DETERMINANT

1. The system of linear equations.

$x + y + z = 2$

$2x + 3y + 2z = 5$

$2x + 3y + (a^2 - 1)z = a + 1$

(1) has infinitely many solutions for a = 4

(2) is inconsistent when $|a| = \sqrt{3}$

(3) is inconsistent when a = 4

(4) has a unique solution for $|a| = \sqrt{3}$

2. If the system of linear equations

$x - 4y + 7z = g$

$3y - 5z = h$

$-2x + 5y - 9z = k$

is consistent, then :

(1) $g + h + k = 0$

(2) $2g + h + k = 0$

(3) $g + h + 2k = 0$

(4) $g + 2h + k = 0$

3. If the system of equations

$x+y+z = 5$

$x+2y+3z = 9$

$x+3y+\alpha z = \beta$

has infinitely many solutions, then $\beta - \alpha$ equals:

(1) 5 (2) 18 (3) 21 (4) 8

4. Let $d \in R$, and

$$A = \begin{bmatrix} -2 & 4+d & (\sin\theta)-2 \\ 1 & (\sin\theta)+2 & d \\ 5 & (2\sin\theta)-d & (-\sin\theta)+2+2d \end{bmatrix},$$

$\theta \in [0, 2\pi]$. If the minimum value of det(A) is 8, then a value of d is :

(1) –7 (2) $2\left(\sqrt{2}+2\right)$

(3) –5 (4) $2\left(\sqrt{2}+1\right)$

5. Let $a_1, a_2, a_3, \ldots, a_{10}$ be in G.P. with $a_i > 0$ for $i = 1, 2, \ldots, 10$ and S be the set of pairs (r,k), r, $k \in N$ (the set of natural numbers) for which

$$\begin{vmatrix} \log_e a_1^r a_2^k & \log_e a_2^r a_3^k & \log_e a_3^r a_4^k \\ \log_e a_4^r a_5^k & \log_e a_5^r a_6^k & \log_e a_6^r a_7^k \\ \log_e a_7^r a_8^k & \log_e a_8^r a_9^k & \log_e a_9^r a_{10}^k \end{vmatrix} = 0$$

Then the number of elements in S, is :

(1) Infinitely many (2) 4

(3) 10 (4) 2

6. The number of values of $\theta \in (0, \pi)$ for which the system of linear equations

$x + 3y + 7z = 0$

$-x + 4y + 7z = 0$

$(\sin 3\theta)x + (\cos 2\theta)y + 2z = 0$

has a non-trivial solution, is :

(1) One (2) Three (3) Four (4) Two

7. If the system of linear equations

$2x + 2y + 3z = a$

$3x - y + 5z = b$

$x - 3y + 2z = c$

where a, b, c are non-zero real numbers, has more then one solution, then :

(1) $b - c - a = 0$ (2) $a + b + c = 0$

(3) $b + c - a = 0$ (4) $b - c + a = 0$

8. If $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)$ $(x + a + b + c)^2$, $x \neq 0$ and $a + b + c \neq 0$, then x is equal to :-

(1) $-(a + b + c)$ (2) $2(a + b + c)$

(3) abc (4) $-2(a + b + c)$

9. An ordered pair(α, β) for which the system of linear equations

$(1+\alpha)x + \beta y + z = 2$

$\alpha x + (1+\beta)y + z = 3$

$\alpha x + \beta y + 2z = 2$ has a unique solution is

(1) (1,–3) (2) (–3,1)

(3) (2, 4) (4) (–4, 2)

10. The set of all values of λ for which the system of linear equations.

$x - 2y - 2z = \lambda x$

$x + 2y + z = \lambda y$

$-x - y = \lambda z$

has a non-trivial solution.

(1) contains more than two elements

(2) is a singleton

(3) is an empty set

(4) contains exactly two elements

11. The greatest value of $c \in R$ for which the system of linear equations

$x - cy - cz = 0$

$cx - y + cz = 0$

$cx + cy - z = 0$

has a non-trivial solution, is :

(1) $\frac{1}{2}$ (2) -1

(3) 0 (4) 2

12. If the system of linear equations

$x - 2y + kz = 1$

$2x + y + z = 2$

$3x - y - kz = 3$

has a solution (x,y,z), $z \neq 0$, then (x,y) lies on the straight line whose equation is :

(1) $3x - 4y - 1 = 0$ (2) $3x - 4y - 4 = 0$

(3) $4x - 3y - 4 = 0$ (4) $4x - 3y - 1 = 0$

13. If the system of equations $2x + 3y - z = 0$, $x + ky - 2z = 0$ and $2x - y + z = 0$ has a non-trival solution (x, y, z), then $\frac{x}{y}+\frac{y}{z}+\frac{z}{x}+k$ is equal to:-

(1) $\frac{3}{4}$ (2) -4 (3) $\frac{1}{2}$ (4) $-\frac{1}{4}$

14. If $\Delta_1 = \begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix}$ and

$\Delta_2 = \begin{vmatrix} x & \sin 2\theta & \cos 2\theta \\ -\sin 2\theta & -x & 1 \\ \cos 2\theta & 1 & x \end{vmatrix}$, $x \neq 0$; then for all $\theta \in \left(0, \frac{\pi}{2}\right)$:

(1) $\Delta_1 - \Delta_2 = x(\cos 2\theta - \cos 4\theta)$

(2) $\Delta_1 + \Delta_2 = -2x^3$

(3) $\Delta_1 - \Delta_2 = -2x^3$

(4) $\Delta_1 + \Delta_2 = -2(x^3 + x - 1)$

15. Let λ be a real number for which the system of linear equations

$x + y + z = 6$

$4x + \lambda y - \lambda z = \lambda - 2$

$3x + 2y - 4z = -5$

has infinitely many solutions. Then λ is a root of the quadratic equation.

(1) $\lambda^2 - 3\lambda - 4 = 0$ (2) $\lambda^2 - \lambda - 6 = 0$

(3) $\lambda^2 + 3\lambda - 4 = 0$ (4) $\lambda^2 + \lambda - 6 = 0$

16. The sum of the real roots of the equatuion

$\begin{vmatrix} x & -6 & -1 \\ 2 & -3x & x-3 \\ -3 & 2x & x+2 \end{vmatrix} = 0$, is equal to :

(1) 6 (2) 1 (3) 0 (4) – 4

17. A value of $\theta \in (0, \pi/3)$, for which

$\begin{vmatrix} 1+\cos^2\theta & \sin^2\theta & 4\cos 6\theta \\ \cos^2\theta & 1+\sin^2\theta & 4\cos 6\theta \\ \cos^2\theta & \sin^2\theta & 1+4\cos 6\theta \end{vmatrix} = 0$, is :

(1) $\frac{7\pi}{24}$ (2) $\frac{\pi}{18}$ (3) $\frac{\pi}{9}$ (4) $\frac{7\pi}{36}$

18. If [x] denotes the greatest integer $\leq$ x, then the system of linear equations $[\sin\theta]x + [-\cos\theta]y = 0$

$[\cot\theta]x + y = 0$

(1) have infinitely many solutions if $\theta \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right) \cup \left(\pi, \frac{7\pi}{6}\right)$

(2) have infinitely many solutions if $\theta \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$ and has a unique solution if $\theta \in \left(\pi, \frac{7\pi}{6}\right)$

(3) has a unique solution if $\theta \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$ and have infinitely many solutions if $\theta \in \left(\pi, \frac{7\pi}{6}\right)$

(4) has a unique solution if $\theta \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right) \cup \left(\pi, \frac{7\pi}{6}\right)$

SOLUTION

1. Ans. (2)

$$D=\begin{vmatrix}1 & 1 & 1\\ 2 & 3 & 2\\ 2 & 3 & a^2-1\end{vmatrix}=a^2-3$$

$$D_1=\begin{vmatrix}2 & 1 & 1\\ 5 & 3 & 2\\ a+1 & 3 & a^2-1\end{vmatrix}=a^2-a+1$$

$$D_2=\begin{vmatrix}1 & 2 & 1\\ 2 & 5 & 2\\ 2 & a+1 & a^2-1\end{vmatrix}=a^2-3$$

$$D_3=\begin{vmatrix}1 & 1 & 2\\ 2 & 3 & 5\\ 2 & 3 & a+1\end{vmatrix}=a-4$$

D = 0 at |a| = $\sqrt{3}$ but D_3 = $\pm\sqrt{3}-4\neq 0$

So the system is Inconsistant for $|a|=\sqrt{3}$

2. Ans. (2)

$P_1 \equiv x - 4y + 7z - g = 0$

$P_2 \equiv 3x - 5y - h = 0$

$P_3 \equiv -2x + 5y - 9z - k = 0$

Here $\Delta = 0$

$2P_1 + P_2 + P_3 = 0$ when $2g + h + k = 0$

3. Ans. (4)

$$D=\begin{vmatrix}1 & 1 & 1\\ 1 & 2 & 3\\ 1 & 3 & \alpha\end{vmatrix}=\begin{vmatrix}1 & 1 & 1\\ 0 & 1 & 2\\ 0 & 2 & \alpha-1\end{vmatrix}=(\alpha-1)-4=(\alpha-5)$$

for infinite solutions $D = 0 \Rightarrow \alpha = 5$

$$D_x=0\Rightarrow\begin{vmatrix}5 & 1 & 1\\ 9 & 2 & 3\\ \beta & 3 & 5\end{vmatrix}=0$$

$$\Rightarrow\begin{vmatrix}0 & 0 & 1\\ -1 & -1 & 3\\ \beta-15 & -2 & 5\end{vmatrix}=0$$

$\Rightarrow 2+\beta-15=0\Rightarrow\beta-13=0$

on $\beta = 13$ we get $D_y = D_z = 0$

$\alpha = 5,\ \beta = 13$

4. Ans. (3)

$$\det A=\begin{vmatrix}-2 & 4+d & \sin\theta-2\\ 1 & \sin\theta+2 & d\\ 5 & 2\sin\theta-d & -\sin\theta+2+2d\end{vmatrix}$$

$(R_1 \to R_1 + R_3 - 2R_2)$

$$=\begin{vmatrix}1 & 0 & 0\\ 1 & \sin\theta+2 & d\\ 5 & 2\sin\theta-d & 2+2d-\sin\theta\end{vmatrix}$$

$=(2+\sin\theta)(2+2d-\sin\theta)-d(2\sin\theta-d)$

$=4 + 4d - 2\sin\theta + 2\sin\theta+2d\sin\theta - \sin^2\theta$
$\qquad -2d\sin\theta+d^2$

$=d^2 + 4d + 4 - \sin^2\theta$

$=(d + 2)^2 - \sin^2\theta$

For a given d, minimum value of

$\det(A) = (d + 2)^2 - 1 = 8$

$\Rightarrow d = 1$ or -5

5. Ans. (1)

Apply

$C_3 \to C_3 - C_2$

$C_2 \to C_2 - C_1$

We get $D = 0$

Option (1)

6. Ans. (4)

$$\begin{vmatrix}1 & 3 & 7\\ -1 & 4 & 7\\ \sin 3\theta & \cos 2\theta & 2\end{vmatrix}=0$$

$(8 - 7\cos 2\theta) - 3(-2 - 7\sin 3\theta)$
$\qquad + 7(-\cos 2\theta - 4\sin 3\theta) = 0$

$14 - 7\cos 2\theta + 21\sin 3\theta - 7\cos 2\theta$
$\qquad - 28\sin 3\theta = 0$

$14 - 7\sin 3\theta - 14\cos 2\theta = 0$

$14 - 7(3\sin\theta - 4\sin^3\theta) - 14(1 - 2\sin^2\theta) = 0$

$-21\sin\theta + 28\sin^3\theta + 28\sin^2\theta = 0$

$7\sin\theta\,[-3 + 4\sin^2\theta + 4\sin\theta] = 0$

$\sin\theta,\ 4\sin^2\theta + 6\sin\theta - 2\sin\theta - 3 = 0$

$2\sin\theta(2\sin\theta + 3) - 1(2\sin\theta + 3) = 0$

$\sin\theta = \frac{-3}{2};\ \sin\theta = \frac{1}{2}$

Hence, 2 solutions in $(0, \pi)$

Option (4)

7. Ans. (1)

$P_1 : 2x + 2y + 3z = a$

$P_2 : 3x - y + 5z = b$

$P_3 : x - 3y + 2z = c$

We find

$P_1 + P_3 = P_2 \Rightarrow a + c = b$

8. Ans. (4)

$$\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

$R_1 \to R_1 + R_2 + R_3$

$$= \begin{vmatrix} a+b+c & a+b+c & a+b+c \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

$$= (a+b+c)\begin{vmatrix} 1 & 0 & 0 \\ 2b & -(a+b+c) & 0 \\ 2c & 2c & c-a-b \end{vmatrix}$$

$= (a + b + c)(a + b + c)^2$

$\Rightarrow x = -2(a + b + c)$

9. Ans. (3)

For unique solution

$$\Delta \neq 0 \Rightarrow \begin{vmatrix} 1+\alpha & \beta & 1 \\ \alpha & 1+\beta & 1 \\ \alpha & \beta & 2 \end{vmatrix} \neq 0$$

$$\begin{vmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ \alpha & \beta & 2 \end{vmatrix} \neq 0 \Rightarrow \alpha + \beta \neq -2$$

10. Ans. (2)

$$\begin{vmatrix} \lambda-1 & 2 & 2 \\ 1 & 2-\lambda & 1 \\ 1 & 1 & 1 \end{vmatrix} = 0 \Rightarrow (\lambda-1)^3 = 0 \Rightarrow \lambda = 1$$

11. Official Ans. by NTA (1)

Sol. For non-trivial solution

D = 0

$$\begin{vmatrix} 1 & -c & -c \\ c & -1 & c \\ c & c & -1 \end{vmatrix} = 0 \Rightarrow 2c^3 - 3c^2 - 1 = 0$$

$\Rightarrow \quad (c + 1)^2 (2c - 1) = 0$

$\therefore$ Greatest value of c is $\frac{1}{2}$

12. Official Ans. by NTA (3)

Sol. $x - 2y + kz = 1 \quad ...(1)$

$2x + y + z = 2 \quad ...(2)$

$3x - y - kz = 3 \quad ...(3)$

(1) +(3)

$\Rightarrow 4x - 3y = 4$

13. Official Ans. by NTA (3)

Sol. $$\begin{vmatrix} 2 & 3 & -1 \\ 1 & K & -2 \\ 2 & -1 & 1 \end{vmatrix} = 0$$

By solving $K = \frac{9}{2}$

$2x + 3y - z = 0 \quad \ldots(1)$

$x + \frac{9}{2}y - 2z = 0 \quad \ldots(2)$

$2x - y + z = 0 \quad \ldots(3)$

(1)-(3) $\Rightarrow 4y - 2z = 0$

$2y = z \quad \ldots(4)$

$\boxed{\frac{y}{z} = \frac{1}{2}} \quad \ldots(5)$

put z from eqn. (4) into (1)

$2x + 3y - 2y = 0$

$2x + y = 0$

$\boxed{\frac{x}{y} = -\frac{1}{2}} \quad \ldots(6)$

$\frac{(6)}{(5)} \boxed{\frac{z}{x} = -4}$

$\frac{x}{y} + \frac{y}{z} + \frac{z}{x} + K = \frac{1}{2}$

14. Official Ans. by NTA (2)

Sol. $$\Delta_1 = f(\theta) = \begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix} = -x^3$$

and $$\Delta_2 = f(2\theta) = \begin{vmatrix} x & \sin 2\theta & \cos 2\theta \\ -\sin 2\theta & -x & 1 \\ \cos 2\theta & 1 & x \end{vmatrix} = -x^3$$

So $\Delta_1 + \Delta_2 = -2x^3$

15. Official Ans. by NTA (2)

Sol. D = 0

$$\begin{vmatrix} 1 & 1 & 1 \\ 4 & \lambda & -\lambda \\ 3 & 2 & -4 \end{vmatrix} = 0 \Rightarrow \lambda = 3$$

16. Official Ans. by NTA (3)

Sol. By expansion, we get

$-5x^3 + 30x - 30 + 5x = 0$

$\Rightarrow -5x^3 + 35x - 30 = 0$

$\Rightarrow x^3 - 7x + 6 = 0$, All roots are real

So, sum of roots = 0

17. Official Ans. by NTA (3)

Sol. $R_1 \to R_1 - R_2$

$$\begin{vmatrix} 1 & -1 & 0 \\ \cos^2\theta & 1+\sin^2\theta & 4\cos 6\theta \\ \cos^2\theta & \sin^2\theta & 1+4\cos 6\theta \end{vmatrix} = 0$$

$R_2 \to R_2 - R_3$

$$\begin{vmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ \cos^2\theta & \sin^2\theta & 1+4\cos 6\theta \end{vmatrix} = 0$$

$\Rightarrow (1 + 4\cos 6\theta) + \sin^2\theta + 1(\cos^2\theta) = 0$

$1 + 2\cos 6\theta = 0 \Rightarrow \cos 6\theta = -1/2$

$$6\theta = \frac{2\pi}{3} \Rightarrow \boxed{\theta = \frac{\pi}{9}}$$

18. Official Ans. by NTA (2)

Sol. $[\sin\theta]x + [-\cos\theta]y = 0$ and $[\cos\theta]x + y = 0$

for infinite many solution

$$\begin{vmatrix} [\sin\theta] & [-\cos\theta] \\ [\cos\theta] & 1 \end{vmatrix} = 0$$

ie $[\sin\theta] = -[\cos\theta][\cot\theta]$ (1)

when $\theta \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right) \Rightarrow \sin\theta \in \left(0, \frac{1}{2}\right)$

$-\cos\theta \in \left(0, \frac{1}{2}\right)$

$\cot\theta \in \left(-\frac{1}{\sqrt{3}}, 0\right)$

when $\theta \in \left(\pi, \frac{7\pi}{6}\right) \Rightarrow \sin\theta \in \left(-\frac{1}{2}, 0\right)$

$-\cos\theta \in \left(\frac{\sqrt{3}}{2}, 1\right)$

$\cot\theta \in \left(\sqrt{3}, \infty\right)$

when $\theta \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$ then equation (i) satisfied there fore infinite many solution.

when $\theta \in \left(\pi, \frac{7\pi}{6}\right)$ then equation (i) not satisfied there fore infinite unique solution.