DIFFERENTIABILITY

1. Let f be a differentiable function from R to R such that $|f(x)-f(y)| \le 2|x-y|^{\frac{3}{2}}$, for all x, y ε R. If $f(0) = 1$ then $\int_0^1 f^2(x)dx$ is equal to

(1) 0 (2) $\frac{1}{2}$ (3) 2 (4) 1

2. Let $f(x)=\begin{cases}\max\{|x|,x^2\}, & |x|\le 2\\ 8-2|x|, & 2<|x|\le 4\end{cases}$

Let S be the set of points in the interval (–4,4) at which f is not differentiable. Then S:
(1) is an empty set
(2) equals {–2, –1, 1, 2}
(3) equals {–2, –1, 0, 1, 2}
(4) equals {–2, 2}

3. Let f : (–1,1)→R be a function defined by $f(x) = \max\left\{-|x|,-\sqrt{1-x^2}\right\}$. If K be the set of all points at which f is not differentiable, then K has exactly :
(1) Three elements (2) One element
(3) Five elements (4) Two elements

4. Let K be the set of all real values of x where the function $f(x) = \sin|x| - |x| + 2(x-\pi)\cos|x|$ is not differentiable. Then the set K is equal to :-
(1) {π} (2) {0}
(3) φ (an empty set) (4) {0, π}

5. Let S be the set of all points in (–π,π) at which the function, f(x) = min {sinx, cosx} is not differentiable. Then S is a subset of which of the following?

(1) $\left\{-\frac{3\pi}{4},-\frac{\pi}{4},\frac{3\pi}{4},\frac{\pi}{4}\right\}$

(2) $\left\{-\frac{3\pi}{4},-\frac{\pi}{2},\frac{\pi}{2},\frac{3\pi}{4}\right\}$

(3) $\left\{-\frac{\pi}{2},-\frac{\pi}{4},\frac{\pi}{4},\frac{\pi}{2}\right\}$

(4) $\left\{-\frac{\pi}{4},0,\frac{\pi}{4}\right\}$

6. Let $f(x) = \begin{cases}-1, & -2 \le x < 0\\ x^2-1, & 0 \le x \le 2\end{cases}$ and $g(x) = |f(x)| + f(|x|)$. Then, in the interval 1(–2, 2), g is :-
(1) differentiable at all points
(2) not differentiable at two points
(3) not continuous
(4) not differentiable at one point

7. Let $f(x) = 15 - |x-10|$; $x \in R$. Then the set of all values of x, at which the function, $g(x) = f(f(x))$ is not differentiable, is :
(1) {5,10,15,20} (2) {10,15}
(3) {5,10,15} (4) {10}

8. Let $f : R \to R$ be differentiable at $c \in R$ and $f(c) = 0$. If $g(x) = |f(x)|$, then at x = c, g is :
(1) differentiable if $f'(c) = 0$
(2) not differentiable
(3) differentiable if $f'(c) \neq 0$
(4) not differentiable if $f'(c) = 0$

SOLUTION

1. Ans. (4)

$|f(x) - f(y)| \le 2|x - y|^{3/2}$

divide both sides by $|x - y|$

$$\left|\frac{f(x)-f(y)}{x-y}\right| \le 2.|x-y|^{1/2}$$

apply limit $x \to y$

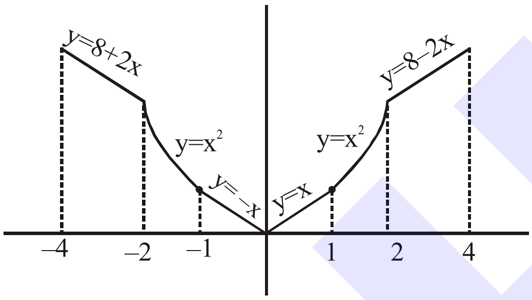
$|f'(y)| \le 0 \Rightarrow f'(y) = 0$

$\Rightarrow f(y) = c \Rightarrow f(x) = 1$

$$\int_0^1 1.dx = 1$$

2. Ans. (3)

$$f(x) = \begin{cases} 8+2x, & -4 \le x < -2 \\ x^2, & -2 \le x \le -1 \\ |x|, & -1 < x < 1 \\ x^2, & 1 \le x \le 2 \\ 8-2x, & 2 < x \le 4 \end{cases}$$



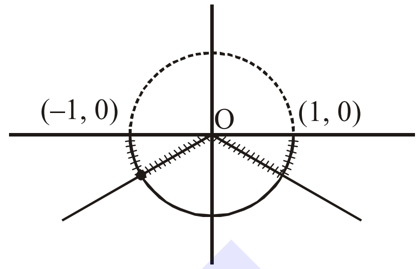
$f(x)$ is not differentiable at $x = \{-2,-1,0,1,2\}$

$\Rightarrow S = \{-2, -1, 0, 1, 2\}$

3. Ans. (1)

$f : (-1, 1) \to R$

$$f(x) = \max\left\{-|x|, -\sqrt{1-x^2}\right\}$$



Non-derivable at 3 points in $(-1, 1)$

Option (1)

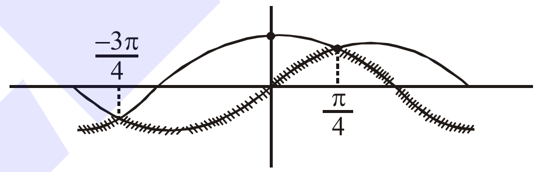
4. Ans. (3)

$f(x) = \sin|x| - |x| + 2(x - \pi)\cos x$

$\because \sin|x| - |x|$ is differentiable function at x=0

$\therefore\ k = \phi$

5. Ans. (1)



6. Ans. (4)

$$|f(x)| = \begin{cases} 1 & , \quad -2 \le x < 0 \\ 1-x^2 & , \quad 0 \le x < 1 \\ x^2-1 & , \quad 1 \le x \le 2 \end{cases}$$

and $f(|x|) = x^2 - 1,\ x \in [-2, 2]$

$$\text{Hence } g(x) = \begin{cases} x^2 & , \quad x \in [-2,0) \\ 0 & , \quad x \in [0,1) \\ 2(x^2-1) & , \quad x \in [1,2] \end{cases}$$

It is not differentiable at x = 1

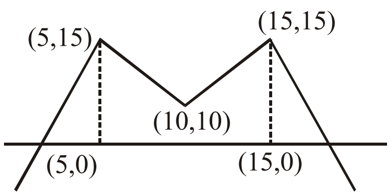
7. Official Ans. by NTA (3)

Sol. $f(x) = 15 - |x - 10|, x \in R$

$f(f(x)) = 15 - |f(x) - 10|$

$= 15 - |15 - |x - 10| - 10|$

$= 15 - |5 - |x - 10||$



x = 5, 10, 15 are points of non differentiability

Aliter :

At x = 10 $f(x)$ is non differentiable

also, when $15 - |x - 10| = 10$

$\Rightarrow x = 5, 15$

$\therefore$ non differentiability points are {5, 10, 15}

8. Official Ans. by NTA (1)

Sol. $g'(c) = \lim_{h \to 0} \frac{|f(c+h)| - |f(c)|}{h}$

$= \lim_{h \to 0} \frac{|f(c+h)|}{h} = \lim_{h \to 0} \frac{|f(c+h) - f(c)|}{h}$

$= \lim_{h \to 0} \left|\frac{f(c+h) - f(c)}{h}\right| \frac{|h|}{h}$

$= \lim_{h \to 0} |f'(c)| \frac{|h|}{h} = 0$, if f '(c) = 0

i.e., g(x) is differentiable at x = c, if f '(c) = 0