frac{1}{e}$ (2) $\frac{1}{2}-e$ (3) e – 2 (4) $\frac{1}{2}-e$

15. Let y = y(x) be the solution of the differential equation, $\frac{dy}{dx}+y\tan x = 2x + x^2\tan x$, $x\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$, such that y(0) = 1. Then :

(1) $y'\left(\frac{\pi}{4}\right)+y'\left(\frac{-\pi}{4}\right)=-\sqrt{2}$

(2) $y'\left(\frac{\pi}{4}\right)-y'\left(\frac{-\pi}{4}\right)=\pi-\sqrt{2}$

(3) $y\left(\frac{\pi}{4}\right)-y\left(-\frac{\pi}{4}\right)=\sqrt{2}$

(4) $y\left(\frac{\pi}{4}\right)+y\left(-\frac{\pi}{4}\right)=\frac{\pi^2}{2}+2$

16. Consider the differential equation, $y^2dx+\left(x-\frac{1}{y}\right)dy=0$. If value of y is 1 when x = 1, the the value of x for which y = 2, is :

(1) $\frac{1}{2}+\frac{1}{\sqrt{e}}$ (2) $\frac{3}{2}-\sqrt{e}$

(3) $\frac{5}{2}+\frac{1}{\sqrt{e}}$ (4) $\frac{3}{2}-\frac{1}{\sqrt{e}}$

17. The general solution of the differential equation $(y^2 - x^3)\,dx - xydy = 0$ $(x \neq 0)$ is :

(where c is a constant of integration)

(1) $y^2 + 2x^3 + cx^2 = 0$

(2) $y^2 + 2x^2 + cx^3 = 0$

(3) $y^2 - 2x^3 + cx^2 = 0$

(4) $y^2 - 2x^2 + cx^3 = 0$

SOLUTION

1. Ans. (3)

$$\frac{dy}{dx}+\left(\frac{2}{x}\right)y=x$$

$\Rightarrow$ I.F. $= x^2$

$\therefore\ yx^2 = \frac{x^4}{4}+\frac{3}{4}$ (As, $y(1) = 1$)

$\therefore\ y\left(x=\frac{1}{2}\right) = \frac{49}{16}$

2. Ans. (2)

$f(xy) = f(x).\ f(y)$

$f(0) = 1$ as $f(0) \neq 0$

$\Rightarrow f(x) = 1$

$$\frac{dy}{dx}=f(x)=1$$

$\Rightarrow y=x+c$

At, $x = 0$, $y = 1 \Rightarrow c = 1$

$y = x + 1$

$$\Rightarrow y\left(\frac{1}{4}\right)+y\left(\frac{3}{4}\right)=\frac{1}{4}+1+\frac{3}{4}+1=3$$

3. Ans. (1)

$$\frac{dy}{dx}+3\sec^2 x.y=\sec^2 x$$

I.F. $= e^{3\int \sec^2 x dx}=e^{3\tan x}$

or $y.e^{3\tan x}=\int \sec^2 x.e^{3\tan x}dx$

or $y.e^{3\tan x}=\frac{1}{3}e^{3\tan x}+C$ ....(1)

Given

$y\left(\frac{\pi}{4}\right)=\frac{4}{3}$

$\therefore\ \frac{4}{3}.e^3=\frac{1}{3}e^3+C$

$\therefore\ C = e^3$

Now put $x = -\frac{\pi}{4}$ in equation (1)

$\therefore\ y.e^{-3}=\frac{1}{3}e^{-3}+e^3$

$\therefore\ y=\frac{1}{3}+e^6$

$\therefore\ y\left(-\frac{\pi}{4}\right)=\frac{1}{3}+e^6$

4. Ans. (1)

$f'(x) = 7 - \frac{3}{4}\frac{f(x)}{x} \qquad (x > 0)$

Given $f(1) \neq 4 \qquad \lim_{x\to 0^+} xf\left(\frac{1}{x}\right) = ?$

$\frac{dy}{dx} + \frac{3}{4}\frac{y}{x} = 7$ (This is LDE)

IF = $e^{\int \frac{3}{4x}dx} = e^{\frac{3}{4}\ln|x|} = x^{\frac{3}{4}}$

$y.x^{\frac{3}{4}} = \int 7.x^{\frac{3}{4}}dx$

$y.x^{\frac{3}{4}} = 7.\frac{x^{\frac{7}{4}}}{\frac{7}{4}} + C$

$f(x) = 4x + C.x^{-\frac{3}{4}}$

$f\left(\frac{1}{x}\right) = \frac{4}{x} + C.x^{\frac{3}{4}}$

$\lim_{x\to 0^+} xf\left(\frac{1}{x}\right) = \lim_{x\to 0^+}\left(4 + C.x^{\frac{7}{4}}\right) = 4$

$\therefore$ Option (1)

5. Ans. (2)

$(x^2 - y^2)\,dx + 2xy\,dy = 0$

$\frac{dy}{dx} = \frac{y^2 - x^2}{2xy}$

Put $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

Solving we get,

$\int \frac{2v}{v^2+1}dv = \int -\frac{dx}{x}$

$\ln(v^2 + 1) = -\ln x + C$

$(y^2 + x^2) = Cx$

$1 + 1 = C \Rightarrow C = 2$

$\boxed{y^2 + x^2 = 2x}$

$\therefore$ Option (2)

6. Ans. (2)

$\frac{dy}{dx} + \left(\frac{2x+1}{x}\right)y = e^{-2x}$

$I.F. = e^{\int\left(\frac{2x+1}{x}\right)dx} = e^{\int\left(2+\frac{1}{x}\right)dx} = e^{2x+\ell nx} = e^{2x}.x$

So, $y(xe^{2x}) = \int e^{-2x}.xe^{2x} + C$

$\Rightarrow xye^{2x} = \int x\,dx + C$

$\Rightarrow 2xye^{2x} = x^2 + 2C$

It passess through $\left(1, \frac{1}{2}e^{-2}\right)$ we get C = 0

$y = \frac{xe^{-2x}}{2}$

$\Rightarrow \frac{dy}{dx} = \frac{1}{2}e^{-2x}(-2x+1)$

$\Rightarrow f(x)$ is decreasing in $\left(\frac{1}{2}, 1\right)$

$y(\log_e 2) = \frac{(\log_e 2)e^{-2(\log_e 2)}}{2}$

$= \frac{1}{8}\log_e 2$

7. Ans. (4)

$x - y = t \Rightarrow \frac{dy}{dx} = 1 - \frac{dt}{dx}$

$\Rightarrow 1 - \frac{dt}{dx} = t^2 \Rightarrow \int \frac{dt}{1-t^2} = \int 1dx$

$\Rightarrow \frac{1}{2}\ell n\left(\frac{1+t}{1-t}\right) = x + \lambda$

$\Rightarrow \frac{1}{2}\ell n\left(\frac{1+x-y}{1-x+y}\right) = x + \lambda$ given $y(1) = 1$

$\Rightarrow \frac{1}{2}\ell n(1) = 1 + \lambda \Rightarrow \lambda = -1$

$\Rightarrow \ell n\left(\frac{1+x-y}{1-x+y}\right) = 2(x-1)$

$\Rightarrow -\ell n\left(\frac{1-x+y}{1+x-y}\right) = 2(x-1)$

8. Ans. (2)

$$\frac{dy}{dx} = \frac{y}{x} = \ell n x$$

$$e^{\int \frac{1}{x}dx} = x$$

$$xy = \int x\ell nx + C$$

$$\ell n x\frac{x^2}{2} - \int \frac{1}{x}\cdot\frac{x^2}{2}$$

$xy = \frac{x}{2}\ell n x - \frac{x^2}{4}$ +C, for $2y(2)= 2\ell n2 - 1$

$\Rightarrow C = 0$

$$y = \frac{x}{2}\ell n x - \frac{x}{4}$$

$$y(e) = \frac{e}{4}$$

9. Ans. (2)

$\frac{dy}{dx} = \frac{x^2 - 2y}{x}$ (Given)

$$\frac{dy}{dx} + 2\frac{y}{x} = x$$

$$\text{I.F} = e^{\int \frac{2}{x}dx} = x^2$$

$$\therefore\ y.x^2 = \int x.x^2dx + C = \frac{x^4}{y} + C$$

hence bpasses through (1, –2) $\Rightarrow$ $C = -\frac{9}{4}$

$$\therefore\ yx^2 = \frac{x^4}{4} - \frac{9}{4}$$

Now check option(s) , Which is satisly by option (ii)

10. Official Ans. by NTA (2)

Sol. $\frac{dy}{dx} + \left(\frac{2x}{x^2+1}\right)y = \frac{1}{(x^2+1)^2}$

(Linear differential equation)

$\therefore$ I.F. = $e^{\ell n\left(x^2+1\right)} = (x^2+1)$

So, general solution is $y.(x^2 + 1) = \tan^{-1}x + c$

As $y(0) = 0 \Rightarrow c = 0$

$$\therefore\ y(x) = \frac{\tan^{-1}x}{x^2+1}$$

As, $\sqrt{a}.\ y(1) = \frac{\pi}{32}$

$$\Rightarrow \sqrt{a} = \frac{1}{4} \Rightarrow a = \frac{1}{16}$$

11. Official Ans. by NTA (1)

Sol. given $\frac{dy}{dx} = \frac{2y}{x^2}$

$$\Rightarrow \int\frac{dy}{2y} = \int\frac{dx}{x^2}$$

$$\Rightarrow \frac{1}{2}\ell ny = -\frac{1}{x} + c$$

passes through centre (1,1)

$\Rightarrow c = 1$

$\Rightarrow x\ell ny = 2(x-1)$

12. Official Ans. by NTA (4)

Sol. $x\frac{dy}{dx} + 2y = x^2 : y(1) = 1$

$\frac{dy}{dx} + \left(\frac{2}{x}\right)y = x$ (LDE in y)

$$IF = e^{\int\frac{2}{x}dx} = e^{2\ell nx} = x^2$$

$$y.\left(x^2\right) = \int x.x^2dx = \frac{x^4}{4} + C$$

y(1) =1

$$1 = \frac{1}{4} + C \Rightarrow C = 1 - \frac{1}{4} = \frac{3}{4}$$

$$yx^2 = \frac{x^4}{4} + \frac{3}{4}$$

$$y = \frac{x^2}{4} + \frac{3}{4x^2}$$

13. Official Ans. by NTA (3)

Sol. $\frac{dy}{dx} - y\tan x = 6x\sec x$

$y\left(\frac{\pi}{3}\right) = 0\,;\, y\left(\frac{\pi}{6}\right) = 7$

$e^{\int pdx} = e^{-\int \tan x dx} = e^{\ell n \cos x} = \cos x$

$y \cdot \cos x = \int 6x\sec x \cos x\, dx$

$y.\cos x = \frac{6x^2}{2} + C$

$y = 3x^2 \sec x + C\sec x$

$0 = 3\cdot\frac{\pi^2}{9}\cdot(2) + C(2)$

$2C = \frac{-2\pi^2}{3} \Rightarrow \boxed{C = -\frac{\pi^2}{3}}$

$y(\pi/6) = 3\cdot\frac{\pi^2}{36}\cdot\left(\frac{2}{\sqrt{3}}\right) + \left(\frac{2}{\sqrt{3}}\right)\cdot\left(-\frac{\pi^2}{3}\right)$

$\Rightarrow\ y = -\frac{\pi^2}{2\sqrt{3}}$

14. Official Ans. by NTA (3)

Sol. $\frac{dy}{dx} = (\tan x - y)\sec^2 x$

Now, put $\tan x = t \Rightarrow \frac{dt}{dx} = \sec^2 x$

So $\frac{dy}{dt} + y = t$

On solving, we get $ye^t = e^t(t-1) + c$

$\Rightarrow y = (\tan x - 1) + ce^{-\tan x}$

$\Rightarrow y(0) = 0 \Rightarrow c = 1$

$\Rightarrow y = \tan x - 1 + e^{-\tan x}$

So $y\left(-\frac{\pi}{4}\right) = e - 2$

15. Official Ans. by NTA (2)

Sol. $\frac{dy}{dx} + y(\tan x) = 2x + x^2\tan x$

$I.F = e^{\int \tan x\, dx} = e^{\ln .\sec x} = \sec x$

$\therefore\ y \cdot \sec x = \int\left(2x + x^2\tan x\right)\sec x.dx$

$= \int 2x\sec x\, dx + \int x^2(\sec x.\tan x)dx$

$y\sec x = x^2\sec x + \lambda$

$\Rightarrow y = x^2 + \lambda\cos x$

$y(0) = 0 + \lambda = 1 \qquad \Rightarrow \lambda = 1$

$y = x^2 + \cos x$

$y\left(\frac{\pi}{4}\right) = \frac{\pi^2}{16} + \frac{1}{\sqrt{2}}$

$y\left(-\frac{\pi}{4}\right) = \frac{\pi^2}{16} + \frac{1}{\sqrt{2}}$

$y'(x) = 2x - \sin x$

$y'\left(\frac{\pi}{4}\right) = \frac{\pi}{2} - \frac{1}{\sqrt{2}}$

$y'\left(\frac{-\pi}{4}\right) = \frac{-\pi}{2} + \frac{1}{\sqrt{2}}$

$y'\left(\frac{\pi}{4}\right) - y'\left(\frac{-\pi}{4}\right) = \pi - \sqrt{2}$

16. Official Ans. by NTA (4)

Sol. $y^2dx + xdy = \frac{dy}{y}$

$$\frac{dx}{dy}+\frac{x}{y^2}=\frac{1}{y^3}$$

$$\text{IF} = e^{\int \frac{1}{y^2}dy} = e^{-\frac{1}{y}}$$

$$e^{-\frac{1}{y}}\cdot x=\int e^{-\frac{1}{y}}\cdot\frac{1}{y^3}dy+C$$

$$xe^{-\frac{1}{y}}=e^{-\frac{1}{y}}+\frac{e^{-\frac{1}{y}}}{y}+C$$

$$C = -\frac{1}{e}$$

$x = \frac{3}{2}-\frac{1}{\sqrt{e}}$ when $y = 2$

17. Official Ans. by NTA (1)

Sol. $xy\frac{dy}{dx}-y^2+x^3=0$

put $y^2=k \Rightarrow y\frac{dy}{dx}=\frac{1}{2}\frac{dk}{dx}$

$\therefore$ given differential equation becomes

$$\frac{dk}{dx}+k\left(-\frac{2}{x}\right)=-2x^2$$

$$\text{I.F.} = e^{\int -\frac{2}{x}dx}=\frac{1}{x^2}$$

$\therefore$ solution is $k\cdot\frac{1}{x^2}=\int -2x^2\cdot\frac{1}{x^2}dx+\lambda$

$$y^2 + 2x^3 = \lambda x^2$$

take $\lambda = -c$ (integration constant)