CONTINUITY

1. Let f : R → R be a function defined as :

$$f(x)=\begin{cases}5, & \text{if } x\le 1\\ a+bx, & \text{if } 1<x<3\\ b+5x, & \text{if } 3\le x<5\\ 30, & \text{if } x\ge 5\end{cases}$$

Then, f is :
(1) continuous if a = 5 and b = 5
(2) continuous if a = –5 and b = 10
(3) continuous if a = 0 and b = 5
(4) not continuous for any values of a and b

2. Let f : [–1,3] → R be defined as

$$f(x)=\begin{cases}|x|+[x], & -1\le x<1\\ x+|x|, & 1\le x<2\\ x+[x], & 2\le x\le 3\end{cases},$$

where [t] denotes the greatest integer less than or equal to t. Then, f is discontinuous at:
(1) four or more points
(2) only one point
(3) only two points
(4) only three points

3. If the function f defined on $\left(\frac{\pi}{6},\frac{\pi}{3}\right)$ by

$$f(x)=\begin{cases}\dfrac{\sqrt{2}\cos x-1}{\cot x-1}, & x\ne\dfrac{\pi}{4}\\ k, & x=\dfrac{\pi}{4}\end{cases}$$ is continuous,

then k is equal to

(1) $\frac{1}{2}$ (2) 1 (3) $\frac{1}{\sqrt{2}}$ (4) 2

4. If the function $f(x)=\begin{cases}a|\pi-x|+1, & x\le 5\\ b|x-\pi|+3, & x>5\end{cases}$ is continuous at x = 5, then the value of a – b is:-

(1) $\frac{2}{5-\pi}$ (2) $\frac{2}{\pi-5}$
(3) $\frac{2}{\pi+5}$ (4) $\frac{-2}{\pi+5}$

5. If $f(x)=\begin{cases}\dfrac{\sin(p+1)+\sin x}{x}, & x<0\\ q, & x=0\\ \dfrac{\sqrt{x+x^2}-\sqrt{x}}{x^{3/2}}, & x>0\end{cases}$

is continuous at x = 0, then the ordered pair (p,q) is equal to :

(1) $\left(\frac{5}{2},\frac{1}{2}\right)$ (2) $\left(-\frac{3}{2},-\frac{1}{2}\right)$
(3) $\left(-\frac{1}{2},\frac{3}{2}\right)$ (4) $\left(-\frac{3}{2},\frac{1}{2}\right)$

SOLUTION

1. Ans. (4)

$$f(x)=\begin{cases}5 & \text{if} \quad x\le 1\\ a+bx & \text{if} \quad 1<x<3\\ b+5x & \text{if} \quad 3\le x<5\\ 30 & \text{if} \quad x\ge 5\end{cases}$$

$f(1)=5,\ \ f(1^-)=5,\ \ f(1^+)=a+b$

$f(3^-)=a+3b,\ f(3)=b+15,\ f(3^+)=b+15$

$f(5^-)=b+25\ ;\ f(5)=30 \quad f(5^+)=30$

from above we concluded that f is not continuous for any values of a and b.

2. Official Ans. by NTA (4)

Sol. $$f(x)=\begin{cases}-(x+1) & , \quad -1\le x<0\\ x & , \quad 0\le x<1\\ 2x & , \quad 1\le x<2\\ x+2 & , \quad 2\le x<3\\ x+3 & , \quad x=3\end{cases}$$

function discontinuous at $x=0,1,3$

3. Official Ans. by NTA (1)

Sol. $\therefore$ function should be continuous at $x=\dfrac{\pi}{4}$

$$\therefore \lim_{x\to\frac{\pi}{4}} f(x)=f\left(\frac{\pi}{4}\right)$$

$$\Rightarrow \lim_{x\to\frac{\pi}{4}}\frac{\sqrt{2}\cos x-1}{\cot x-1}=k$$

$$\Rightarrow \lim_{x\to\frac{\pi}{4}}\frac{-\sqrt{2}\sin x}{-\operatorname{cosec}^2 x}=k \quad \text{(Using L'Hôpital rule)}$$

$$\lim_{x\to\frac{\pi}{4}}\sqrt{2}\sin^3 x=k$$

$$\Rightarrow k=\sqrt{2}\left(\frac{1}{\sqrt{2}}\right)^3=\frac{1}{2}$$

4. Official Ans. by NTA (1)

Sol. $f(x)$ → $a|\pi-x|+1;x\ge 5$; $b|\pi-x|+3;x>5$

$a|\pi-5|+1=b|5-\pi|+3$

$a(5-\pi)+1=b(5-\pi)+3$

$(a-b)(5-\pi)=2$

$$a-b=\frac{2}{5-\pi}$$

5. Official Ans. by NTA (4)

Sol. $$\text{RHL}=\lim_{x\to 0^+}\frac{\sqrt{x+x^2}-\sqrt{x}}{x^{\frac{3}{2}}}=\lim_{x\to 0^+}\frac{\sqrt{1+x}-1}{x}=\frac{1}{2}$$

$$\text{LHL}=\lim_{x\to 0}\frac{\sin(p+1)x+\sin x}{x}=(p+1)+1=p+2$$

for continuity LHL = RHL = f(0)

$$\Rightarrow (p,q)=\left(\frac{-3}{2},\frac{1}{2}\right)$$