DEFINITE INTEGRATION

1. The value of $\int_0^{\pi} |\cos x|^3\,dx$

(1) 2/3 (2) 0 (3) –4/3 (4) 4/3

2. If $\int_0^{\pi/3} \frac{\tan\theta}{\sqrt{2k\sec\theta}}d\theta = 1-\frac{1}{\sqrt{2}},(k>0)$, then the value of k is :

(1) 2 (2) $\frac{1}{2}$ (3) 4 (4) 1

3. Let $I=\int_a^b (x^4-2x^2)dx$. If I is minimum then the ordered pair (a, b) is :

(1) $\left(-\sqrt{2},0\right)$ (2) $\left(-\sqrt{2},\sqrt{2}\right)$

(3) $\left(0,\sqrt{2}\right)$ (4) $\left(\sqrt{2},-\sqrt{2}\right)$

4. The value of $\int_{-\pi/2}^{\pi/2} \frac{dx}{[x]+[\sin x]+4}$, where [t] denotes the greatest integer less than or equal to t, is :

(1) $\frac{1}{12}(7\pi+5)$

(2) $\frac{3}{10}(4\pi-3)$

(3) $\frac{1}{12}(7\pi-5)$

(4) $\frac{3}{20}(4\pi-3)$

5. If $\int_0^x f(t)dt = x^2+\int_x^1 t^2 f(t)dt$, then f'(1/2) is :

(1) $\frac{6}{25}$ (2) $\frac{24}{25}$ (3) $\frac{18}{25}$ (4) $\frac{4}{5}$

6. The value of the integral $\int_{-2}^{2} \frac{\sin^2 x}{\left[\frac{x}{\pi}\right]+\frac{1}{2}}dx$

(where [x] denotes the greatest integer less than or equal to x) is :

(1) 4 (2) 4 – sin4 (3) sin 4 (4) 0

7. The integral $\int_{\pi/6}^{\pi/4} \frac{dx}{\sin 2x(\tan^5 x+\cot^5 x)}$ equals :

(1) $\frac{1}{10}\left(\frac{\pi}{4}-\tan^{-1}\left(\frac{1}{9\sqrt{3}}\right)\right)$

(2) $\frac{1}{5}\left(\frac{\pi}{4}-\tan^{-1}\left(\frac{1}{3\sqrt{3}}\right)\right)$

(3) $\frac{\pi}{10}$

(4) $\frac{1}{20}\tan^{-1}\left(\frac{1}{9\sqrt{3}}\right)$

8. Let f and g be continuous functions on [0, a] such that f(x) = f(a–x) and g(x)+g(a–x)=4, then $\int_0^a f(x)g(x)dx$ is equal to :-

(1) $4\int_0^a f(x)dx$ (2) $2\int_0^a f(x)dx$

(3) $-3\int_0^a f(x)dx$ (4) $\int_0^a f(x)dx$

9. The integral $\int_1^e \left\{\left(\frac{x}{e}\right)^{2x}-\left(\frac{e}{x}\right)^{x}\right\}\log_e x\,dx$ is equal to :

(1) $\frac{1}{2}-e-\frac{1}{e^2}$ (2) $\frac{3}{2}-\frac{1}{e}-\frac{1}{2e^2}$

(3) $-\frac{1}{2}+\frac{1}{e}-\frac{1}{2e^2}$ (4) $\frac{3}{2}-e-\frac{1}{2e^2}$

10. $\lim_{n\to\infty}\left(\frac{n}{n^2+1^2}+\frac{n}{n^2+2^2}+\frac{n}{n^2+3^2}+....+\frac{1}{5n}\right)$ is equal to :

(1) $\frac{\pi}{4}$ (2) $\tan^{-1}(2)$ (3) $\tan^{-1}(3)$ (4) $\frac{\pi}{2}$

11. If $f(x)=\frac{2-x\cos x}{2+x\cos x}$ and g(x) = $\log_e x$, (x > 0)

then the value of integral $\int_{-\pi/4}^{\pi/4} g(f(x))dx$ is :

(1) $\log_e 3$ (2) $\log_e 2$

(3) $\log_e e$ (4) $\log_e 1$

12. Let $f(x)=\int_0^x g(t)dt$, where g is a non-zero even function. If $f(x+5)=g(x)$, then $\int_0^x f(t)dt$ equals-

(1) $\int_{x+5}^{5} g(t)dt$ (2) $5\int_{x+5}^{5} g(t)dt$

(3) $\int_{5}^{x+5} g(t)dt$ (4) $2\int_{5}^{x+5} g(t)dt$

13. The value of $\int_0^{\pi/2} \frac{\sin^3 x}{\sin x+\cos x}dx$ is

(1) $\frac{\pi-2}{4}$ (2) $\frac{\pi-2}{8}$ (3) $\frac{\pi-1}{4}$ (4) $\frac{\pi-1}{2}$

14. If f : R → R is a differentiable function and f(2) = 6, then $\lim_{x\to 2}\int_6^{f(x)} \frac{2tdt}{(x-2)}$ is :-

(1) 0 (2) 2f'(2)

(3) 12f '(2) (4) 24f '(2)

15. The value of $\int_0^{2\pi}[\sin 2x(1+\cos 3x)]dx$, where [t] denotes the greatest integer function, is :

(1) -2π (2) π (3) $-\pi$ (4) 2π

16. $\lim_{n\to\infty}\left(\frac{(n+1)^{1/3}}{n^{4/3}}+\frac{(n+2)^{1/3}}{n^{4/3}}+......+\frac{(2n)^{1/3}}{n^{4/3}}\right)$ is equal to :

(1) $\frac{4}{3}(2)^{4/3}$ (2) $\frac{3}{4}(2)^{4/3}-\frac{4}{3}$

(3) $\frac{3}{4}(2)^{4/3}-\frac{3}{4}$ (4) $\frac{4}{3}(2)^{3/4}$

17. The integral $\int_{\pi/6}^{\pi/3}\sec^{2/3}x\,\text{cosec}^{4/3}x\,dx$ equal to:

(1) $3^{7/6}-3^{5/6}$ (2) $3^{5/3}-3^{1/3}$

(3) $3^{4/3}-3^{1/3}$ (4) $3^{5/6}-3^{2/3}$

18. If $\int_0^{\frac{\pi}{2}}\frac{\cot x}{\cot x+\text{cosec}x}dx=m(\pi+n)$, then m . n is equal to :

(1) –1 (2) 1 (3) $\frac{1}{2}$ (4) $-\frac{1}{2}$

19. The value of the integral $\int_0^1 x\cot^{-1}(1-x^2+x^4)dx$ is :-

(1) $\frac{\pi}{4}-\frac{1}{2}\log_e 2$ (2) $\frac{\pi}{2}-\log_e 2$

(3) $\frac{\pi}{2}-\frac{1}{2}\log_e 2$ (4) $\frac{\pi}{4}-\log_e 2$

20. Let f : R → R be a continuously differentiable function wuch that f(2) = 6 and f(2) = $\frac{1}{48}$.

If $\int_6^{f(x)}4t^3dt=(x-2)g(x)$, then $\lim_{x\to 2}g(x)$ is equal to :

(1) 24 (2) 36 (3) 12 (4) 18

21. A value of α such that

$$\int_{\alpha}^{\alpha+1}\frac{dx}{(x+\alpha)(x+\alpha+1)}=\log_e\left(\frac{9}{8}\right)$$ is :

(1) $\frac{1}{2}$ (2) 2 (3) $-\frac{1}{2}$ (4) – 2

SOLUTION

1. Ans. (4)

$$\int_0^{\pi} |\cos x|^3\,dx = \int_0^{\pi/2} \cos^3 x\,dx - \int_{\pi/2}^{\pi} \cos^3 x\,dx$$

$$= \int_0^{\pi/2} \left(\frac{\cos 3x + 3\cos x}{4}\right)dx - \int_{\pi/2}^{\pi}\left(\frac{\cos 3x + 3\cos x}{4}\right)dx$$

$$= \frac{1}{4}\left[\left(\frac{\sin 3x}{3} + 3\sin x\right)_0^{\pi/2} - \left(\frac{\sin 3x}{3} + 3\sin x\right)_{\pi/2}^{\pi}\right]$$

$$= \frac{1}{4}\left[\left(\frac{-1}{3}+3\right) - (0+0) - \left\{(0+0) - \left(\frac{-1}{3}+3\right)\right\}\right]$$

$$= \frac{4}{3}$$

2. Ans. (1)

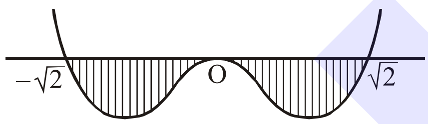
$$\frac{1}{\sqrt{2k}}\int_0^{\pi/3} \frac{\tan\theta}{\sqrt{\sec\theta}}\,d\theta = \frac{1}{\sqrt{2k}}\int_0^{\pi/3}\frac{\sin\theta}{\sqrt{\cos\theta}}\,d\theta$$

$$= -\frac{1}{\sqrt{2k}}\,2\sqrt{\cos\theta}\Big|_0^{\pi/3} = -\frac{\sqrt{2}}{\sqrt{k}}\left(\frac{1}{\sqrt{2}} - 1\right)$$

given it is $1 - \frac{1}{\sqrt{2}} \Rightarrow k = 2$

3. Ans. (2)

Let $f(x) = x^2(x^2 - 2)$



As long as $f(x)$ lie below the x-axis, definite integral will remain negative,

so correct value of (a, b) is $(-\sqrt{2}, \sqrt{2})$ for minimum of I

4. Ans. (4)

$$I = \int_{-\pi/2}^{\pi/2} \frac{dx}{[x] + [\sin x] + 4}$$

$$= \int_{-\pi/2}^{-1}\frac{dx}{-2-1+4} + \int_{-1}^{0}\frac{dx}{-1-1+4} + \int_0^1 \frac{dx}{0+0+4} + \int_1^{\pi/2}\frac{dx}{1+0+4}$$

$$\int_{-\pi/2}^{-1}\frac{dx}{1} + \int_{-1}^{0}\frac{dx}{2} + \int_0^1\frac{dx}{4} + \int_1^{\pi/2}\frac{dx}{5}$$

$$\left(-1+\frac{\pi}{2}\right) + \frac{1}{2}(0+1) + \frac{1}{4} + \frac{1}{5}\left(\frac{\pi}{2} - 1\right)$$

$$-1 + \frac{1}{2} + \frac{1}{4} - \frac{1}{5} + \frac{\pi}{2} + \frac{\pi}{10}$$

$$\frac{-20+10+5-4}{20} + \frac{6\pi}{10}$$

$$\frac{-9}{20} + \frac{3\pi}{5}$$

Option (4)

5. Ans. (2)

$$\int_0^x f(t)\,dt = x^2 + \int_x^1 t^2 f(t)\,dt \qquad f'\left(\frac{1}{2}\right) = ?$$

Differentiate w.r.t. 'x'

$f(x) = 2x + 0 - x^2 f(x)$

$$f(x) = \frac{2x}{1+x^2} \Rightarrow f'(x) = \frac{(1+x^2)2 - 2x(2x)}{(1+x^2)^2}$$

$$f'(x) = \frac{2x^2 - 4x^2 + 2}{(1+x^2)^2}$$

$$f'\left(\frac{1}{2}\right) = \frac{2 - 2\left(\frac{1}{4}\right)}{\left(1+\frac{1}{4}\right)^2} = \frac{\left(\frac{3}{2}\right)}{\frac{25}{16}} = \frac{48}{50} = \frac{24}{25}$$

Option (2)

6. Ans. (4)

$$I=\int_{-2}^{2}\frac{\sin^2 x}{\left[\frac{x}{\pi}\right]+\frac{1}{2}}dx$$

$$I=\int_{0}^{2}\left(\frac{\sin^2 x}{\left[\frac{x}{\pi}\right]+\frac{1}{2}}+\frac{\sin^2(-x)}{\left[-\frac{x}{\pi}\right]+\frac{1}{2}}\right)dx$$

$$\left(\left[\frac{x}{\pi}\right]+\left[-\frac{x}{\pi}\right]=-1 \text{ as } \quad x\neq n\pi\right)$$

$$I=\int_{0}^{2}\left(\frac{\sin^2 x}{\left[\frac{x}{\pi}\right]+\frac{1}{2}}+\frac{\sin^2 x}{-1-\left[\frac{x}{\pi}\right]+\frac{1}{2}}\right)dx=0$$

7. Ans. (1)

$$I=\int_{\pi/6}^{\pi/4}\frac{dx}{\sin 2x\left(\tan^5 x+\cot^5 x\right)}$$

$$I=\frac{1}{2}\int_{\pi/6}^{\pi/4}\frac{\tan^4 x\sec^2 x\,dx}{\left(1+\tan^{10}x\right)} \quad \text{Put } \tan^5 x = t$$

$$I=\frac{1}{10}\int_{\left(\frac{1}{\sqrt{3}}\right)^5}^{1}\frac{dt}{1+t^2}=\frac{1}{10}\left(\frac{\pi}{4}-\tan^{-1}\frac{1}{9\sqrt{3}}\right)$$

8. Ans. (2)

$$I=\int_0^a f(x)g(x)dx$$

$$I=\int_0^a f(a-x)g(a-x)dx$$

$$I=\int_0^a f(x)(4-g(x)dx$$

$$I=4\int_0^a f(x)dx-I$$

$$\Rightarrow\ I=2\int_0^a f(x)dx$$

9. Ans. (4)

$$\int_1^e\left(\frac{x}{e}\right)^{2x}\log_e x.dx-\int_1^e\left(\frac{e}{x}\right)\log_e x.dx$$

$$\text{Let }\left(\frac{x}{e}\right)^{2x}=t,\ \left(\frac{e}{x}\right)^{x}=v$$

$$=\frac{1}{2}\int_{\left(\frac{1}{e}\right)^2}^{1}dt\ +\int_e^1 dv$$

$$=\frac{1}{2}\left(1-\frac{1}{e^2}\right)+(1-e)=\frac{3}{2}-\frac{1}{2e^2}-e$$

10. Ans. (2)

$$\lim_{x\to\infty}\sum_{r=1}^{2n}\frac{n}{n^2+r^2}$$

$$\lim_{x\to\infty}\sum_{r=1}^{2n}\frac{1}{n\left(1+\frac{r^2}{n^2}\right)}=\int_0^2\frac{dx}{1+x^2}=\tan^{-1}2$$

11. Official Ans. by NTA (4)

Sol. $g(f(x))=\ell n(f(x))=\ell n\left(\frac{2-x.\cos x}{2+x.\cos x}\right)$

$$\therefore\quad I=\int_{-\pi/4}^{\pi/4}\ell n\left(\frac{2-x.\cos x}{2+x.\cos x}\right)dx$$

$$=\int_0^{\pi/4}\left(\ell n\left(\frac{2-x.\cos x}{2+x.\cos x}\right)+\ell n\left(\frac{2+x.\cos x}{2-x.\cos x}\right)\right)dx$$

$$=\int_0^{\pi/2}(0)dx=0=\log_e(1)$$

12. Official Ans. by NTA (1)

Sol. $f(x)=\int_0^x g(t)dt$

$f(-x)=\int_0^{-x} g(t)dt$

put $t=-u$

$=-\int_0^x g(-u)du$

$=-\int_0^x g(u)d(u)=-f(x)$

$\Rightarrow f(-x)=-f(x)$

$\Rightarrow f(x)$ is an odd function

Also $f(5+x)=g(x)$

$f(5-x)=g(-x)=g(x)=f(5+x)$

$\Rightarrow f(5-x)=f(5+x)$

Now

$I=\int_0^x f(t)dt$

$t=u+5$

$I=\int_{-5}^{x-5} f(u+5)du$

$=\int_{-5}^{x-5} g(u)du$

$=\int_{-5}^{x-5} f'(u)du$

$=f(x-5)-f(-5)$

$=-f(5-x)+f(5)$

$=f(5)-f(5+x)$

$=\int_{5+x}^{5} f'(t)dt=\int_{5+x}^{5} g(t)dt$

13. Official Ans. by NTA (3)

Sol. $I=\int_0^{\pi/2}\frac{\sin^3 x}{\sin x+\cos x}dx$

$\Rightarrow I=\int_0^{\pi/4}\frac{\sin^3 x+\cos^3 x}{\sin x+\cos x}dx$

$=\int_0^{\pi/4}(1-\sin x\cos x)\,dx$

$=\left(x-\frac{\sin^2 x}{2}\right)_0^{\pi/4}$

$=\frac{\pi}{4}-\frac{1}{4}$

$=\frac{\pi-1}{4}$

14. Official Ans. by NTA (3)

Sol. $\lim\limits_{x\to 2}\frac{\int_6^{f(x)} 2t\,dt}{x-2}$

L Hopital Rule

$\lim\limits_{x\to 2}\frac{2f(x)f'(x)}{1}=2f(2)=f'(2)=12f'(2)$

15. Official Ans. by NTA (3)

Sol. $I=\int_0^{2\pi}\left[\sin 2x(1+\cos 3x)\right]dx$

$I=\int_0^{\pi}([\sin 2x+\sin 2x\cos 3x]+[-\sin 2x-\sin 2x\cos 3x])dx$

$=\int_0^{\pi}-dx=-\pi$

16. Official Ans. by NTA (3)

Sol. $\lim\limits_{n\to\infty}\sum_{r=1}^{n}\frac{1}{n}\left(\frac{n+r}{n}\right)^{1/3}$

$=\int_0^1(1+x)^{1/3}dx=\frac{3}{4}\left(2^{4/3}-1\right)$

17. Official Ans. by NTA (1)

Sol. $I = \int \frac{1}{\cos^{2/3} x \sin^{1/3} x . \sin x} dx$

$= \int \frac{\tan^{2/3} x}{\tan^2 x} . \sec^2 x . dx$

$= \int \frac{\sec^2 x}{\tan^{4/3} x} . dx \qquad \{\tan x = t, \sec^2 x dx = dt\}$

$= \int \frac{dt}{t^{4/3}} = \frac{t^{-1/3}}{-1/3} = -3\left(t^{-1/3}\right)$

$\Rightarrow I = -3\tan(x)^{-1/3}$

$\Rightarrow I = \frac{3}{(\tan x)^{1/3}}\Bigg|_{\pi/6}^{\pi/3} = -3\left(\frac{1}{\left(\sqrt{3}\right)^{1/3}} - \left(\sqrt{3}\right)^{1/3}\right)$

$= 3\left(3^{1/3} - \frac{1}{3^{1/6}}\right) = 3^{7/6} - 3^{5/6}$

18. Official Ans. by NTA (1)

Sol. $\int_0^{\pi/2} \frac{\cot x dx}{\cot x + \operatorname{cosec} x}$

$\int_0^{\pi/2} \frac{\cos x}{\cos x + 1} = \int \frac{2\cos^2 \frac{x}{2} - 1}{2\cos^2 \frac{x}{2}}$

$\int_0^{\pi/2} \left(1 - \frac{1}{2}\sec^2 \frac{x}{2}\right) dx$

$\left[x - \tan \frac{x}{2}\right]_0^{\frac{\pi}{2}}$

$\frac{1}{2}[\pi - 2] \qquad m = \frac{1}{2}, \; n = -2$

$mn = -1$

19. Official Ans. by NTA (1)

Sol. $I = \int_0^1 x \tan\left(\frac{1}{1 + x^2(x^2 - 1)}\right) dx$

$I = \int_0^1 x\left(\tan^{-1} x^2 - \tan^{-1}(x^2 - 1)\right) dx$

$x^2 = t \Rightarrow 2xdx = dt$

$I = \frac{1}{2}\int_0^1 \left(\tan^{-1} t - \tan^{-1}(t - 1)\right) dx$

$= \frac{1}{2}\int_0^1 \tan^{-1} t \, dt - \frac{1}{2}\int_0^1 \tan^{-1}(t - 1) \, dt$

$= \frac{1}{2}\int_0^1 \tan^{-1} t \, dt - \frac{1}{2}\int_0^1 \tan^{-1} dt = \int_0^1 \tan^{-1} dt$

$\tan^{-1} t = \theta \Rightarrow t = \tan\theta$

$dt = \sec^2\theta d\theta$

$\int_0^{\pi/4} \theta \cdot \sec^2\theta \, d\theta$

$I = (\theta . \tan\theta)\big|_0^{\pi/4} - \int_0^{\pi/4} \tan\theta \, d\theta$

$= \left(\frac{\pi}{4} - 0\right) - \ln(\sec\theta)\Big|_0^{\pi/4}$

$= \frac{\pi}{4} - \left(\ell n\sqrt{2} - 0\right)$

$= \frac{\pi}{4} - \frac{1}{2}\ell n 2$

20. Official Ans. by NTA (4)

Sol. $\lim_{x \to 2} g(x) = \lim_{x \to 2} \frac{\int_6^{f(x)} 4t^3 dt}{x - 2}$

$= \lim_{x \to 2} \frac{4.f^3(x) \cdot f'(x)}{1}$

$= 4f^3(2) \, f'(2) = 18$

21. Official Ans. by NTA (4)

Sol. $\int_{\alpha}^{\alpha+1} \frac{(x+\alpha+1)-(x+\alpha)}{(x+\alpha)(x+\alpha+1)} dx = \left(\ell n|x+\alpha| - \ell n|x+\alpha+1|\right)_{\alpha}^{\alpha+1}$

$= \ell n\left|\frac{2\alpha+1}{2\alpha+2} \times \frac{2\alpha+1}{2\alpha}\right| = \ell n\frac{9}{8}$

$\Rightarrow \alpha = -2, 1$